Information Bulletin

Applied Mathematics 30

Diploma Examinations Program

Archived Curriculum Standards and Example Questions

Alberta Education

This document was written primarily for:

Students	✓
Teachers	✓ of Applied Mathematics 30
Administrators	✓
Parents	
General Audience	
Others	

Contents

Introduction

In this bulletin are examples of questions that students should be able to answer in order to demonstrate the *acceptable standard* and the *standard of excellence*. The examples provided are by no means exhaustive; they are intended to provide a profile of acceptable and excellent achievement. Some examples and solutions were developed and validated by classroom teachers of mathematics but have not been validated by students. Other examples were taken from previous diploma examinations. All examples model the types of questions and problems that students should be able to solve and the types of activities that students should be able to perform in order to meet the specific outcomes to which the questions are linked.

To meet the outcomes of Applied Mathematics 30, students will need access to an approved graphing calculator and a computer with a spreadsheet program. In most classrooms, students will use a graphing calculator on a daily basis. For a list of approved graphing calculators, refer to the *Applied Mathematics 30 Information Bulletin* for the current school year or go to the Alberta Education website at www.education.alberta.ca, then follow the pathway: Teachers > (Additional Programs and Services) Diploma Exams > Information Bulletins > Calculator Policy

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Learner Assessment would like to recognize and thank the many teachers throughout the province who helped prepare this document. We would also like to thank the Curriculum Branch and the Learning Technologies Branch for their input and assistance in reviewing these standards.

Matrices and Pathways

General Outcomes

Describe and apply operations on matrices to solve problems, using technology as required.

Solve problems based on the counting of sets, using techniques such as the fundamental counting principle, permutations and combinations.

General Notes:

- A node is an intersection of two or more pathways on a grid.
- The intent of the matrices section is that students will recognize situations in which matrices may be applied. This section should not be an in-depth study of matrix operations. However, students will need to understand the basic operations and procedures of addition, subtraction, and multiplication of matrices in order to solve problems in a given context.
- Inverse matrices, determinants, and matrix solutions to linear systems are all beyond the scope of Applied Mathematics 30.
- A formal study of permutations and combinations is beyond the scope of Applied Mathematics 30.

Specific Outcomes

Specific Outcome 1.1

Solve pathway problems, interpreting and applying any constraints. [PS, R]

1.1 Note:

- Organizational tools, such as Pascal's triangle, should be used to present solutions. However, a formal discussion of Pascal's triangle is not necessary.

Refer to examples 12 and 13.

Specific Outcome 1.2

Use the fundamental counting principle to determine the number of different ways to perform multistep operations. [PS, R]

1.2 Note:
- Questions should be kept at a level where students who achieve the *acceptable standard* can be successful.

Refer to examples 1 and 2.

Specific Outcome 1.3

Perform, using technology only for larger matrices, the matrix operations of addition, subtraction, matrix multiplication and multiplication by a scalar. [C, E, R, T, V]

1.3 Note:
- Paper and pencil calculations involving operations on matrices should be limited to those that can be expressed as a matrix no larger than 3 by 3.

Refer to examples 3, 6, and 15.

Specific Outcome 1.4

Model and solve consumer and network problems, performing matrix operations and using algebraic solution strategies as needed. [CN, PS, T, V]

1.4 Notes:
- Problems involving operations on matrices should be limited to those that can be expressed as a matrix no larger than 4 by 4.
- Transition problems may begin with an initial probability matrix or with an initial number matrix.
- Students need to be able to interpret elements in resultant matrices.

Refer to examples 4, 7, 8, and 9 (consumer problems), 10 and 11 (network problems), and 5 and 14 (transition matrix problems).

Acceptable Standard	*Standard of Excellence*
The student can	The student can also
• solve simple pathway problems	• solve complex pathway problems
• use the fundamental counting principle to determine the number of different ways to perform multistep operations	
• build a matrix from a given table	• build a matrix from a given context and solve problems associated with it
• recognize and produce a row matrix and column matrix	
• describe the dimensions of a matrix	
• identify conditions required to perform operations on matrices	
• identify the required matrix operation for a given context	
• perform operations of addition, subtraction, matrix multiplication, and scalar multiplication on matrices	
• identify elements of a matrix or a matrix operation as they relate to a specific problem	
• make modifications to one or two elements of a matrix in response to new scenarios	• make modifications to a matrix in response to new scenarios
• model a simple problem with a matrix	• model a complex problem with a matrix
• solve problems in which the matrix is given	
• participate in and contribute toward the problem-solving process for problems that require the analysis of matrices and pathways studied in Applied Mathematics 30	• complete the solution to problems that require the analysis of matrices and pathways studied in Applied Mathematics 30

Examples

Students who achieve the *acceptable standard* should be able to answer all the following questions, except for any part of a question labelled **SE**. Parts labelled **SE** are appropriate examples for students who achieve the *standard of excellence*.

Use the following information to answer the next question.

The Hawaiian alphabet has only twelve letters:

Vowels	*a*, *e*, *i*, *o*, *u*
Consonants	*h*, *k*, *l*, *m*, *n*, *p*, *w*

1. The number of 3-letter "words" that can be made using the Hawaiian alphabet if every "word" must have the pattern of consonant-vowel-consonant, and if letters can be repeated, is

A. 19

B. 175

C. 245

D. 1 728

Solution:

There are 7 choices for the first letter, 5 choices for the second letter, and 7 choices for the last letter. Using the fundamental counting principle, there are 245 different 3-letter words that can be made.

Numerical Response

2. Jaycen and Kate are to be the first and second students, respectively, in a lineup of 7 students. The number of different orders in which the remaining 5 students can line up behind Jaycen and Kate is _____.

Solution:

The 5 students can line up in $5 \times 4 \times 3 \times 2 \times 1 = 120$ different orders.

3. On which of the following matrices can addition be performed?

A. $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$ and $\begin{bmatrix} g \\ h \end{bmatrix}$

B. $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$ and $\begin{bmatrix} g & h \\ i & j \\ k & l \end{bmatrix}$

C. $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$ and $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

D. $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$ and $\begin{bmatrix} g & h & i \\ j & k & l \end{bmatrix}$

Solution:

In order to add two matrices, they must contain the same number of rows and columns. Therefore, the correct response is D.

Use the following information to answer the next question.

The table below shows the number of vehicles parked in a downtown parking lot over a three-day period.

	Cars	Buses	Bicycles
Thursday	**72**	**6**	**7**
Friday	**81**	**2**	**2**
Saturday	**94**	**3**	**12**

The charge for cars is $6, the charge for buses is $15, and there is no charge for bicycles. The total revenue for each of the three days can be determined from the product of the two matrices below.

$$\begin{bmatrix} 72 & 6 & 7 \\ 81 & 2 & 2 \\ 94 & 3 & 12 \end{bmatrix} \times \begin{bmatrix} 6 \\ 15 \\ 0 \end{bmatrix}$$

4. From this matrix operation, it can be determined that the revenue from

A. cars on Thursday was $522

B. all vehicles on Saturday was $609

C. all vehicles on Thursday was $516

D. buses over the three days was $516

Solution:

$$\begin{bmatrix} 72 & 6 & 7 \\ 81 & 2 & 2 \\ 94 & 3 & 12 \end{bmatrix} \times \begin{bmatrix} 6 \\ 15 \\ 0 \end{bmatrix} = \begin{bmatrix} 522 \\ 516 \\ 609 \end{bmatrix}$$

This matrix operation shows that the revenue from all vehicles on Thursday was $522, on Friday was $516, and on Saturday was $609. Therefore, the correct response is B.

Use the following information to answer the next question.

Three major countries produce cars to be purchased in Canada: Canada, Japan, and Germany. A poll of car owners in Canada revealed that of people who presently own a Canadian-produced car, 51% would purchase another Canadian-produced car the next time they purchase a car. Of people who presently own a Japanese-produced car, 30% would purchase a Canadian-produced car the next time they purchase a car.

The following matrix shows detailed results of the poll.

$$\begin{array}{c} \\ \text{CA} \\ \text{JA} \\ \text{GR} \\ \text{Other} \end{array} \begin{array}{c} \begin{array}{cccc} \text{CA} & \text{JA} & \text{GR} & \text{Other} \end{array} \\ \begin{bmatrix} 0.51 & 0.32 & 0.12 & 0.05 \\ 0.30 & 0.50 & 0.12 & 0.08 \\ 0.35 & 0.15 & 0.40 & 0.10 \\ 0.20 & 0.25 & 0.15 & 0.40 \end{bmatrix} \end{array}$$

5. The entry in row 3 and column 2 indicates that

A. 15% of people who presently own a Japanese-produced car would purchase a German-produced car the next time they purchase a car

B. 12% of people who presently own a German-produced car would purchase a Japanese-produced car the next time they purchase a car

C. 15% of people who presently own a German-produced car would purchase a Japanese-produced car the next time they purchase a car

D. 12% of people who presently own a Japanese-produced car would purchase a German-produced car the next time they purchase a car

Solution:

The entry in row 3 and column 2 is 0.15. This represents the percentage of people who presently own a German-produced car but would purchase a Japanese-produced car the next time they purchase a car. Therefore, the correct response is C.

6. Solve for x in the following matrix.

$$\begin{bmatrix} 2 & 3 & 5 \\ 4 & 7 & 6 \\ 1 & 2 & 8 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ x & 5 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 33 & 26 \\ 53 & 53 \\ 40 & 21 \end{bmatrix}$$

Solution:

$$\begin{aligned} 2(2) + 3(x) + 5(4) &= 33 \\ 4 + 3x + 20 &= 33 \\ 3x &= 9 \\ x &= 3 \end{aligned}$$

7. The matrix below shows the time, in hours, that each of three toys—a car, a truck, and a motorcycle—spend at each of three factory assembly stations, A, B, and C. The "finishing time," in hours, for a given toy is the time required for it to pass through all three stations.

$$\begin{array}{r} \\ \text{Car} \\ \text{Truck} \\ \text{Motorcycle} \end{array} \begin{array}{c} \begin{array}{ccc} \text{A} & \text{B} & \text{C} \end{array} \\ \begin{bmatrix} 0.7 & 0.3 & 0.6 \\ 0.6 & 0.4 & 0.5 \\ 0.4 & 0.3 & 0.4 \end{bmatrix} \end{array}$$

a. A toy car, truck, and motorcycle pass through station A. What is the total time that the three toys spend at station A?

b. What is the "finishing time" required for a truck?

SE **c.** Changes to the truck's design make it necessary to increase its finishing time by 20%. If the time spent on the truck at each station is increased by the same percentage, what is a matrix operation that will model this change?

Solutions:

a. The total time spent at station A is the sum of the elements of the first column in the matrix

$$\begin{bmatrix} 0.7 \\ 0.6 \\ 0.4 \end{bmatrix}$$

$0.7 + 0.6 + 0.4 = 1.7$

The total time that the three toys spend at station A is 1.7 hours.

b. The finishing time for a truck is the sum of the elements of the second row of the matrix

$[0.6 \;\; 0.4 \;\; 0.5]$

$0.6 + 0.4 + 0.5 = 1.5$

A truck's finishing time is 1.5 hours.

SE **c.** **Method One**

Strip out the row that indicates the time the truck requires at each station and make it into a row matrix. This matrix can be multiplied by a scalar of 1.2.

$1.2[0.6 \;\; 0.4 \;\; 0.5] = [0.72 \;\; 0.48 \;\; 0.6]$

After the design changes, the truck will require 0.72 hours at station A, 0.48 hours at station B, and 0.6 hours at station C.

Method Two

Perform the following matrix multiplication.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1.2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0.7 & 0.3 & 0.6 \\ 0.6 & 0.4 & 0.5 \\ 0.4 & 0.3 & 0.4 \end{bmatrix} = \begin{bmatrix} 0.7 & 0.3 & 0.6 \\ 0.72 & 0.48 & 0.6 \\ 0.4 & 0.3 & 0.4 \end{bmatrix}$$

The time that the car and motorcycle spend at each station has not changed, but the time that the truck spends at each station has increased to 0.72 hours at station A, 0.48 hours at station B, and 0.6 hours at station C.

Use the following information to answer the next question.

The owner of a small amusement park represents the number and type of vehicles that are in his parking lot on a particular Thursday, Friday, and Saturday using the matrix below.

Matrix A:

Day of Week	**Type of Vehicle**	
	Car	Bus
T	85	12
F	43	17
S	102	33

He makes a second matrix to indicate the parking cost of 8 per car and 22 per bus.

Matrix B:

Type of Vehicle	**Parking Cost**
Car	8
Bus	22

Written Response—10%

8. **a.** What does the value 33 in matrix A represent?

SOLUTION to part a

There are 33 buses in the parking lot on Saturday.

b. Use matrix multiplication to calculate the revenue for each of the three days. Write a statement that describes the result of this multiplication.

A POSSIBLE SOLUTION to part b

$$\begin{bmatrix} 85 & 12 \\ 43 & 17 \\ 102 & 33 \end{bmatrix} \times \begin{bmatrix} 8 \\ 22 \end{bmatrix} = \begin{bmatrix} 944 \\ 718 \\ 1542 \end{bmatrix}$$

The revenue is \$944, \$718, and \$1 542 for Thursday, Friday, and Saturday, respectively.

c. Use matrix operations to calculate an increase of 10% in the daily parking price. Show all calculations.

A POSSIBLE SOLUTION to part c

$$1.10 \times \begin{bmatrix} 8 \\ 22 \end{bmatrix} = \begin{bmatrix} 8.8 \\ 24.2 \end{bmatrix}$$

The new parking price is \$8.80 for cars and \$24.20 for buses.

or

$$0.1 \times \begin{bmatrix} 8 \\ 22 \end{bmatrix} = \begin{bmatrix} 0.8 \\ 2.2 \end{bmatrix}$$

The parking prices will increase by \$0.80 for cars and \$2.20 for buses.

SE **d.** How much **more** money would the owner have made on Saturday as a result of a 10% price increase?

A POSSIBLE SOLUTION to part d

$$\begin{bmatrix} 85 & 12 \\ 43 & 17 \\ 102 & 33 \end{bmatrix} \times \begin{bmatrix} 8 \\ 22 \end{bmatrix} \times 1.10 = \begin{bmatrix} 85 & 12 \\ 43 & 17 \\ 102 & 33 \end{bmatrix} \times \begin{bmatrix} 8.8 \\ 24.2 \end{bmatrix}$$

$$= \begin{bmatrix} 1038.4 \\ 789.8 \\ 1696.2 \end{bmatrix}$$

or

$$1.1 \times \begin{bmatrix} 944 \\ 718 \\ 1542 \end{bmatrix} = \begin{bmatrix} 1038.4 \\ 789.8 \\ 1696.2 \end{bmatrix}$$

New revenue = \$1 696.20
Original revenue = \$1 542.00

\$1 696.20 – \$1 542 = \$154.20

The owner would have made \$154.20 more on Saturday.

Use the following information to answer the next question.

A soccer league collected the following statistics over eight games.

	Win	Tie	Loss
Bulldogs	4	3	1
Titans	7	1	0
Rovers	2	2	4

Each team earns 3 points for a win, 1 point for a tie, and 0 points for a loss.

9. Which of the following matrix operations could be used to determine the points earned by each team after eight games?

A. $\begin{bmatrix} 4 & 3 & 1 \\ 7 & 1 & 0 \\ 2 & 2 & 4 \end{bmatrix} \times \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$

B. $\begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} \times \begin{bmatrix} 4 & 3 & 1 \\ 7 & 1 & 0 \\ 2 & 2 & 4 \end{bmatrix}$

C. $\begin{bmatrix} 4 & 3 & 1 \\ 7 & 1 & 0 \\ 2 & 2 & 4 \end{bmatrix} \times \begin{bmatrix} 3 & 1 & 0 \end{bmatrix}$

D. $\begin{bmatrix} 3 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} 4 & 3 & 1 \\ 7 & 1 & 0 \\ 2 & 2 & 4 \end{bmatrix}$

Solution:

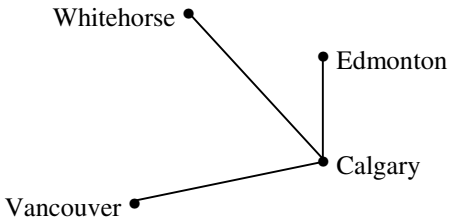
To get a 3×1 matrix that shows the total number of points earned by each team, the number of wins, ties and losses for each team $\begin{bmatrix} 4 & 3 & 1 \\ 7 & 1 & 0 \\ 2 & 2 & 4 \end{bmatrix}$ needs to be multiplied by the points awarded for each $\begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$.

In order to perform matrix multiplication, the number of columns in the first matrix must match the number of rows in the second matrix. Therefore, the correct response is A.

10. A particular company's air freight is transported among four cities in Western Canada. Calgary is the hub of the network, which means that all air freight either originates in Calgary or is transported through Calgary.

a. Set up a network matrix, A, for the cities given below. Use 1 to indicate that there is a direct freight transport between two cities and 0 to indicate that there is no direct freight transport between two cities.

Cities in Network A



b. If matrix A is squared, matrix A^2 can represent the network matrix for one-way routes and round-trip routes that have exactly one stopover. A stopover occurs when the airplane stops in a city en route to its final destination. Evaluate matrix A^2.

SE **c.** Explain why there is one element equal to 3, nine elements equal to 1, and six elements equal to 0 in matrix A^2.

Solutions:

a. **Matrix A**

$$\begin{array}{c} \\ C \\ E \\ W \\ V \end{array}\begin{array}{c} \begin{array}{cccc} C & E & W & V \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Calgary is the hub of the network, and Edmonton, Whitehorse, and Vancouver are all spokes.

b. **Matrix A^2**

$$\begin{array}{c} \\ C \\ E \\ W \\ V \end{array}\begin{array}{c} \begin{array}{cccc} C & E & W & V \end{array} \\ \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

Each entry in A^2 represents the number of one-way routes and round-trip routes that have exactly one stopover.

SE **c.** There is only one element equal to 3 because there are 3 routes with exactly one stopover. You can fly to any one of the three spokes from the Calgary hub and back, with one stopover.

There are nine elements equal to 1 because you can fly from any of the spokes, Edmonton, Whitehorse, or Vancouver, and back with one stopover in Calgary (3 routes), or you can fly between any two of the cities (E, W, or V) with one stopover in Calgary (6 routes). This gives a total of 9 routes.

There are six elements equal to 0 because there is no way to fly from the hub to any spoke, or from any spoke to the hub, with exactly one stopover.

11. The matrix below shows the distances of airline routes between Vancouver, Calgary, and Edmonton, with Calgary as the hub.

$$\begin{array}{c} \\ C \\ E \\ V \end{array} \begin{array}{c} \begin{array}{ccc} C & E & V \end{array} \\ \begin{bmatrix} 0 & 300 & 800 \\ 300 & 0 & 0 \\ 800 & 0 & 0 \end{bmatrix} \end{array}$$

Given that Calgary is the hub, then the sum of the distances from the hub to each of the spoke cities and back is (300 + 800 + 300 + 800) or 2 200 km.

a. The matrix below shows the distances of airline routes between Vancouver, Calgary, and Edmonton, with Vancouver as the hub.

$$\begin{array}{c} \\ V \\ E \\ C \end{array} \begin{array}{c} \begin{array}{ccc} V & E & C \end{array} \\ \begin{bmatrix} 0 & 1000 & 800 \\ 1000 & 0 & 0 \\ 800 & 0 & 0 \end{bmatrix} \end{array}$$

Given that Vancouver is the hub, then the sum of the distances from the hub to each of the spoke cities and back is _____.

b. The matrix below shows the distances of airline routes between Vancouver, Calgary, and Edmonton, with Edmonton as the hub.

$$\begin{array}{c} \\ E \\ V \\ C \end{array} \begin{array}{c} \begin{array}{ccc} E & V & C \end{array} \\ \begin{bmatrix} 0 & 1000 & 300 \\ 1000 & 0 & 0 \\ 300 & 0 & 0 \end{bmatrix} \end{array}$$

Given that Edmonton is the hub, then the sum of the distances from the hub to each of the spoke cities and back is _____.

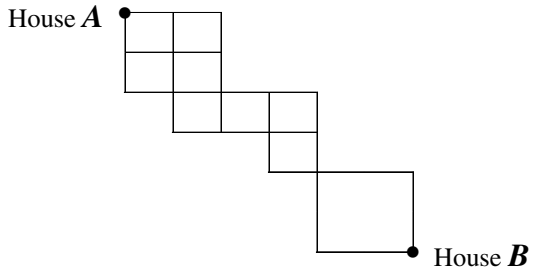
c. Which is the most efficient hub?

Solutions:

a. (1 000 + 800 + 1 000 + 800) or 3 600 km
b. (1 000 + 300 + 1 000 + 300) or 2 600 km
c. Calgary is the most efficient hub, followed by Edmonton and then Vancouver.

Use the following information to answer the next question.

A newspaper carrier who delivers papers on his bike can travel only on the trails represented in the diagram below.

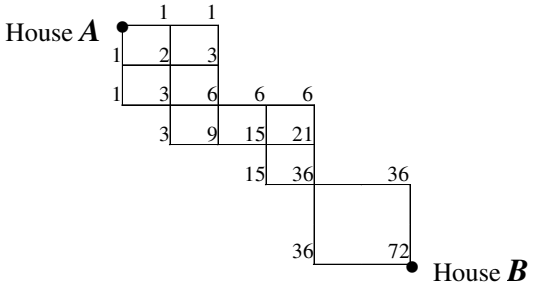


Numerical Response

12. The number of different trails that the newspaper carrier can take to get from house *A* to house *B* without backtracking is _____.

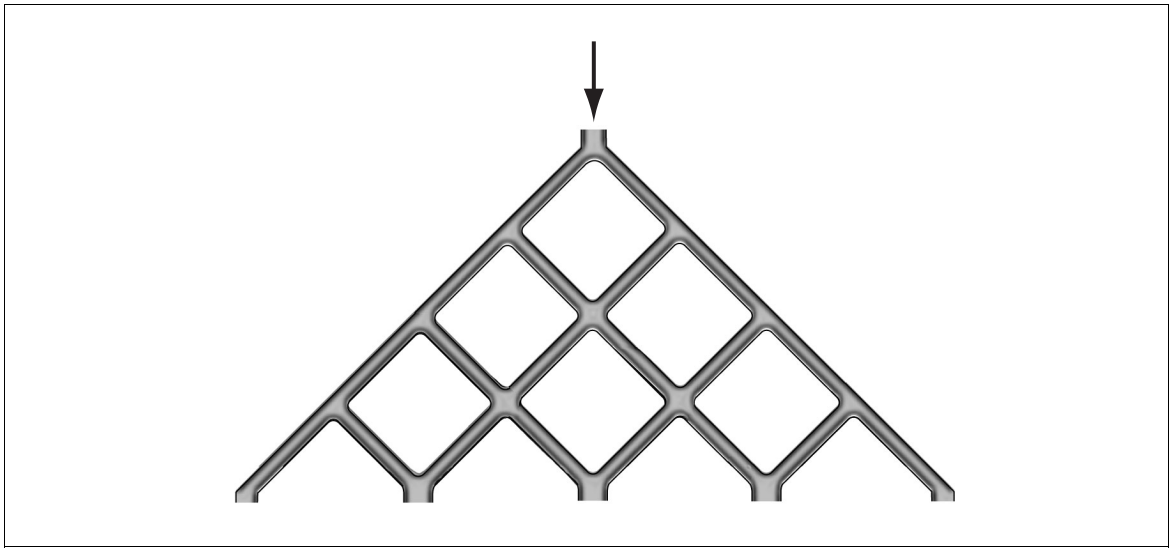
Solution:

The diagram above can be completed in the following manner.



There are 72 different trails that the newspaper carrier can take from house *A* to house *B* without backtracking.

Use the following diagram to answer the next question.

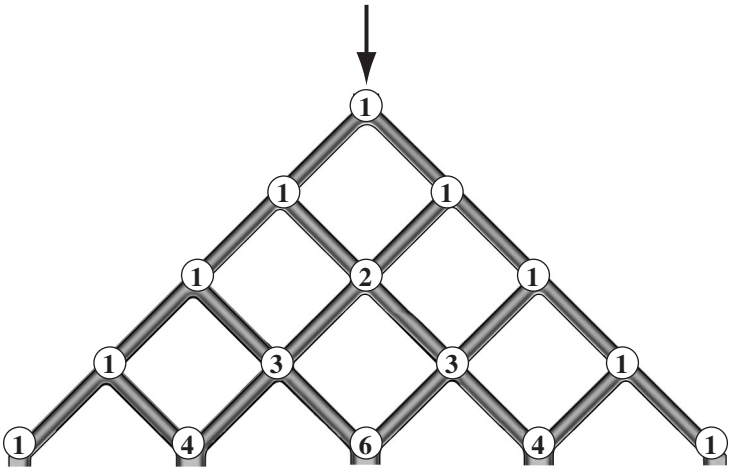


Numerical Response

13. The total number of paths through the maze is _____.

Solution:

The diagram above can be completed in the following manner.



There are $1 + 4 + 6 + 4 + 1 = 16$ different paths through the maze.

Use the following information to answer the next question.

In 1998, a truck dealership sold 300 blue trucks, 100 green trucks, and 200 red trucks. In 1999, sales of blue trucks decreased by 10%, sales of green trucks increased by 20%, and sales of red trucks increased by 30%. This situation can be modelled by the matrix operation shown below.

$$\begin{bmatrix} 0.9 & 0 & 0 \\ 0 & 1.2 & 0 \\ 0 & 0 & 1.3 \end{bmatrix} \times \begin{bmatrix} 300 \\ 100 \\ 200 \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix}$$

Written Response—15%

14. **a.** Calculate the product of the matrices above.

A POSSIBLE SOLUTION to part a

$$\begin{bmatrix} 0.9 & 0 & 0 \\ 0 & 1.2 & 0 \\ 0 & 0 & 1.3 \end{bmatrix} \times \begin{bmatrix} 300 \\ 100 \\ 200 \end{bmatrix} = \begin{bmatrix} 270 \\ 120 \\ 260 \end{bmatrix}$$

b. How many green trucks were sold in 1999?

A POSSIBLE SOLUTION to part b

$100 \times 1.2 = 120$ green trucks sold.

c. Assume that the pattern continues. Use matrix multiplication to determine the sales for each colour of truck in the year 2000.

A POSSIBLE SOLUTION to part c

$$\begin{bmatrix} 0.9 & 0 & 0 \\ 0 & 1.2 & 0 \\ 0 & 0 & 1.3 \end{bmatrix} \times \begin{bmatrix} 270 \\ 120 \\ 260 \end{bmatrix} = \begin{bmatrix} 243 \\ 144 \\ 338 \end{bmatrix}$$

Sales of blue trucks will be 243, green trucks will be 144, and red trucks will be 338.

Use the following additional information to answer the next part of the question.

In 1998, of all vehicles sold at a different vehicle dealership, 800 were trucks and 600 were sport utility vehicles (SUVs).

In 1999, 8% of the truck owners traded in their trucks for new SUVs, and 22% of the SUV owners traded in their SUVs for new trucks.

No SUV or truck owner switched to a different type of vehicle and no owner of other types of vehicles switched to an SUV or a truck.

d. • Represent this information by completing the 2×2 matrix below, and perform the matrix multiplication.

$$\begin{bmatrix} 800 & 600 \end{bmatrix} \times \begin{bmatrix} \_\_\_ & 0.08 \\ 0.22 & \_\_\_ \end{bmatrix}$$

A POSSIBLE SOLUTION to part d, bullet 1

$$\begin{bmatrix} 800 & 600 \end{bmatrix} \times \begin{bmatrix} \underline{0.92} & 0.08 \\ 0.22 & \underline{0.78} \end{bmatrix} = \begin{bmatrix} 868 & 532 \end{bmatrix}$$

or

$$\begin{bmatrix} 800 & 600 \end{bmatrix} \times \begin{bmatrix} \underline{\quad 0 \quad} & 0.08 \\ 0.22 & \underline{\quad 0 \quad} \end{bmatrix} = \begin{bmatrix} 132 & 64 \end{bmatrix}$$

SE

- Explain what the product of the matrix multiplication in the bullet above means in this context.

A POSSIBLE SOLUTION to part d, bullet 2

In 1999, 868 people owned trucks bought from the dealership, and 532 owned SUVs bought from the dealership.

or

There were 132 people who traded their SUVs for trucks and 64 people who traded their trucks for SUVs.

15. What conditions must be met in order to perform matrix multiplication? Justify your answer.

Solution:

To perform matrix multiplication, the number of columns in the first matrix must match the number of rows in the second matrix.
For example:

$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ is a 2 by 3 matrix. (2 rows and 3 columns)

and

$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ is a 3 by 3 matrix. (3 rows and 3 columns)

When A and B are multiplied, the procedure is as follows:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = \begin{bmatrix} 1(1) + 2(4) + 3(7) & 1(2) + 2(5) + 3(8) & 1(3) + 2(6) + 3(9) \\ 4(1) + 5(4) + 6(7) & 4(2) + 5(5) + 6(8) & 4(3) + 5(6) + 6(9) \end{bmatrix}$$

$$= \begin{bmatrix} 30 & 36 & 42 \\ 66 & 81 & 96 \end{bmatrix}$$

It is impossible to multiply two matrices in which the number of columns in the first matrix does not match the number of rows in the second matrix.
For example:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = [1(1) + 2(3) + 3(\underline{\ ?\ })]$$

The number of columns in the first matrix does not equal the number of rows in the second matrix. Since there is a dimensional mismatch, a product cannot be found.

Statistics and Probability

General Outcomes

Use normal and binomial probability distributions to solve problems involving uncertainty.

Model the probability of a compound event, and solve problems based on the combining of simpler probabilities.

General Notes:
- Teachers may want to do the probability section of this unit first (i.e., Outcomes 2.4 to 2.7).
- Students may need to review basic concepts and terms of probability.
- Probability should be expressed as a decimal or fractional value between 0 and 1.
- Teachers may use either tables or technology to solve normal distribution problems.

Specific Outcomes

Specific Outcome 2.1
Find the population standard deviation of a data set, using technology. [CN, E, T, V]

2.1 Notes:
- Students are also expected to have an understanding of what standard deviation is.
- Review measures of central tendency and histograms. Teachers may also need to review the difference between a sample and a population.
- Sample standard deviation will not be used. Population standard deviation is represented by σ.
- Students must be familiar with the procedures required to input values into the list function on their calculators.

Refer to examples 1 and 3.

Specific Outcome 2.2
Use z-scores to solve problems related to the normal distribution. [PS, R, T, V]

2.2 Notes:
- Students should have practice with paper and pencil calculations and sketches before being introduced to technology.
- Teachers and students need to be aware that the z-scores and areas under the curve from the textbook and data table may vary slightly from those values given by the calculator. When writing diploma examinations, students may use either technology **or** z-score tables.

Refer to examples 2, 4, and 7.

Specific Outcome 2.3
Use the normal approximation to the binomial distribution to solve problems involving confidence intervals for large-sample binomial experiments. [CN, E, PS, T]

2.3 Notes:
- A sample is considered to be large if $np > 5$ and if $n(1-p) > 5$. Teachers should ensure that their class examples and examination questions meet these criteria, but students will not be asked to check sample size on the diploma examination.
- Continuity correction will **not** be assessed on the diploma examination.
- This outcome addresses only **discrete** data and is not intended for continuous data.
- Students are expected to calculate 95% confidence intervals only. However, students may benefit from a discussion of the differences between 90%, 95%, and 99% confidence intervals.
- Regular rounding rules do not always apply when rounding confidence intervals. To ensure that a 95% confidence interval contains at least 95% of the data symmetric about the mean, always round the lower limit down and round the upper limit up (see *example 6d*).
- Margin of error is **not** included in this outcome. However, an understanding of how margin of error relates to the size of the confidence interval may be beneficial.

Refer to examples 5, 6c, and 6d.

Specific Outcome 2.4
Construct a sample space for two or three events. [PS, R, V]

2.4 Notes:
- Some discrete sample spaces can also be displayed as the intersection points on a two-dimensional grid.
- Once a sample space has been constructed, students should be able to answer questions based on it by using independent probability calculations or through investigation.

Refer to examples 6a, 6b, and 9.

Specific Outcome 2.5
Classify events as independent or dependent. [C]

Specific Outcome 2.6
Use expressions for $P(A \text{ and } B)$ to solve problems involving independent and dependent events. [CN, PS, R]

Refer to examples 8 and 11.

Specific Outcome 2.7
Solve problems using the probabilities of mutually exclusive and complementary events. [CN, PS, R]

2.7 Notes:
- The formulas for $P(A \text{ or } B)$ should be restricted to the mutually exclusive case only.
- Non-mutually exclusive events are beyond the scope of Applied Mathematics 30.

Refer to examples 10 and 12.

Acceptable Standard	*Standard of Excellence*
The student can	The student can also
• determine population standard deviation and mean using hand-held technology	
• create a histogram, and compare it with a normal distribution	
• rank the standard deviation of various histograms without doing any calculation	• interpret how changing the data can affect the standard deviation and/or mean
• calculate a z-score, using the formula	
• calculate the missing value when given the value of 3 of the 4 variables in the z-score formula	
• sketch a diagram of a normal curve and indicate appropriate shading for a given problem	
• calculate the area under the standard normal curve to the left of a z-score	• calculate the area to the right of a z-score or between two z-scores • use z-scores to compare two sets of data and draw conclusions
• describe the properties of a normal distribution	• make inferences given an area under the standard normal curve
• calculate the mean and the standard deviation when given binomial distribution data and the values of n and p	
• calculate the symmetric 95% confidence interval, and apply it to the normal curve when given binomial distribution data and the values for μ and σ	• calculate the values of μ and σ in a given context to determine the symmetric 95% confidence interval and apply it to the normal curve when given binomial distribution data and the values of n and p
• determine sample spaces for problems involving two or three components	• generate a sample space and use it to draw conclusions about the outcomes of non-routine problems
• draw conclusions about the outcomes of routine problems when given a sample space	
• distinguish between independent and dependent events	• determine the probability of the other event when given the probability of one event and the probability of combined events
• determine $P(A \text{ and } B)$ for independent events	
• determine $P(A \text{ and } B)$ for dependent events where order is specified	
• determine $P(A \text{ or } B)$ for events that are mutually exclusive	
• identify complementary events	
• determine the complement to a particular event	
• determine the probability of the complement when given the probability of an event	
• participate in and contribute toward the problem-solving process for problems that require the analysis of statistics and probability studied in Applied Mathematics 30	• complete the solution to problems that require the analysis of statistics and probability studied in Applied Mathematics 30

Examples

Students who achieve the *acceptable standard* should be able to answer all the following questions, except for any part of a question labelled **SE**. Parts labelled **SE** are appropriate examples for students who achieve the *standard of excellence*.

Use the following information to answer the next question.

The average individual score per round for each of several golfers on the 2000 PGA tour is recorded in the table below.

Name	Average	Name	Average
Mike Weir	70.4	Joe Durrant	70.9
Greg Chalmers	70.5	Tiger Woods	67.8
David Duval	69.4	Scott Dunlop	70.4

Numerical Response

1. The standard deviation, σ, of the golfers' average scores, to the nearest hundredth, is _____.

Solution:

Enter the values into the list function on a graphing calculator and use the single variable statistics function to calculate the population standard deviation $\sigma = 1.04$.

Use the following information to answer the next question.

A medical researcher measured the body temperature of 700 people and found that the temperatures were normally distributed with a mean of 36.8° C and a standard deviation of 0.35° C.

2. The number of people expected to have a body temperature of 37.5° C or lower is

A. 16

B. 68

C. 490

D. 684

Solution:

Method 1

$z = \dfrac{37.5 - 36.8}{0.35}$

$z = 2$

$P(z \le 2) = 0.9772$ (from table)

$0.9772 \times 700 = 684.04$

Method 2

Normalcdf (0, 37.5, 36.8, 0.35) = 0.97724…

$700 \times 0.97724\ldots = 684.07\ldots$

Method 3

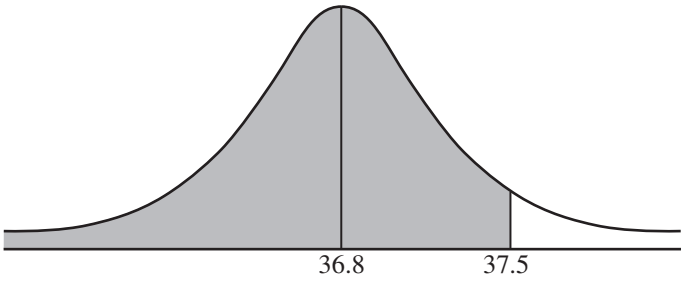
$z = \dfrac{37.5 - 36.8}{0.35}$

$z = 2$

shadenorm(–5, 2)

area = 0.97725

$0.97725 \times 700 = 684.075$



The number of people expected to have a body temperature of 37.5°C or lower is 684.

Note: Since these data are continuous, the probabilities $P(x \le 37.5)$ or $P(x < 37.5)$ would both be calculated using any of the three methods shown above.

Use the following information to answer the next question.

Henry played 24 golf games on the same course during each of two seasons. In the first season, his mean score was 78 with a standard deviation of 2.1. In the second season, his mean score was 74 with a standard deviation of 3.8.

3. The **standard deviation** of Henry's scores for the two seasons indicates that his

A. scores were more consistent in the first season

B. scores were more consistent in the second season

C. average score was better in the first season

D. average score was better in the second season

Solution:

Standard deviation is a measure of the extent to which the data are spread from the mean. Since Henry's second season scores have a larger standard deviation, his scores in the first season were more consistent.

SE **4.** In a particular town, 70% of the students are bussed to school. In a random sample of 1 000 students, the mean of the number of students bussed to school is expected to be 700, with a standard deviation of 14.49. The probability that in any given sample of 1 000 students, 720 or more students are bussed to school is

A. 0.08
B. 0.38
C. 0.62
D. 0.92

Solution:

Method 1

$x = 720$

$\mu = 700$

$\sigma = 14.49$

$z = \dfrac{720 - 700}{14.49}$

$z \doteq 1.38$

$P(z \geq 1.38) = 1 - 0.9162$
(using table)
$= 0.0838$

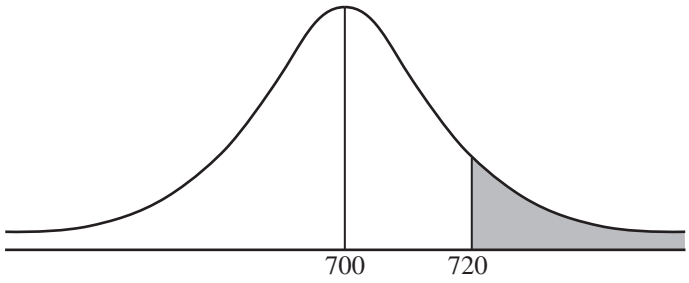
Method 2

Normalcdf (720,1000,700,14.49)

$= 0.0837\ldots$

Method 3

$x = 720$
$\mu = 700$
$\sigma = 14.49$
$z = \dfrac{720 - 700}{14.49}$
$z \doteq 1.38$



shadenorm(1.38, 5)
area = 0.0838

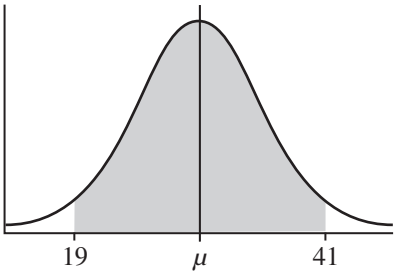
The probability that 720 or more students are bussed to school is 0.08.

Use the following information to answer the next question.

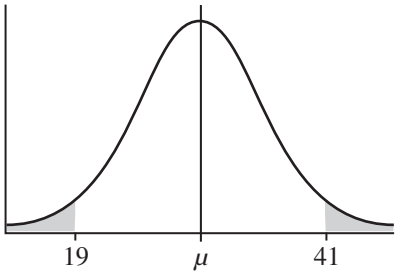
Quality control testing on an assembly line has determined that the probability of a particular battery being defective is 0.02. In a shipment of 1 500 batteries, the symmetric 95% confidence interval for the number of defective batteries is from 19 to 41.

5. Which of the following graphs has a shaded region indicating this 95% confidence interval?

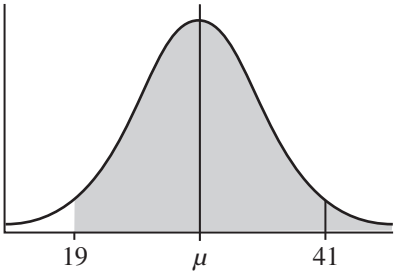
A.



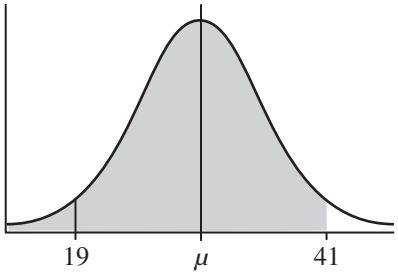
B.



C.



D.



Solution:

The symmetric 95% confidence interval includes all of the data that have a z-score between –1.96 and 1.96. Therefore, the correct response is A.

Use the following information to answer the next question.

In humans, some characteristics are genetically determined by genes, which occur in pairs. Each pair consists of any combination of dominant (*E*) and recessive (*e*) genes. For example, a person may have two dominant genes (*EE*), two recessive genes (*ee*), or a dominant and a recessive gene (*Ee*).

If a person has one or two dominant genes, then the detached earlobe characteristic is displayed.

Attached ear-lobe

Detached ear-lobe

Written Response—10%

6. **a.** Complete the Punnett square below to show the sample space for the offspring of two parents who both carry one dominant (*E*) gene for the detached earlobe.

A POSSIBLE SOLUTION to part a

Mother

Father		***E***	***e***
	E	*EE*	*Ee*
	e	*Ee*	*ee*

b. What is the probability that one offspring from these parents will have detached earlobes?

A POSSIBLE SOLUTION to part b

The probability that one offspring from these parents will have detached earlobes is $\frac{3}{4}$ or 0.75.

Use the following information to answer the next part of the question.

Approximately 39% of the population exhibits the attached earlobe characteristic.

c. Use the normal approximation to the binomial distribution to calculate the mean and standard deviation for the number of people in a sample of 8 748 that will have attached earlobes. Round your answers to the nearest hundredth for standard deviation and to the nearest whole number for mean.

A POSSIBLE SOLUTION to part c

$n = 8\,748$	$\mu = np$	$\sigma = \sqrt{np(1-p)}$
$p = 0.39$	$\mu = 8\,748\,(0.39)$	$\sigma = \sqrt{8\,748\,(0.39)(0.61)}$
$1 - p = 0.61$	$\mu = 3\,411.72$	$\sigma = 45.6196\ldots$

The mean is 3 412 and the standard deviation is 45.62.

d. Calculate the 95% confidence interval for the number of people in this sample that will have attached earlobes.

A POSSIBLE SOLUTION to part d

95% confidence int. $= \mu \pm 1.96\sigma$
$= 3\,412 \pm 1.96(45.62)$
$= 3\,412 \pm 89.4152$
$=$ 3 322.3048, 3 501.1352

It can be predicted, with 95% confidence, that between 3 322 and 3 502 people in this sample will have attached earlobes.

Note: Always round the lower limit of the interval down and the upper limit of the interval up.

SE **7.** In the general population, the IQ scores of individuals are normally distributed with a mean of 110 and a standard deviation of 10. If a large group of people is tested,

a. what proportion will be expected to have IQs between 100 and 120?

b. what is the probability that an individual in the group has an IQ greater than 120?

c. what minimum IQ is necessary to be within the top 5% of the sample group?

Solutions:

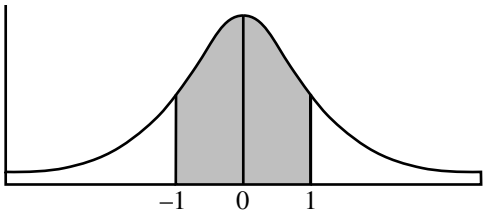
a.

$$z = \frac{120 - 110}{10} = 1$$

$$P(s < 120) = 0.8413$$

$$z = \frac{100 - 110}{10} = -1$$

$$P(s < 100) = 0.1587$$



$$\therefore P(100 < s < 120) = 0.8413 - 0.1587$$

$$P(100 < s < 120) = 0.6826$$

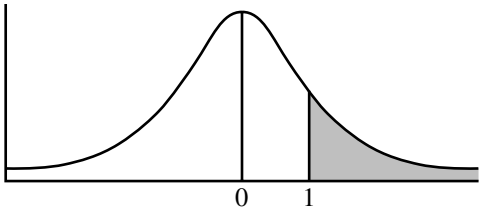
The proportion of this group with an IQ between 100 and 120 is 0.6826.

b.

$$P(s < 120) = 0.8413$$

$$\therefore P(s > 120) = 1 - 0.8413$$

$$= 0.1587$$



The probability of a group member having an IQ greater than 120 is 0.1587.

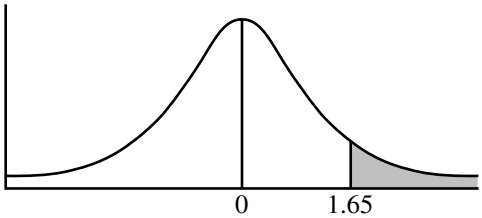
c.

$$P(s < x) = 0.95$$

$$z \doteq 1.65$$

$$1.65 \doteq \frac{x - 110}{10}$$

$$x = 126.5$$



The minimum IQ necessary to be within the top 5% is 126.5.

Use the following information to answer the next question.

A box contains 6 blue balls and 4 red balls. Two balls are drawn from the box, one after the other, without replacement.

Numerical Response

8. The probability, to the nearest hundredth, that the first ball drawn is blue and the second ball drawn is red is _____.

Solution:

Since the first ball is not replaced before the second one is drawn, the probability of the second event depends on the results of the first event.

$$P(\text{Blue and then Red}) = \frac{6}{10} \times \frac{4}{9}$$
$$= 0.266\ldots$$

The probability is 0.27.

Use the following information to answer the next question.

The numbers 1 through 5 are each written on a separate slip of paper, and the papers are placed in a box. The letters A, B, C, and D are each written on a separate slip of paper, and the papers are placed into a **different** box. Jodi draws one slip of paper from each box.

9. The number of elements in the sample space for this trial is

A. 51

B. 20

C. 9

D. 2

Solution:

There are 5 numbers and 4 letters. Therefore, there are 20 elements in the sample space.

1A 2A 3A 4A 5A
1B 2B 3B 4B 5B
1C 2C 3C 4C 5C
1D 2D 3D 4D 5D

Use the following information to answer the next question.

A particular traffic light at the outskirts of a town is red for 30 s, green for 25 s, and yellow for 5 s in every minute.

10. The probability that the traffic light will **not** be green when a motorist first sees it is

A. $\frac{1}{2}$

B. $\frac{1}{12}$

C. $\frac{5}{12}$

D. $\frac{7}{12}$

Solution:

Method 1

$$\begin{aligned} P(\text{not green}) &= 1 - P(\text{green}) \\ P(\text{green}) &= \frac{25}{60} \\ &= \frac{5}{12} \\ 1 - P(\text{green}) &= 1 - \frac{5}{12} \\ &= \frac{7}{12} \end{aligned}$$

Method 2

$$\begin{aligned} P(\text{not green}) &= P(\text{red or yellow}) \\ &= \frac{30}{60} + \frac{5}{60} \\ &= \frac{35}{60} \\ &= \frac{7}{12} \end{aligned}$$

Use the following information to answer the next question.

Given their previous performance, the probability of a particular baseball team winning any given game is $\frac{4}{5}$.

11. The probability that the team will win their next 2 games is

A. $\frac{8}{5}$

B. $\frac{16}{25}$

C. $\frac{2}{5}$

D. $\frac{1}{25}$

Solution:

$$\begin{aligned} \text{P(winning next } 2 \text{ games)} &= \text{P(winning next game)} \times \text{P(winning second game)} \\ &= \frac{4}{5} \bullet \frac{4}{5} \\ &= \frac{16}{25} \end{aligned}$$

Use the following information to answer the next question.

Malaga, Spain, lies in a region of Europe known as the Costa Del Sol (Coast of the Sun). The probability of sunshine on any given day in this region is approximately 0.89.

Numerical Response

12. In a non-leap year of 365 days, the average number of days of the year that a tourist could expect to experience weather **other** than sunshine, to the nearest whole number, is _____.

(Record your answer in the numerical-response section on the answer sheet.)

Solution:

Probability of weather other than sunshine = 1– 0.89
= 0.11

$\mu = np$
$\mu = 365(0.11)$
$\mu = 40.15$

A tourist should expect weather other than sunshine on 40 days a year.

Finance and Spreadsheets

General Outcome

Design or use a spreadsheet to make and justify financial decisions.

General Notes:

- Teachers may wish to check with the computer system administrator about log-ons for students before beginning this unit.
- This unit is intended to focus on the analysis of financial situations using spreadsheets, but is not intended to be an intense, in-depth study of spreadsheets.
- Students are not expected to master specialized computer functions such as *If-Then-Else* statements.
- Teachers should be aware of time constraints during this unit as some analyses can become quite lengthy. The resource package provides templates of spreadsheets for analytical use.
- Teachers should spend approximately one-quarter of the time devoted to this unit on design of the spreadsheet and the remaining time on analyses.
- Students should be familiar with absolute cell references (example B3).
- Some calculators have financial application capabilities that can be used to solve a variety of problems in this unit. Not all calculators on the approved list have these financial applications.
- Most questions assume a constant interest rate and payment throughout the duration of the loan or mortgage. Interesting discussions may result when changes in rate and payment are considered.
- Discussion regarding different types of investments may be beneficial.

Specific Outcomes

Specific Outcome 3.1

Design a financial spreadsheet template to allow users to input their own variables. [C, PS, T]

3.1 Note:

- This outcome is intended to be achieved using computer spreadsheet technology rather than functions on some graphing calculators.

Refer to examples 1, 7, and 8.

Specific Outcome 3.2

Analyze the costs and benefits of renting or buying an increasing asset, such as a home. [C, CN, PS, T]

3.2 Notes:

- Communicating the results of the analysis is a key idea behind specific outcome 3.2.
- Although calculating mortgage or loan payments using the finance applications of a graphing calculator may be beneficial to classroom discussion, it will not be assessed on the diploma examination. Not all calculators on the approved list have this financial application function.
- Diploma examination questions relating to this outcome will focus on situations where the mortgage payment is known or where it can be calculated using a given table or chart.
- Much of the analysis can be done using an exponential regression model. For an increasing asset, the value of b in $y = a \cdot b^x$ is greater than 1.
- Students are not required to calculate effective interest rates.

Refer to example 10.

Specific Outcome 3.3

Analyze the costs and benefits of leasing or buying a decreasing asset, such as a vehicle or a computer. [C, CN, PS, T]

3.3 Notes:

- Communicating the results of the analysis is a key idea behind specific outcome 3.3.
- Limit spreadsheet construction (as this is not the focus of this outcome) to examples where the compounding period matches the payment period.
- Much of the analysis can be done using an exponential regression model. For a decreasing asset, the value of b in $y = a \cdot b^x$ is between 0 and 1.

Refer to examples 4, 5, and 6.

Specific Outcome 3.4

Analyze an investment portfolio, applying such concepts as interest rate, rate of return and total return. [C, CN, PS, T]

3.4 Notes:

- Students should be able to recognize that compound interest can be represented by an exponential function of the form $y = a \cdot b^x$, where b is equal to $1 + i$.
- The rate of return of lump-sum investments can be calculated using exponential regression; however, in a diversified portfolio, the average rate of return can be calculated with a weighted approach.

Refer to examples 2, 3, and 9.

Acceptable Standard	*Standard of Excellence*
The student can when given	The student can also
• distinguish the difference between entered values, fixed values, and computed values in a spreadsheet	
• identify appropriate algebraic functions used in the solution of spreadsheet problems	• interpret a written problem and develop an appropriate spreadsheet
• work within the parameters of an existing spreadsheet to solve non-recursive or recursive problems	• modify an existing spreadsheet to accommodate changing needs, and analyze differing factors in each model
• design a simple spreadsheet by using formulas and functions to solve non-recursive problems, such as billing and design calculations, or recursive problems such as loan calculations	• design more complex spreadsheets
• determine the algebraic formulas used to construct the spreadsheet when given a spreadsheet	
• correctly identify the more appropriate payment model when given two schedules, and give appropriate justification for the choice in terms of cost	• set up a spreadsheet that can help in cost-and-benefit analysis
• offer a few pros and cons with respect to the chosen payment model	• offer pros and cons that show a thorough understanding with respect to the chosen payment model
• use correct terminology to discuss payment options	
• determine whether renting or buying is a more appropriate choice, given specific circumstances	
• discuss the appropriateness of a regression model to a given context	
• recognize the value of an asset as either increasing or decreasing	
• recognize that the value of an increasing asset can be represented by the regression model $y = a \cdot b^x$, where $b > 1$	
• recognize that the value of a decreasing asset can be represented by the regression model $y = a \cdot b^x$, where $0 < b < 1$	
• use an exponential regression model to make predictions	
• calculate average annual percentage appreciation by using exponential regression	• calculate average annual percentage depreciation by using exponential regression
• use correct terminology to describe an investment portfolio	• compare two similar portfolios and make comments with regard to interest rate, rate of return, and total return
• calculate the total return and the average annual rate of return of an investment	
• participate in and contribute toward the problem-solving process for problems that require the analysis of finance and spreadsheets studied in Applied Mathematics 30	• complete the solution to problems that require the analysis of finance and spreadsheets studied in Applied Mathematics 30

Examples

Students who achieve the *acceptable standard* should be able to answer all the following questions, except for any part of a question labelled **SE**. Parts labelled **SE** are appropriate examples for students who achieve the s*tandard of excellence*.

1. For the following invoice, develop a spreadsheet that calculates the parts and labour costs, and that requires the operator to input a minimum number of entries.

Acme Auto Parts

Customer Inquiries

Item No.	Auto Parts	Quantity	Unit Price	Total	Labour	
1	Brake Pads	1	$26.34	$26.34	OH/Front Brakes (0.9 hrs. @ $53.00/hr.)	$47.70
2	Wheel Seals	2	$5.25	$10.50	Machined and Replaced Rotor (flat rate)	$10.00
3	Rotor	1	$30.16	$30.16		
			Total Parts	$67.00	Total Labour	$57.70
					Subtotal	$124.70
					GST (6%)	$7.46
					Total	$132.18

Solution:

	A	B	C	D	E	F	G
1			**Acme Auto Parts**				
2	**Customer Inquiries**						
3	Item No.	Auto Parts	Quantity	Unit Price	Total	Labour	
4	1	Brake Pads	1	$26.34	=C4*D4	OH/Front Brakes (0.9 hrs. @ $53.00/hr.)	$47.70
5							
6	2	Wheel Seals	2	$5.25	=C6*D6	Machined and Replaced Rotor (flat rate)	$10.00
7							
8							
9	3	Rotor	1	$30.16	=C9*D9		
10							
11				Total Parts	=SUM(E4:E10)	Total Labour	=SUM(G4:G10)
12						Subtotal	=E11+G11
13						GST (6%)	=(G12)*0.06
14						Total	=G12+G13

Use the following information to answer the next two questions.

Owen invested $2 000 three years ago. During this time, he has been tracking the investment by recording its value at the end of each year.

Year	Value
0	$2 000.00
1	$2 160.00
2	$2 309.04
3	$2 477.98

2. The average annual rate of return on this investment, to the nearest tenth of a percentage, is

A. 0.7%/a

B. 1.1%/a

C. 1.2%/a

D. 7.4%/a

Solution:

Method 1

Enter values and perform an exponential regression:

$y = 2004.59\ldots(1.0735.)^x$

In this expression, $y = a \cdot b^x$, the value of b is 1.0735…. This value indicates that the investment is increasing at an average annual rate of 7.35…% (7.4%).

Method 2

Year	Percentage Growth
0–1	8%
1–2	6.9%
2–3	7.3%

Average Percentage Growth

$$\frac{8 + 6.9 + 7.3}{3} = 7.4\%$$

Note: Method 2 works only because this is a lump-sum investment into a single account. However, this method would not work for example 9.

3. The total **return** on this investment after year 3 is

A. $168.94

B. $477.98

C. $6 947.02

D. $8 947.02

Solution:

At the end of year 3, the $2 000 investment is worth $2 477.98. Therefore, the total return is $477.98.

Use the following information to answer the next question.

John is considering whether to buy or lease a car. The price of the car he is interested in is $25 500. Regardless of the option he chooses, John will make a down payment of $3 000.

John's first option is to purchase the car. He will have to make payments of $557.25/month for 48 months.

John's second option is to lease the car. He will have to make payments of $307.50/month for 48 months, after which time he will still owe $12 850.

4. *If John wants to own the car at the end of 48 months, he should* ___***i***___ *the car because he will save* ___***ii***___.

The statement above is completed by the information in row

Row	***i***	***ii***
A.	lease	$862
B.	buy	$862
C.	lease	$11 988
D.	buy	$11 988

Solution:

Purchase: $\$557.25 \times 48 = \$26\,748$
Lease: $\$307.50 \times 48 + \$12\,850 = \$27\,610$
If John wants to own the car at the end of 48 months, he should buy the car because he will save $862.

Use the following information to answer the next question.

It is predicted that the value of John's car will depreciate every year for the first four years according to the table below.

Year	Value
0	$25 500
1	$17 850
2	$16 065
3	$14 459
4	$13 012

SE **5.** If John uses an exponential regression equation to determine the value of his car in the future, then he will find that the average rate of **depreciation** is

A. 14.4% per year

B. 49.0% per year

C. 51.0% per year

D. 85.6% per year

Solution:

The data can be modelled by the exponential function $y = 23\,061.45\ldots(0.8558\ldots)^x$. The average annual rate of depreciation is $1 - 0.8558\ldots = 0.14412\ldots$ (14.4%).

6. Chris is looking into the purchase/lease of a car. The car he has chosen is priced at $21 720.00 plus 6% GST, and financing is calculated at 7.5%/a, compounded monthly. Chris will make a down payment of $2 000.00 and has the following two options available.

Option 1: (Lease with buyout)

Amount financed	$19 720
Monthly lease payments	$377.55 + GST on monthly payment + $\frac{\text{GST on down payment}}{36}$ = $403.54
Number of months	36
Final buyout	$9 394.06 + GST = $9 957.70

Option 2: (Finance full amount)

Purchase price	$21 720 + GST = $23 023.20
Amount financed	$23 023.20 – 2 000.00 = $21 023.20
Monthly payment	$653.95
Number of months	36

Notes: When leasing a car, the 6% GST is calculated on the monthly payment. The 6% GST on the down payment is divided by the number of months and also added to the monthly payment. Lease payments must be made at the beginning of the period.

The information for option 1 and option 2 could be entered into a spreadsheet (see template on p.175, *Applied Mathematics 12*).

a. How much does Chris spend in total for monthly payments over the term of the financing for each option? Why is there a difference between the two amounts? Explain.

b. What is the total amount spent by Chris in each option if he would like to own the vehicle at the end of 36 months? What is the difference in final price?

c. Which option would you consider to be better for Chris and in which circumstances? Explain.

SE **d.** How much would Chris have to save each month at 6%/a, compounded monthly, to pay for the $9 957.70 buyout at the end of 36 months? Would this decrease his final cost for the lease or not? Explain.

Solutions:

a. **Option 1** $403.54 \times 36 = \$14\ 527.44$

Option 2 $653.95 \times 36 = \$23\ 542.20$

The difference is that in option 2, Chris owns the car at the end of 36 months, and in option 1, Chris does not own the car at the end of 36 months.

b. The total cost for option 1 would be $26 485.14, which includes all payments, the down payment of $2 000, and the final buyout of $9 957.70. It is assumed that Chris has the $9 957.70 for the final buyout and does not have to finance this portion as well. The total amount spent by Chris in option 2 would be $25 542.20, which includes all payments plus the down payment of $2 000. The difference in final price to purchase the car would be $942.94.

c. If Chris can afford the monthly payment of $653.95 for 36 months, he will save $942.58. If Chris cannot afford the higher payment, then the lease option is better, but he must remember to save the money for the buyout over the next 36 months.

SE **d.** To accumulate the buyout amount over 36 months at 6%/a, compounded monthly, Chris would have to save $253.15 per month; this would amount to $9 957.93, including interest, which is $0.23 more than is required to pay for the buyout. The total cost for option 1 would then be $25 640.61, which includes the $2 000 down payment, $14 527.44 in lease payments, $9 113.40 in savings for 36 months, and a subtraction of $0.23 that the monthly savings has over and above the required buyout amount.

Note: The solution to part d requires the use of the finance application of a graphing calculator.

Use the following information to answer the next question.

The following spreadsheet shows the beginning of an amortization table for 5 equal monthly payments to be made on a $900 loan with an interest rate of 1% per month.

	A	B	C	D	E
1	**Payment Number**	**Payment**	**Interest Payment**	**Payment to Principal**	**Balance Remaining**
2	1	$185.44	$9.00	$176.44	$723.56
3	2	$185.44	$7.24	$178.20	$545.36
4	3	$185.44			
5	4				
6	5				

7. Which of the following formulas can be used to calculate the value of cell D4?

A. =B4 + C4

B. =E3 – B4

C. =B4 – 0.1*E3

D. =B4 – 0.01*E3

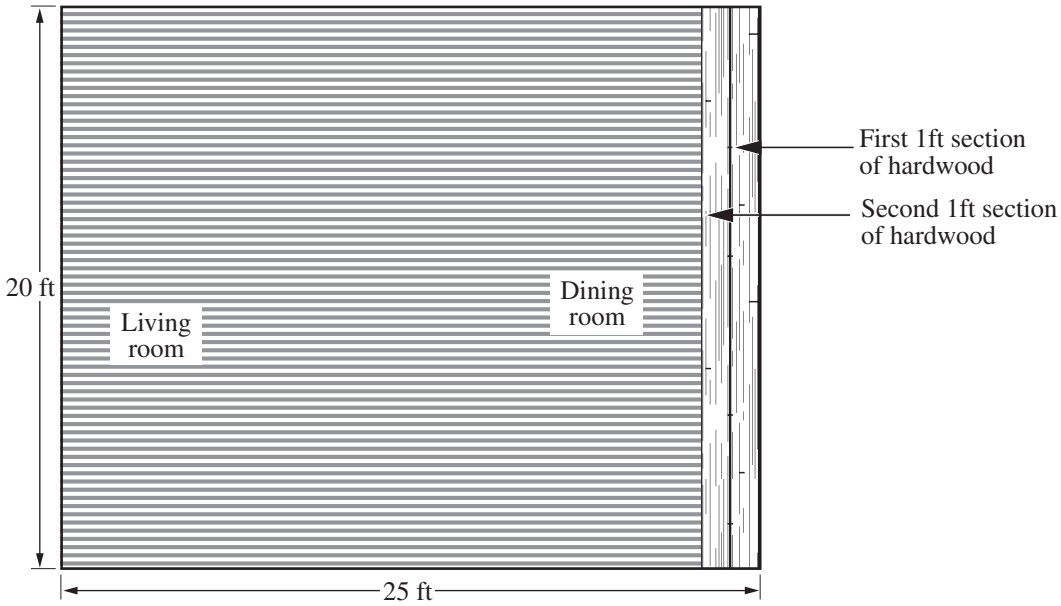
Solution:

Payment to principal is determined by subtracting the interest paid (1%) from the monthly payment. Therefore, a formula that will work is =B4 – 0.01 *E3.

Use the following information to answer the next question.

A family is planning to cover their $20\text{ ft} \times 25\text{ ft}$ living room/dining room with carpet and hardwood. Their budget for this project is $5 000. The cost of the carpet is $\$7.25/\text{ft}^2$, and the cost of the hardwood is $\$12.50/\text{ft}^2$.

The family would like to cover the entire area with hardwood, but they know that it would cost more than the budgeted amount. In order to determine the maximum area of hardwood that they can afford, they calculated the total cost of the flooring each time a 1 ft wide section of hardwood was added to the room, as shown below.



The family used the following spreadsheet to calculate costs and to determine the maximum area of hardwood that they could afford.

	A	B	C	D	E
1	**Area of floor with hardwood (ft^2)**	**Area of floor with carpet (ft^2)**	**Cost of hardwood @ \$12.50/ft^2**	**Cost of carpet @ \$7.25/ft^2**	**Total cost of flooring**
2	500	0	\$6 250	\$0	\$6 250
3	480	20	\$6 000	\$145	\$6 145
4	460	40	\$5 750	\$290	\$6 040
5	440	60	\$5 500	\$435	\$5 935
6	420	80	\$5 250	\$580	\$5 830
7	400	100	\$5 000	\$725	\$5 725
8	380	120	\$4 750	\$870	\$5 620
9	360	140	\$4 500	\$1 015	\$5 515
10	340	160	\$4 250	\$1 160	\$5 410
11	320	180	\$4 000	\$1 305	\$5 305
12	300	200	\$3 750	\$1 450	\$5 200
13	280	220	\$3 500	\$1 595	\$5 095
14	260	240	\$3 250	\$1 740	\$4 990
15	240	260	\$3 000	\$1 885	\$4 885
16	220	280	\$2 750	\$2 030	\$4 780
17	200	300	\$2 500	\$2 175	\$4 675
18	180	320	\$2 250	\$2 320	\$4 570
19	160	340	\$2 000	\$2 465	\$4 465
20	140	360	\$1 750	\$2 610	\$4 360
21	120	380	\$1 500	\$2 755	\$4 255
22	100	400	\$1 250	\$2 900	\$4 150
23	80	420	\$1 000	\$3 045	\$4 045
24	60	440	\$750	\$3 190	\$3 940
25	40	460	\$500	\$3 335	\$3 835
26	20	480	\$250	\$3 480	\$3 730
27	0	500	0	\$3 625	\$3 625

Written Response—10%

8. **a.** Explain the relationship between the values in columns A and B.

A POSSIBLE SOLUTION to part a

The sum of column A and the sum of column B each equal 500 ft^2 (the total floor area of the living room and dining room).

b. Show, by writing a statement or a formula, how the value in cell E9 ($5 515) was calculated. Make reference to other cells in row 9.

A POSSIBLE SOLUTION to part b

Cell C9 is calculated by multiplying A9, the total area of hardwood, by $12.50. Cell D9 is calculated by multiplying B9, the total area of carpet, by $7.25. Cell E9 is then calculated by adding cells C9 and D9 to find the total cost of flooring for the given areas of carpet and hardwood.

or

E9 = A9 * 12.50 + B9 * 7.25

or

C9 = A9 * 12.50
D9 = B9 * 7.25
C9 + D9 = E9

or

C9 + D9 = E9

c. If the family is to remain within their budget, what is the maximum area of hardwood that they can place into this living room/dining room area?

A POSSIBLE SOLUTION to part c

To remain within budget, the maximum area of hardwood is a 13 ft $\times$ 20 ft piece or 260 ft^2.

d. • What is the total cost for this plan?

POSSIBLE SOLUTIONS to part d, bullet 1
Option 1: The family could have 250 ft^2 each of carpet and hardwood for a total cost of $4 937.50. **Option 2:** reference row 14 260 ft^2 of hardwood and 240 ft^2 of carpet for a total cost of $4 990.

• Will the family remain within their budget? Explain.

A POSSIBLE SOLUTION to part d, bullet 2
The total cost is less than $5 000, so the family will remain within their budget.

Use the following information to answer the next question.

A person has invested $20 000 with an investment firm. In the first year, her portfolio gives the following returns:

Type	Percent Invested	First-Year Profit
Guaranteed certificates	20%	4%
Blue-chip stocks	50%	9.25%
High-risk stocks	30%	–7.5%

9. For this portfolio, the rate of return, to the nearest tenth of a percentage, is

A. 20.8%

B. 5.8%

C. 3.2%

D. 1.9%

Solution:

$$\text{Guaranteed certificates} = (0.2 \times 20\,000) \times 0.04 = \$160$$
$$\text{Blue-chip stocks} = (0.5 \times 20\,000) \times 0.0925 = \$925$$
$$\text{High-risk stocks} = (0.3 \times 20\,000) \times -0.075 = -\$450$$

$$\text{First Year Profit} = \$635$$
$$\text{Rate of Return} = \frac{635}{20\,000}$$
$$= 0.03175 \text{ or } 3.2\%$$

Use the following information to answer the next question.

A couple plans to buy a house for $175 000. They will make a $30 000 down payment and take out a 25-year mortgage at 7.5% per annum, compounded semi-annually. The monthly payments on the mortgage will be $1 060.75 per month.

10. The total amount of interest that the couple will pay for this mortgage is

A. $188 125

B. $173 225

C. $155 875

D. $143 225

Solution:

$$\begin{aligned}\text{Amount of mortgage} &= \$175\,000 - \$30\,000 \\ &= \$145\,000\end{aligned}$$

$$\begin{aligned}\text{Total payments} &= 25 \times 12 \times \$1\,060.75 \\ &= \$318\,225\end{aligned}$$

$$\begin{aligned}\text{Total interest} &= \$318\,225 - \$145\,000 \\ &= \$173\,225\end{aligned}$$

Cyclic, Recursive, and Fractal Patterns

General Outcome

Generate and analyze cyclic, recursive and fractal patterns.

General Notes:

- It is not the intent of this unit to do an indepth study on sinusoidal curves and the characteristics of fractal patterns, but to use these, and other functions, as regression models to show patterns from which inferences can be drawn.
- These outcomes require substitution of calculator-generated parameters into the regression equations and an understanding of the significance of these values.
- A fractal is a geometric figure exhibiting self-similarity and is produced by repeated iterations. Each successive fractal in a fractal pattern is more complex than the previous one.

Specific Outcomes

Specific Outcome 4.1

Collect sinusoidal data, graph the data using technology, and represent the data with a best-fit equation of the form:

- $y = \text{a}\sin(bx + c) + d$. [C, CN, PS, T, V]

4.1 Notes:

When using technology to generate a sinusoidal regression model, note the following points.

- Graphing calculators will model sinusoidal regression in the form $y = a \cdot \sin(bx + c) + d$, where x is measured in radians. This means that the period of the function is $\frac{2\pi}{b}$ units, and the horizontal transformation is to the left or right. To graph a sinusoidal regression model, students' calculators must be in radian mode. This means that teachers will have to explain to students that there are two ways to express angle measures: in radians and in degrees. It is not the intent to teach conversions from one measure to another, but simply to make students aware of the two measures.
- A minimum of five points over most of one period is normally sufficient to produce an adequate regression model. Creating a list of eight or more points over two periods produces a better model. It is recommended that when entering values for sinusoidal regression into a graphing calculator, students begin with the lowest x value.
- Sinusoidal regression models on a graphing calculator will occasionally have very small values for one or more of the parameters. Students will require practice recognizing these values and should write values less than or equal to 10^{-4} as zero.
- Students using a TI-83 or TI-83 Plus may be referred to Utility 32 on page 361 of *Applied Mathematics 12* for detailed information on using the SinReg function.
- Recognize that a value of c other than zero is an indication that the initial point of the cycle is to the left or right of the y-axis.

- Determining the exact value of the horizontal transformation $\left(\frac{-c}{b}\right)$ is beyond the scope of Applied Mathematics 30.
- The notation for a sinusoidal regression equation in Applied Mathematics 30 is $y = a \sin(bx + c) + d$, and in Pure Mathematics 30 it is $y = a \sin[b(x - c)] + d$.

Refer to examples 1 and 6.

Specific Outcome 4.2

Use best-fit sinusoidal equations, and their associated graphs, to make predictions by interpolation and extrapolation. [C, CN, PS, T]

4.2 Notes:
- Students should be able to make predictions from equations and graphs.
- Calculated values should not be rounded until the final answer.

Refer to examples 5 and 6.

Specific Outcome 4.3

Describe periodic events, including those represented by sinusoidal curves, using the terms amplitude, period, maximum and minimum values, and vertical and horizontal shift, and relating these terms to the parameters a, b, c, and d described in 4.1. [C, V]

4.3 Notes:
- Graphs should be provided for questions relating to non-sinusoidal periodic events.
- Descriptions of sinusoidal events can be based on a context, a graph, or an equation.
- Students should connect the term vertical shift to median and the term horizontal shift to start point.

Refer to examples 2, 3, 4, and 6.

Specific Outcome 4.4

Use technology to generate and graph sequences that model real-life phenomena.
[PS, T, V]

4.4 Notes:
- Students should be given an opportunity to determine, without the use of technology, an equation that models a sequence.
- Students should be familiar with regression models from Applied Mathematics 10 and 20 and should be able to determine which regression is the "best fit" for a particular set of data.
- Regression models are sufficient for this outcome. Students are not required to be familiar with the SEQ mode on some graphing calculators.

- When determining the best-fit regression equation for a set of data, the context and pattern of the data should be considered along with the correlation coefficient of the regression equation being considered.

Refer to examples 7, 8, 9, 10, 11, and 12.

Specific Outcome 4.5

Use technology to construct a fractal pattern by repeatedly applying a procedure to a geometric figure. [CN, R, V, T]

4.5 Notes:

- The use of technology in this outcome is intended to allow students to test conjectures without intensive labour. Students should, however, be able to generate the first few fractal patterns from simple figures, such as regular polygons. Grid paper will help students create accurate fractal shapes.
- Dynamic software may be appropriate for classwork, but assessment should be with paper and pencil only. Formulas can be generated by regression models.
- Define iterations as repeated recursive calculations or procedures.

Refer to examples 13 and 14.

Specific Outcome 4.6

Use the concept of self-similarity to compare and/or predict the perimeters, areas, and volumes of fractal patterns. [CN, R, T, V]

4.6 Notes:

- Be aware that fractal patterns can become very difficult very quickly. Look for examples that can be modelled by exponential regression. This can usually be accomplished by focusing on the number of new shapes at each iteration rather than on the total number of shapes.
- Two important properties of fractal patterns are
  – self-similarity (characteristics of the shape are maintained under any magnification)
  – increasing complexity of the shape at each iteration
- For area and volume fractal patterns, the pattern does not establish itself between the original and the first iteration.
- Exponential regression cannot be performed on fractal patterns involving increasing areas and volumes.

Refer to examples 15, 16, and 17.

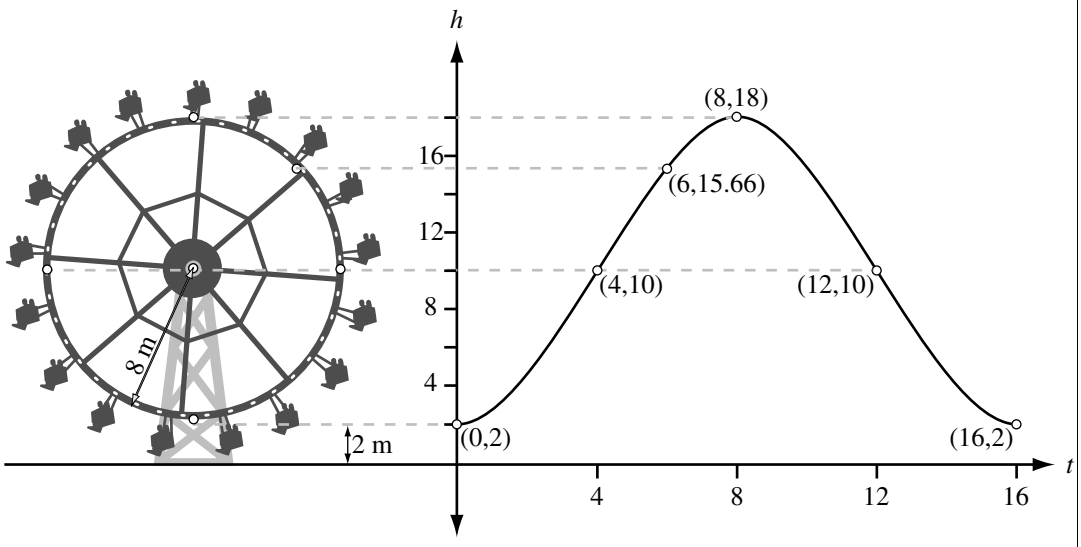
Acceptable Standard The student can	*Standard of Excellence* The student can also
• use technology to graph data that represent a sinusoidal curve	
• state appropriate graphing calculator window settings as an ordered triple in the form x: $[x_{min}, x_{max}, x_{scl}]$ and y: $[y_{min}, y_{max}, y_{scl}]$	
• use the regression model $y = a \sin(bx + c) + d$ to determine a best-fit equation	• provide explanations for trends in data
• use the TRACE or CALC function on a graphing calculator to make predictions	
• describe a periodic event, using correct terminology, given a diagram, a context, an equation, or a graph	• describe the transformational effect(s) of additional information on a periodic event
• make predictions from the derived sinusoidal regression equation	• sketch transformations of a given graph
• relate values of a, b, and d of $y = a \sin(bx + c) + d$ to given window settings	• choose appropriate window settings for a graphing calculator, based on given values for amplitude, period, and vertical shift
• determine the values of a, b, and d in the equation $y = a \sin(bx + c) + d$ or in a given graph	
• recognize that when $c \neq 0$, a horizontal transformation occurs	
• enter data into a list or scatter plot, apply a best-fit model (either linear, quadratic, or exponential), and use the regression equation to make predictions	
• distinguish between patterns that are possibly fractals and those that are clearly not fractals	
• generate the next two iterations of a pattern, given the original shape and a written and visual description of the pattern	
• describe, in writing or by listing a sequence of numbers, the patterns related to length of sides and to number of sides, or to number of vertices of geometric shapes	
• describe the numerical pattern for the perimeter of each shape in a two-dimensional pattern	
• determine an algebraic expression describing the perimeter of the shapes in a pattern	• give a rule for the change in area for the first few iterations of a pattern
• generalize a pattern using an algebraic expression that represents the patterns in length of sides, number of sides, or number of vertices of geometric shapes	• find the volume of the shapes for the first few iterations of a simple pattern and make conjectures
• participate in and contribute toward the problem-solving process for problems that require the analysis of patterns and fractals studied in Applied Mathematics 30	• complete the solution to problems that require the analysis of patterns and fractals studied in Applied Mathematics 30

Examples

Students who achieve the *acceptable standard* should be able to answer all the following questions, except for any part of a question labelled **SE**. Parts labelled **SE** are appropriate examples for students who achieve the *standard of excellence*.

Use the following information to answer the next question.

The height, h, in metres, of a point on a ferris wheel at time, t, in seconds, can be represented by a sinusoidal function in the form $h = a \sin(bt + c) + d$, as shown below.



1. The function that **best** describes the height of the point on the ferris wheel is

A. $h = 8 \sin(0.39t + 1.57) + 10$

B. $h = 8 \sin(0.39t - 1.57) + 10$

C. $h = 8 \sin(t + 4.02) + 10$

D. $h = 8 \sin(t - 4.02) + 10$

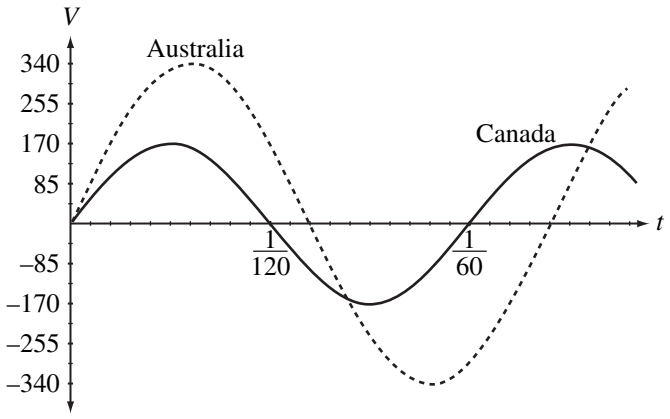
Solution:

Enter values into the list function on a graphing calculator and perform a sinusoidal regression.

$h = 8\ldots\sin(0.39\ldots t + -1.57\ldots) + 10\ldots$

Use the following information to answer the next question.

The sinusoidal function that represents electrical voltage in Australia and the sinusoidal function that represents electrical voltage in Canada are graphed below.



2. The sinusoidal function representing electrical voltage in Australia differs from the sinusoidal function representing electrical voltage in Canada in

A. amplitude and period

B. period and horizontal shift

C. amplitude and vertical shift

D. horizontal shift and vertical shift

Solution:

The function representing electrical voltage in Australia takes longer to complete one cycle and has a higher maximum and lower minimum than the function representing electrical voltage in Canada. Therefore, the two graphs have different periods and amplitudes. The correct response is A.

Use the following information to answer the next three questions.

The function $y = 17.14 \sin (0.52x - 1.75) + 7.11$, where x represents the number of the month, models the average monthly temperature, in degrees Celsius, for Edmonton.

3. The amplitude of this function is

A. 0.52°C
B. 1.75°C
C. 7.11°C
D. 17.14°C

Solution:

The amplitude of a sinusoidal function in the form $y = a \cdot \sin(bx + c) + d$ is equal to the value of a (in this case 17.14).

4. The maximum average monthly temperature in Edmonton can be found by adding 17.14°C and

A. 7.11°C
B. 0.52°C
C. 1.75°C
D. –1.75°C

Solution:

The maximum value of the sinusoidal function can be found by adding 17.14 and 7.11.

5. A tourist is able to determine that the average monthly temperature in Edmonton, to the nearest tenth of a degree, for month 7 (July) is _____°C.

Solution:

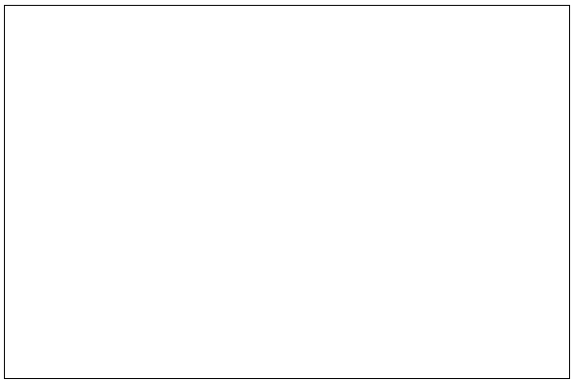
For month 7, $x = 7$
$y = 17.14 \sin[0.52(7) - 1.75] + 7.11$
$y = 23.38\ldots$ or 23.4°C

6. The average daily temperatures over many months in Edmonton, Alberta, are shown in the table below.

Month	Average Temperature
January	–11.7
February	–8.4
March	–2.6
April	5.5
May	11.7
June	15.5
July	17.5
August	16.6
September	11.3
October	5.6
November	–4.1
December	–9.6

a. Use a graphing calculator to plot the data. Let January = 1, February = 2, etc.

b. Determine an approximate sinusoidal function for these data. Express the values to the nearest hundredth. Graph this function in the viewing window provided, and state the window settings.



x: [, ,]

y: [, ,]

function: _____

c. What is the period of this function? How does it compare with your knowledge of annual temperature changes?

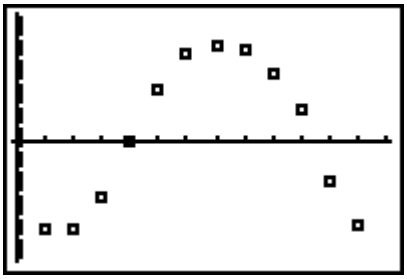
d. What do the parameters a and d in your regression equation represent in terms of the annual mean temperature and range of temperatures in Edmonton, Alberta?

e. The growing season is the part of the year in which the average temperature remains above 5 °C. What is the growing season in Edmonton? Justify your answer by using data from the graph or equation.

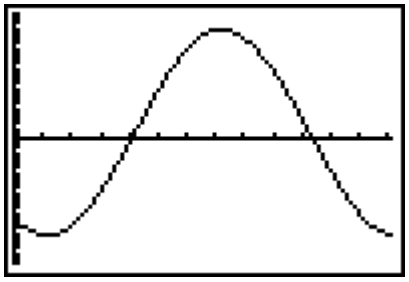
SE **f.** If the average daily temperature rose by 1 °C as a result of global warming, what would be the effect on the sinusoidal function? What would be the effect on the growing season?

Solutions:

a. A possible solution



b.



$x : [0, 13, 1]$ $y : [-20, 20, 1]$

function: $y = 15.39 \sin(0.48x + {}^{-}1.76) + 2.55$
(obtained by inputting a level 5 regression search, and an estimate of period 12 when using a TI-84 Plus: i.e., SinReg 5, L_1, L_2, 12)

or

function: $y = 15.37 \sin(0.48x + {}^{-}1.77) + 2.59$
(obtained when no regression search or period estimate is inputted in the TI-84 Plus)

Note: Depending on the data set, these two methods of calculating a sinusoidal regression may yield slightly different functions, as shown in this example.

c.

$$\text{period} = \frac{2\pi}{b}$$

$$b \doteq 0.48$$

$$\frac{2\pi}{0.48} \doteq 13.1$$

The period of this function is approximately 13.1 months. Since the temperatures should follow approximately the same pattern every year, the period should be approximately 12 months. Since the data in the table is averaged over many months, this may explain why the period calculation is not exactly 12 months.

d. The value of a is approximately 15.4 and the value of d is approximately 2.6. This indicates that the annual mean temperature in Edmonton is about 2.6 °C. The range of the average daily temperature should be from as high as 18.0 °C (2.6 + 15.4) to as low as –12.8 °C (2.6 – 15.4).

e. Graph the line $y = 5$ °C on the same axis as the regression equation. The sinusoidal curve lies above $y = 5$ between $x \doteq 4.0$ and $x \doteq 9.9$. Therefore, the growing season is almost 6 months long.

SE **f.** The value of d in the sinusoidal function would increase by 1, so the new equation would be
$y = 15.39 \sin(0.48x + {}^{-}1.76) + \mathbf{3.55}$ **or** $y = 15.37 \sin(0.48x + {}^{-}1.77) + \mathbf{3.59}$
Since the points of intersection of these curves and $y = 5$ are $x \doteq 3.9$ and $x \doteq 10.0$, the length of the growing season would increase slightly.

Use the following information to answer the next question.

A Set of Data

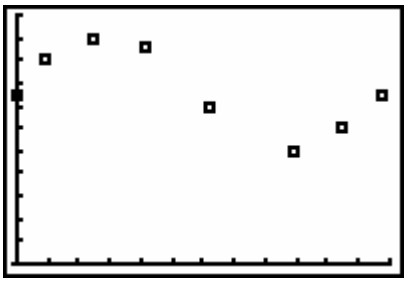
(0, 15)
(9, 18)
(25, 20)
(42, 19)
(62, 14)
(89, 10)
(104, 12)
(118, 15)

7. These data could most appropriately be modelled using

A. a linear regression
B. a quadratic regression
C. a sinusoidal regression
D. an exponential regression

Solution:

When plotted on a graphing calculator, the points form the following pattern:



This is clearly an example of data that models a sinusoidal function.

Use the following information to answer the next question.

A researcher discovered mould growing in a petri dish in her laboratory. When first observed, the mould covered only 12.5% of the dish's surface. After 24 hours, the surface area of the mould doubled in size, as shown in the table below.

Time (h)	Area covered (%)
0	12.5
24	25

Numerical Response

8. If the mould continues to grow at the same rate, the petri dish will be covered with mould after _____ hours.

(Record your answer in the numerical-response section on the answer sheet.)

Solution:

If the mould continues to double every 24 hours, the chart could be completed as follows:

Time (h)	Area covered (%)
0	12.5
24	25
48	**50**
72	**100**

The petri dish will be covered with mould after 72 hours.

Use the following information to answer the next question.

The values of small SUVs as a function of age are shown in the table below.

Age of Vehicle (years)	Value of Vehicle ($)
2	16 000
3	12 000
5	7 800
6	6 200
7	4 500

9. These data could most appropriately be modelled using

A. a linear regression

B. a quadratic regression

C. a sinusoidal regression

D. an exponential regression

Solution:

Although the quadratic regression has a high r^2 value, it is not the most appropriate model. A quadratic function assumes that the data reaches a minimum value and then starts to increase, which is highly unlikely in this context. Generally, vehicles continue to depreciate over time, which is characteristic of an exponential function. Therefore, the correct response is D.

Use the following information to answer the next question.

The median price for houses in Calgary for each two-year period from 1988 to 1998 is given below.

Number of Years after 1988	**0**	**2**	**4**	**6**	**8**	**10**
Price ($)	127 526	127 621	132 868	134 643	157 353	176 316

These figures may be represented by an equation of the form $y = ax^2 + bx + c$.

Numerical Response

10. The value of a in the equation above, rounded to the nearest whole number, is _____.

(Record your answer in the numerical-response section on the answer sheet.)

Solution:

The median price for houses in Calgary over these 10 years can be expressed by a quadratic regression equation in the form

$$y = 733x^2 - 2\,545.41\ldots\, x + 128\,571.57\ldots$$

The value of a in this equation is 733.

Use the following information to answer the next question.

The population in a particular high school was tracked over time.

School Year	Population
0	621
1	852
2	
3	1 250
4	1 324
5	1 348

A student used a quadratic regression equation to represent these data.

11. The student was able to estimate that the population of the school in year 2 was

A. 941
B. 1 051
C. 1 083
D. 1 085

Solution:

These data can be modelled by the quadratic regression equation
$y = -29.84\ldots x^2 + 298.25\ldots x + 607.96\ldots$.

Graph this equation and use the CALC function on the graphing calculator for $x = 2$. This function indicates that $y = 1085.09\ldots$. There were approximately 1 085 students in the school in year 2.

Use the following information to answer the next question.

A group of biologists studied the population of wolves in an area in northern Canada. The biologists found that the number of wolves was directly related to the number of caribou found in the study area. The wolf population for the period of the study is shown in the table below.

Year	Wolf Population
Base year (0)	400
1	548
2	800
3	1 168

Written Response—10%

12. **a.** State the exponential regression equation for these data in the form $y = ab^x$. Round the value of a to the nearest whole number and the value of b to the nearest hundredth.

A POSSIBLE SOLUTION to part a

$y = 392\ (1.43)^x$

b. The biologists found that a herd of approximately 17 800 caribou was needed to sustain 650 wolves. If the wolf population continued increasing at the same rate as it did in years 0 to 3 of the study, how many caribou would be required to sustain the wolf population in year 4?

A POSSIBLE SOLUTION to part b

Using Rounded Values:

$y = 392\ (1.43)^4$
$y = 1639.193476$

$$\frac{17\,800}{650} = \frac{\text{caribou}}{1639.193476}$$

Caribou = 44 888.68288

The caribou should number 44 889.

Using Calculated Values:

Graph the exponential regression equation and use the CALC function to determine the value of y when $x = 4$.

$y = 1651.8014$

$$\frac{17\,800}{650} = \frac{\text{caribou}}{1651.8014}$$

Caribou = 45 233.94603

The caribou should number 45 234.

Use the following additional information to answer the next part of the question.

The area the biologists studied was approximately 14 500 km^2, and they found the maximum population density of wolves to be 18 wolves/100 km^2.

c. • What is the maximum number of wolves that this area can sustain?

A POSSIBLE SOLUTION to part c, bullet 1

$$\frac{14\,500}{100} \times 18 = 2\,610$$

The area can sustain 2 610 wolves.

• If the wolf population continued increasing at the same rate as it did in years 0 to 3 of the study, how much time will have elapsed when this maximum number of wolves is reached?

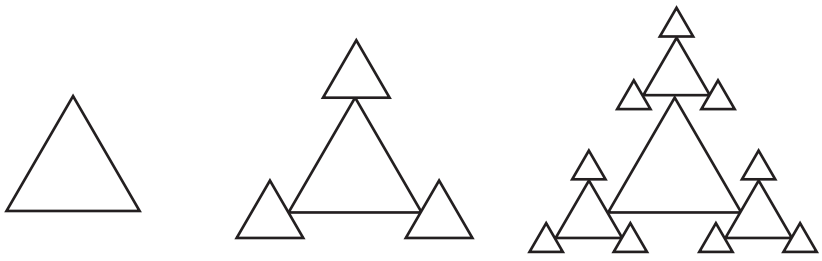
A POSSIBLE SOLUTION to part c, bullet 2

1. Graph $y = 392.44\ldots(1.43\ldots)^x$
2. Graph $y = 2610$
3. Use INTERSECT function on calculator
   when $y = 2\,610$, $x = 5.273\ldots$

The wolf population will reach the maximum sustainable population during year 5.

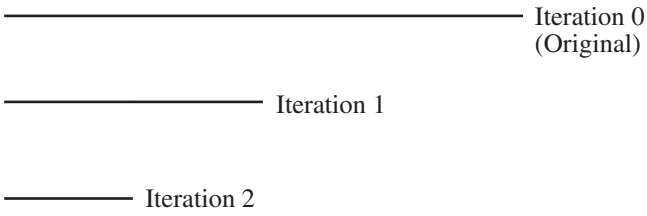
13. Determine which of the following patterns are examples of fractals. Justify your answer.

a.

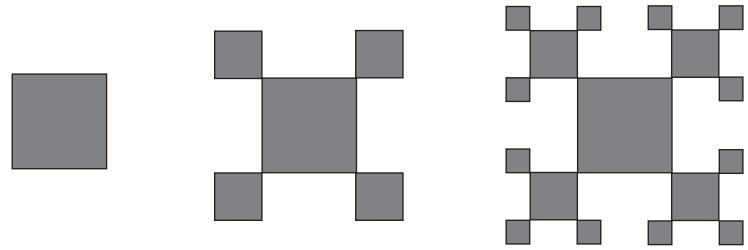


Iteration 0 (Original) Iteration 1 Iteration 2

b.



c.



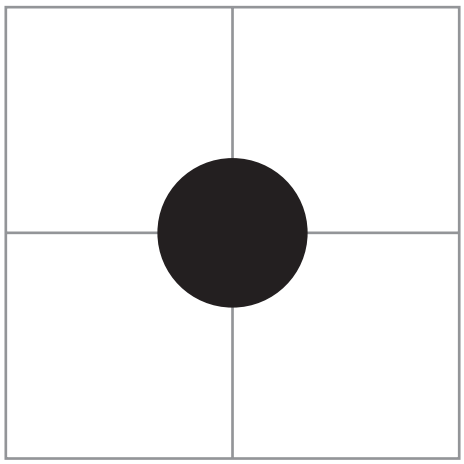
Iteration 0 (Original) Iteration 1 Iteration 2

Solutions:

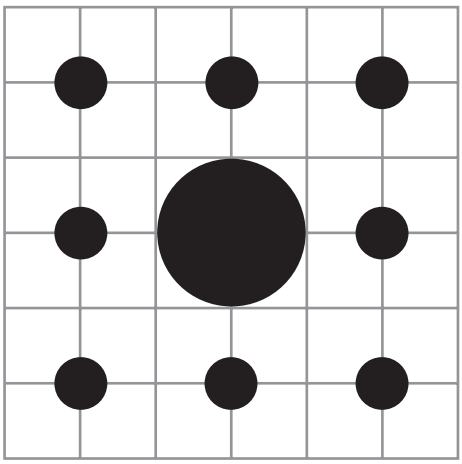
a. This is an example of a fractal pattern. Each iteration is more complex and exhibits self-similarity.
b. This is not a fractal pattern. Each iteration is not more complex than the previous one.
c. This is not a fractal pattern. The iterations do not exhibit self-similarity.

Use the following information to answer the next question.

The diagrams below show the first two iterations of a fractal pattern. In each consecutive iteration, 8 new circles are drawn around the circles created in the previous iteration.



Iteration **1**



Iteration **2**

14. The total number of **new** circles created in iteration 3 will be

A. 17

B. 64

C. 72

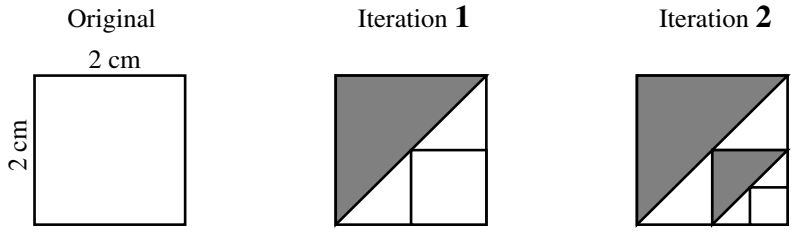
D. 81

Solution:

There were 8 new circles created in iteration 2. In iteration 3, each of these circles will have 8 new circles around it. This means that the total number of new circles created in iteration 3 will be 64.

Use the following information to answer the next question.

A student has a 2 cm by 2 cm square piece of paper. The student draws a diagonal of this square and shades the region of the square above this diagonal. In the unshaded region, the student then draws a second square by using the midpoint of the diagonal as one corner. This process produces iteration 1 shown below. The process is then continued with the smaller square, as shown in iteration 2.



Numerical Response

15. The total area of the shaded regions shown in the diagram for iteration 2, to the nearest tenth, is _____ cm^2.

(Record your answer in the numerical-response section on the answer sheet.)

Solution:

Area of larger shaded triangle: $A_1 = \frac{2 \times 2}{2} = 2 \text{ cm}^2$

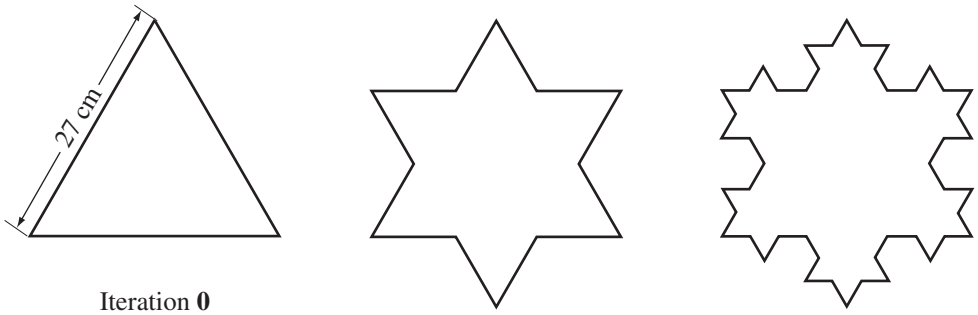
Area of smaller shaded triangle: $A_2 = \frac{1 \times 1}{2} = 0.5 \text{ cm}^2$

Total area: $2 \text{ cm}^2 + 0.5 \text{ cm}^2$

$= 2.5 \text{ cm}^2$

Use the following information to answer the next two questions.

The following diagrams show the original triangle and the first two iterations in the construction of a Koch snowflake. The original triangle is an equilateral triangle with 27 cm sides.



Iteration **0** (Original) Iteration **1** Iteration **2**

The following chart shows the relationship between the added perimeter and the total perimeter for the Koch snowflake.

Iteration	Added Perimeter (cm)	Total Perimeter (cm)
0	0	81
1	27	108
2	36	144
3	48	

Numerical Response

16. The total perimeter of iteration 3, to the nearest centimetre, will be _____ cm.

(Record your answer in the numerical-response section on the answer sheet.)

Solution:

Total perimeter $= 144 \text{ cm}^2 + 48 \text{ cm}^2$
$= 192 \text{ cm}^2$

Use the following additional information to answer the next question.

Exponential regression can be used to relate iteration number to **total** perimeter and to determine the perimeter of a particular iteration of this Koch snowflake.

17. Based on an exponential regression of the perimeters of iterations 0, 1, and 2, the perimeter, to the nearest centimetre, of iteration 10 will be

A. 1 918 cm

B. 1 081 cm

C. 1 438 cm

D. 123 cm

Solution:

Exponential regression equation: $y = 81(1.\overline{3})^x$

Iteration 10: $y = 81(1.\overline{3})^{10}$

$y = 1438.37\ldots\text{cm}$

Vectors

General Outcome

Solve problems involving polygons and vectors, including both 3-D and 2-D applications.

General Notes:

- Students should be reminded that their calculator needs to be in degree mode before doing problems in this unit. Some calculators default to radian mode when they are reset.
- Before starting the unit, it is strongly recommended that teachers review with their students Sine Law, Cosine Law, and right-angle trigonometry, and bridge the trigonometry problems to appropriate vector terminology and diagrams. Most problems are applications of the primary trigonometric ratios, the Cosine Law, and the Sine Law.
- The concept of angles related to transversals and parallel lines is crucial to succeed in this unit and will need to be reviewed.
- Students will need to understand the difference between scalar quantities, such as distance and speed, which identify only magnitude (e.g., 50 m, 100 km/h) and vector quantities, such as displacement and velocity, which identify direction as well as magnitude (e.g., 100 km/h in a direction 10° south of east).
- Angle measurements for direction of vectors will be dealt with in the context of the situation. Students should be prepared to work with both bearing notation (030° or 249° clockwise from north) and directional (heading) notation (W35°S means 35° south of west). We assume north to always be pointing to the top of the page. Since directional notation is not covered in the textbook, this area may need to be supplemented.
- Angles in standard position are beyond the scope of Applied Mathematics 30. Unless otherwise stated, angle measures should be stated to the nearest degree and resultant magnitudes to the nearest tenth.
- Application problems will be limited to displacement, velocity, and force.
- The mathematical process of visualization should be addressed when working with vectors. For example, when doing vector addition, students should draw the vectors. This can be done using technology and/or student modelling of vectors of displacement, velocity, or force. Students are allowed to use rulers and protractors on their diploma examination.
- Teachers may want to make a connection with Physics 20 and Science 20 concepts. However, some terminology and methods may differ.
- *Geometer's Sketchpad*, *Calculator-Based Rangers*, isometric dot paper, and engineering paper may all be helpful tools in teaching this unit.

Specific Outcomes

Specific Outcome 5.1
Use appropriate terminology to describe:
- vectors quantities
- scalar quantities [C, CN]

5.1 Notes:
- Use appropriate notation for identification of
  - vectors. Example: $\vec{a}$
  - magnitude of a vector. Example: $|\vec{a}|$
- Use a directed line segment of relative magnitude to draw vector diagrams.
- Appropriately label and use bearing notation and directional notation (headings) for angles in vector problems.

Refer to examples 1, 2, and 4.

Specific Outcome 5.2
Assign meaning to the multiplication of a vector by a scalar. [CN]

5.2 Note:
- Students should be encouraged to use visualization and scale models when working with vectors.

Refer to example 3.

Specific Outcome 5.3
Determine the magnitude and direction of a resultant vector, using triangle or parallelogram methods. [CN, R, T, V]

5.3 Notes:
- Students should be comfortable using both parallelograms (tail-to-tail) and triangles (head-to-tail) to solve problems involving vectors.
- When a force acts on a body, it should be represented by a vector whose initial point is the body and whose terminal point is in the direction that the force will move away from the body.
- Students will require a great deal of practice drawing appropriate vector diagrams and resultant vectors, both parallelograms (tail-to-tail) and triangles (head-to-tail).
- The intent of the study of vectors in Applied Mathematics 30 is that students become aware that vectors represent real values, and therefore, resultant vectors should make sense physically.

Refer to examples 5 and 9.

Specific Outcome 5.4

Model and solve problems in 2-D and simple 3-D, using vector diagrams and technology. [CN, PS, T, V]

5.4 Notes:

- Students should only be presented with illustrated 3-D problems that use multiple right triangles in two dimensions. Refer to *example 11* for clarification.
- Teachers may wish to use rectangular prisms to help students model 3-D vectors.
- Students need to be made aware that the trigonometric solution they calculate for a problem may need to be **adapted** to the context of the vector problem. (See *example 6*)

Refer to examples 6, 7, 8, 10, and 11.

Acceptable Standard	*Standard of Excellence*
The student can	The student can also
• identify properties of a parallelogram and use the properties to draw appropriate vector diagrams	
• construct an appropriate vector diagram from given information	
• describe vectors and scalar quantities by using appropriate terminology	
• describe an angle using a heading and a bearing	
• distinguish between a scalar quantity and a vector quantity	
• determine relative magnitude of vectors by comparing lengths	
• recognize that multiplying a vector by a positive scalar changes magnitude only	
• recognize that multiplying a vector by a negative scalar reverses direction	
• solve problems involving scalar multiplication	
• solve 2-D problems involving right triangles	
• solve 2-D problems involving oblique triangles in which a vector diagram is given	• solve 2-D problems involving oblique triangles without being given a parallelogram or head-to-tail diagram
• model 3-D problems	• model and solve 3-D problems • solve 3-D problems that have been represented by two 2-D diagrams (one in a horizontal plane and one in a vertical plane)
• participate in and contribute toward the problem-solving process for problems that require the analysis of vectors studied in Applied Mathematics 30	• complete the solution to problems that require the analysis of vectors studied in Applied Mathematics 30

Examples

Students who achieve the *acceptable standard* should be able to answer all the following questions, except for any part of a question labelled **SE**. Parts labelled **SE** are appropriate examples for students who achieve the *standard of excellence*.

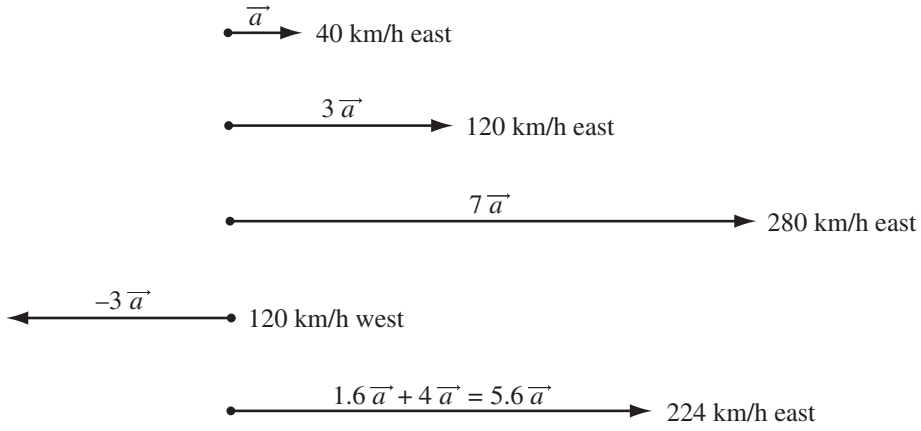
1. A vector $\vec{a}$ represents a velocity of 40 km/h east. Make a scale drawing of each of the following vectors.

a. $3\vec{a}$
b. $7\vec{a}$
c. $-3\vec{a}$
d. $1.6\vec{a} + 4\vec{a}$

What velocities do each of these vectors represent?

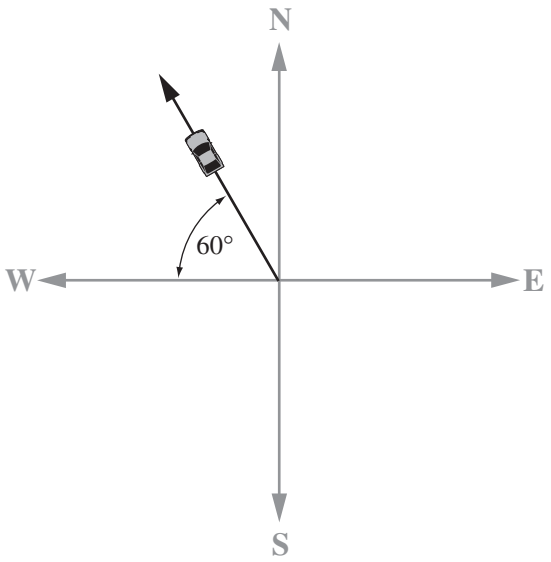
Solution:

Scale used: 1 cm = 40 km/h



Use the following information to answer the next question.

The path of a particular vehicle is shown below.



2. The direction in which the vehicle is travelling is

A. N30°W

B. N60°W

C. on a bearing of 60°

D. on a bearing of 300°

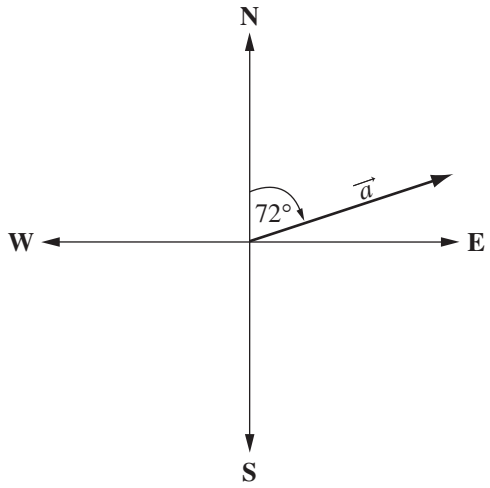
Solution:

The vehicle is travelling in a direction of 30° west of north, which can be expressed as N30°W. The direction could also be expressed as 60° north of west.

Note: For C, "on a bearing of 60°" could also be written as [060°], and D could also be written as [300°].

Use the following information to answer the next question.

Vector $\vec{a}$ has a bearing of $72°$, as shown in the diagram below.



3. If vector $\vec{a}$ is multiplied by a negative scalar, then the new vector will have a bearing of

A. 108°
B. 144°
C. 252°
D. 288°

Solution:

The new vector will have a bearing of $72° + 180° = 252°$.

4. Which of the following statements describes a vector quantity?

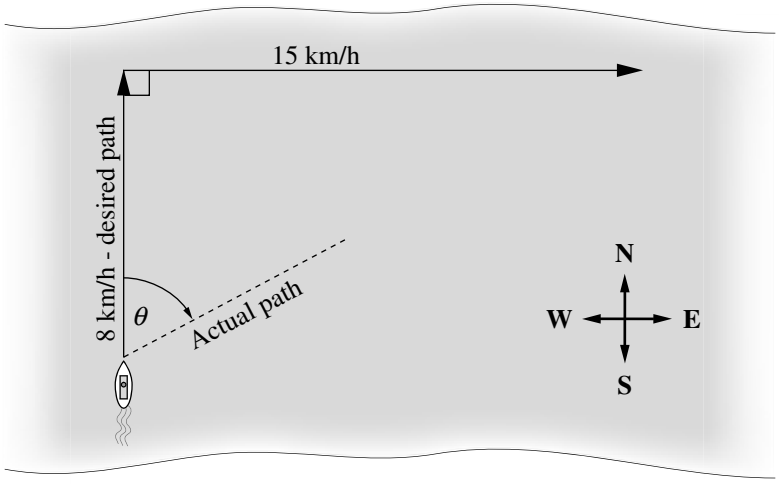
A. A car travelled north.
B. A car travelled at 100 km/h.
C. A car travelled north at 100 km/h.
D. A car travelled 200 km at 100 km/h.

Solution:

Vectors must have both magnitude and direction. The only response that meets these criteria is C.

Use the following information to answer the next question.

A person steers a boat north across a river at a speed of 8 km/h. The boat is being pushed off-course by a 15 km/h current flowing toward the east, as shown below.



5. *Relative to the shore, the actual speed of the boat is* ***i*** *km/h, and relative to the desired path, the boat is travelling off-course by an angle, θ, of* ***ii***.

The statement above is completed by the information in row

Row	*i*	*ii*
A.	17	62°
B.	17	28°
C.	13	62°
D.	13	28°

Solution:

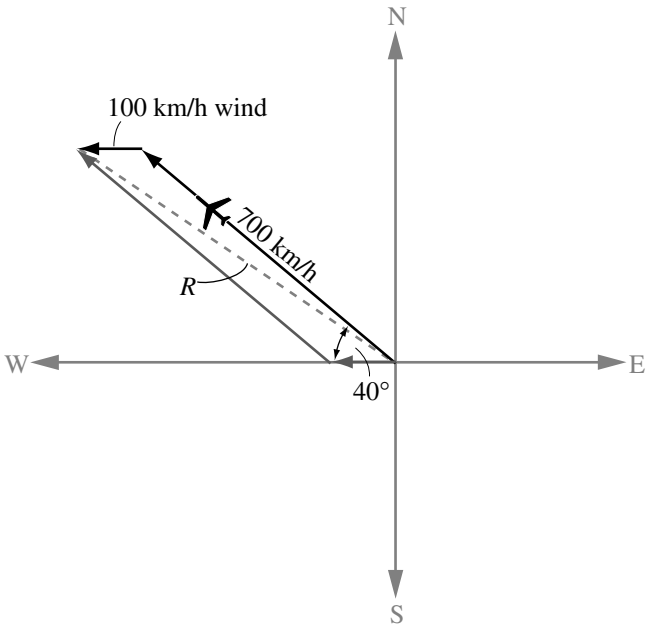
$$\sqrt{8^2+15^2} = \text{actual path } (i)$$
$$17 = i$$

$$\tan\theta = \frac{15}{8}$$
$$\theta \doteq 62^\circ$$

Relative to the shore, the actual speed of the boat is 17 km/h, and relative to the desired path, the boat is travelling off-course by an angle, θ, of 62°.

Use the following information to answer the next question.

A 737 aircraft is set on a course at a bearing of 310° with an air speed of 700 km/h. It is being affected by a 100 km/h wind blowing from the east, as shown in the diagram below.



6. The distance, R, relative to the ground, that the plane has travelled after 2 hours is, to the nearest kilometre,

A. 1 254 km

B. 1 280 km

C. 1 414 km

D. 1 559 km

Solution:

The angle between the two labelled vectors is 140°.

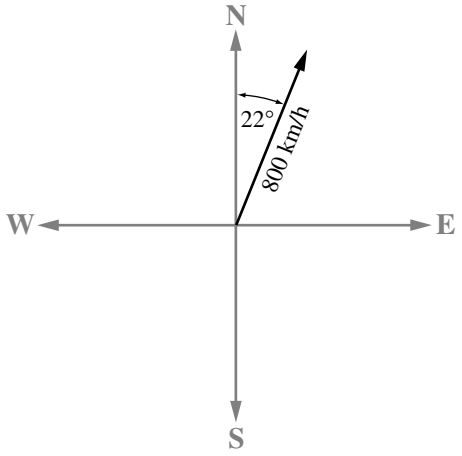
$R^2 = 700^2 + 100^2 - 2(700)(100)(\cos 140°)$

$R = 779.2600\ldots$

After 1 hour, the plane has travelled 779.26… km, so after 2 hours it has travelled 1 559 km.

Use the following information to answer the next question.

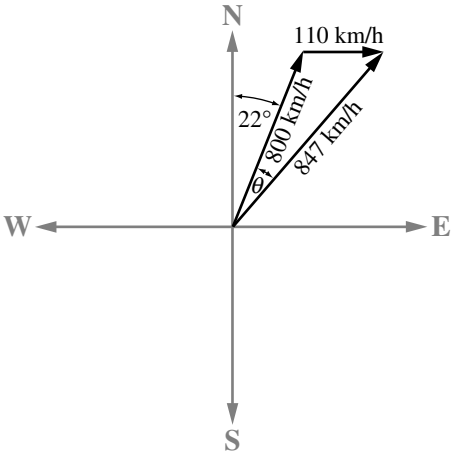
An aircraft sets a course with a bearing of 22° and a speed of 800 km/h, as shown in the diagram below. A horizontal wind blowing from the west at a speed of 110 km/h causes the aircraft to fly off-course with a speed of 847 km/h by an angle θ.



SE **7.** The angle, θ, by which the aircraft is flying off-course is, to the nearest degree,

A. 7°
B. 22°
C. 61°
D. 68°

Solution:

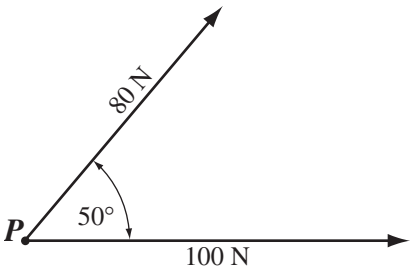


$$\cos\theta = \frac{847^2 + 800^2 - 110^2}{2(847)(800)}$$
$$\cos\theta = 0.9927...$$
$$\theta = 6.9266...$$

The aircraft is flying off-course at an angle of 7°.

Use the following information to answer the next question.

Two forces act on a point P. The first force has a magnitude of 100 N. The second force has a magnitude of 80 N and is at an angle of 50° to the first force, as shown below.

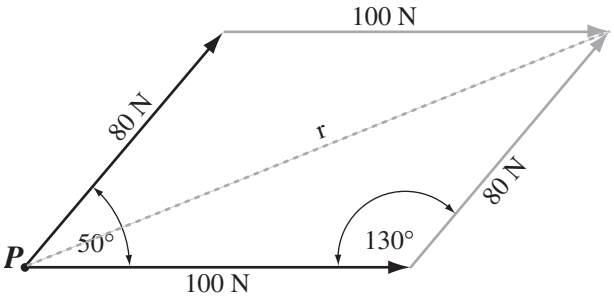


SE **8.** The magnitude of the resultant force, to the nearest newton, is

A. 31 N
B. 128 N
C. 163 N
D. 180 N

Solution:

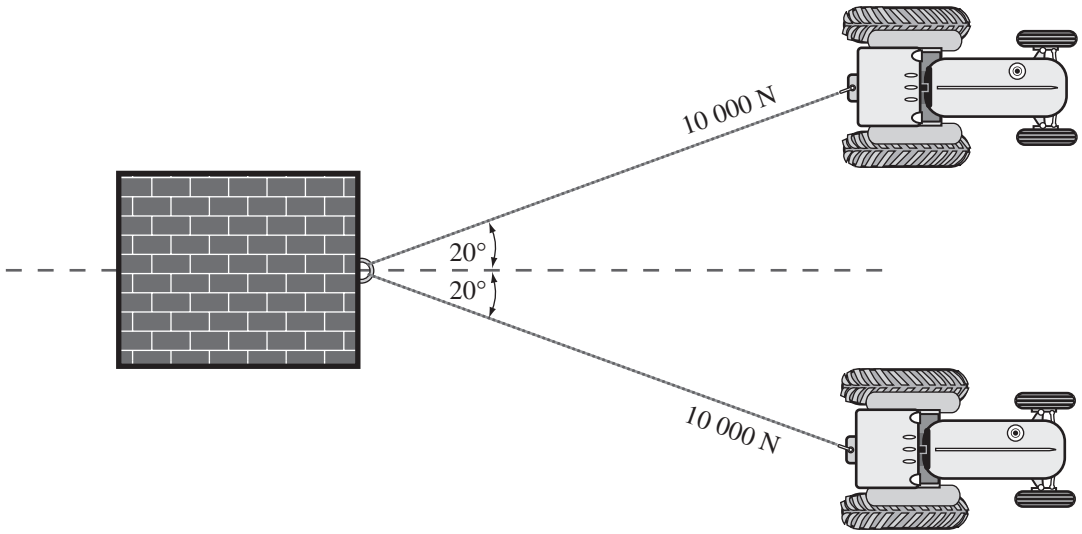
Sketch an appropriate parallelogram diagram and solve for r.



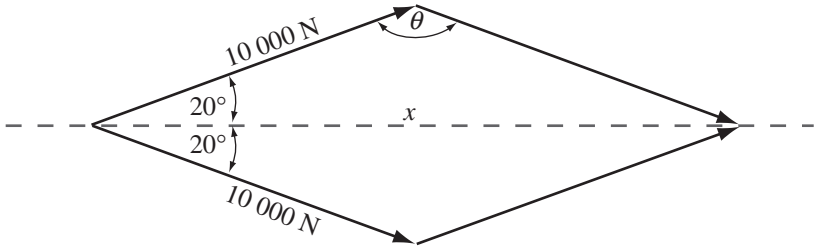
$r^2 = 100^2 + 80^2 - 2(100)(80)\cos 130^\circ$
$r^2 = 26\,684.601\,75$
$r = 163.354\,2\ldots$
The magnitude of the resultant force is 163 N.

Use the following information to answer the next question.

Two tractors pull on a large block with a force of 10 000 N each. Both of the tractors pull at a 20° angle from the direction the block is to move in, as shown below.



The force exerted by the tractors can be modelled by the vector diagram shown below.



Written Response—5 marks

9. **a.** Determine the measure of angle θ in the vector diagram.

A POSSIBLE SOLUTION to part a.

$\theta = 180 - 40$
$\theta = 140^{\circ}$

b. Calculate the magnitude, x, of the resultant force that the tractors exert on the block.

A POSSIBLE SOLUTION to part b.

$$x = \sqrt{10\,000^2 + 10\,000^2 - 2(10\,000)^2 \cos 140^\circ}$$

$$x = 18\,793.85$$

The tractors exert a force of 18 793.9 N.

c. If a force of 18 000 N is required to move the block, will the tractors be able to do so? Explain and justify your answer mathematically.

A POSSIBLE SOLUTION to part c.

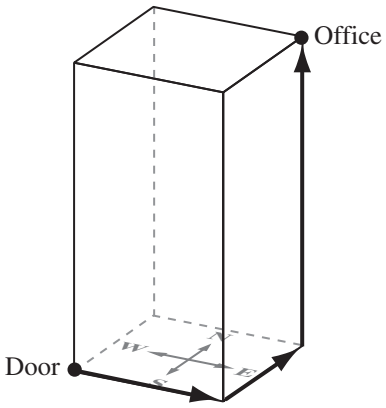
Yes, the tractors exceed the required force by 793.9 N.

Use the following information to answer the next question.

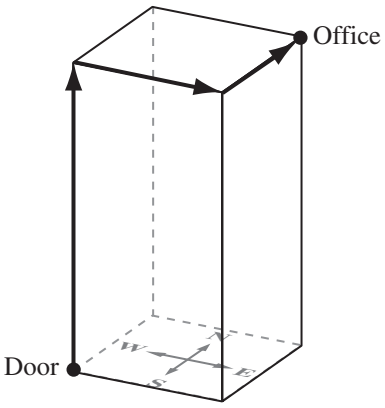
As Norma walks into her office tower, she is facing north. She walks due east from the front door to the elevator. She takes the elevator up 10 floors, then walks due north to her office.

10. Which of the following 3-D diagrams could be used to model Norma's path?

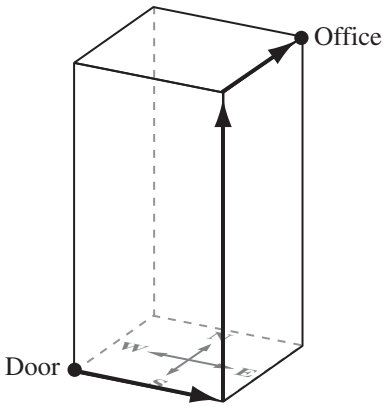
A.



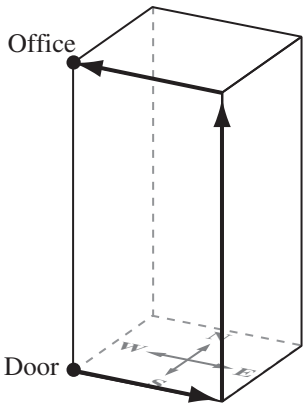
B.



C.



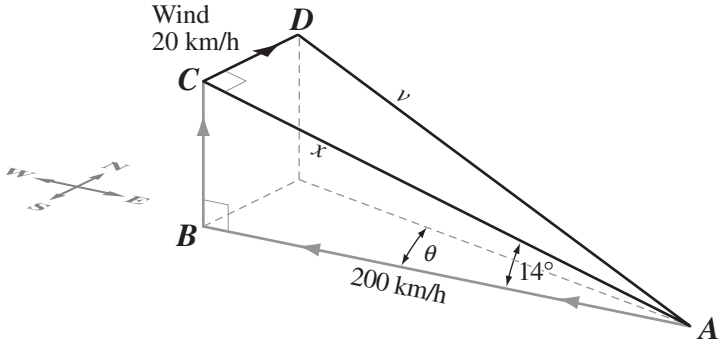
D.



Solution:

The 3-D diagram that correctly illustrates Norma's path is C.

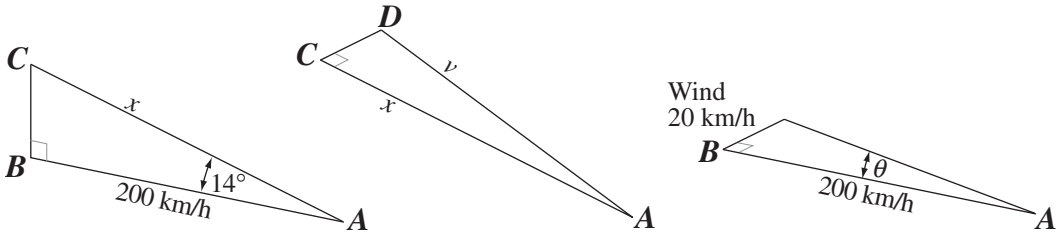
SE **11.** The pilot of an aircraft sets a course to fly with a ground speed of 200 km/h due west while climbing at an angle of 14° to the horizontal. From the time of takeoff, the aircraft is affected by a horizontal 20 km/h wind blowing from the south, as shown in the diagram below.



a. Model this problem using 2-D diagrams.
b. Determine the resultant velocity of the aircraft.
c. Approximately what course correction should the pilot set to remain on track?

Solutions:

a.



b.

$$\cos 14° = \frac{200}{x} \qquad \tan\theta = \frac{20}{200}$$

$$x = \frac{200}{\cos 14°} \qquad \theta = 5.71\ldots°$$

$$x = 206.1227\ldots \text{ km/h}$$

$$v = \sqrt{x^2 + (20)^2}$$

$$v = \sqrt{(206.1227...)^2 + (20)^2}$$

$$v = 207.09...\text{km/h}$$

The resultant air speed of the aircraft is 207.1 km/h, and the aircraft is flying on a bearing of 276°.

c. To correct the path, the pilot should set the course correction on a bearing of 264°.

Design

General Outcome

Analyze objects, shapes and processes to solve cost and design problems.

General Notes:

- The objectives for this unit are covered in the project book, **not** the textbook.
- Students have done considerable work with perimeter, area, scaling, and volume in Applied Mathematics 10 and 20.
- This unit should concentrate on the use of dimensions, unit prices, and estimation to cost projects.
- This unit should include projects wherein a design is given, projects wherein a design is not given, projects wherein a budget is specified, and projects wherein a budget is not specified.
- Generally, the difference between the student at the *acceptable standard* and the student at the *standard of excellence* is the quality of the solution and the extent to which mathematical processes are demonstrated.
- Outcomes 6.1 and 6.2 are primarily review and are not intended to be the main focus of this unit. They are more of a preparation for outcomes 6.3 and 6.4.
- Be aware of the amount of time spent on this unit—the general outcome is large because of its exploratory nature.
- Students should be reminded to evaluate the reasonableness of their answers.

Specific Outcomes

Specific Outcome 6.1
Use dimensions and unit prices to solve problems involving perimeter, area and volume. [E, PS, V]

Refer to examples 3, 4, and 5.

Specific Outcome 6.2
Solve problems involving estimation and cost for objects, shapes or processes when a design is given. [C, E, PS]

Refer to examples 1 and 6.

Specific Outcome 6.3
Use appropriate variables to design an object, shape, layout or process within a specified budget. [C, PS, R, V]

6.3 Note:
- In this outcome, skills from many other areas may be required. Some problems may require various function types on the regression menu of the graphing calculator. It is better to do a few large design projects rather than many small questions.

Refer to example 8.

Specific Outcome 6.4
Use mathematical models to estimate the solutions to complex measurement problems. [E, V]

Refer to examples 2 and 7.

Acceptable Standard	*Standard of Excellence*
The student can	The student can also
• calculate perimeter, surface area, and volume, and can determine costs of composite designs when calculations (i.e., Pythagorean theorem, trigonometry, solving for any variable in familiar formulas, etc.) are required to find values of necessary dimensions	
• draw sketches and scale diagrams in 2-D from various views when a written description of a design is given	• draw sketches and scale diagrams in 3-D from various views when a written description of a design is given
• solve cost and estimation problems related to perimeter, surface area, and volume consistent with a diagram he or she has drawn	• communicate effectively the meaning from his or her solutions and justify the procedures used
• design objects based on previously learned mathematics by using given information	
• make allowances for changes in given problems	
• describe an estimation process that yields a reasonable result	• design and communicate the most efficient process to yield a desired result
• participate in and contribute toward the problem-solving process for problems that require the analysis of design studied in Applied Mathematics 30	• complete the solution to problems that require the analysis of design studied in Applied Mathematics 30

Examples

Students who achieve the *acceptable standard* should be able to answer all the following questions, except for any part of a question labelled **SE**. Parts labelled **SE** are appropriate examples for students who achieve the *standard of excellence*.

The following example could be used as a project.

1. A manufacturer of cylindrical cans uses tin plate that comes in sheets that are 240 cm by 160 cm and that cost \$3.20 per sheet. Cans are 6 cm in diameter and 11 cm high, and they have 3 seals each. Seals cost 0.8¢ each to make. All costs include GST.

Currently, the manufacturer uses two sheets to make the sides and one sheet to make the end pieces.

a. How many ends can be made from one sheet?

b. How many sides can be made from two sheets?

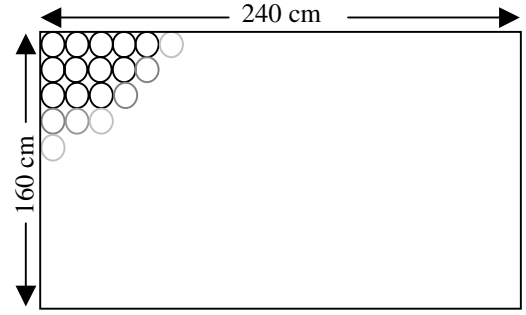
c. Use your answers from part a and part b to determine how many cans can be made from three sheets.

d. How much does it cost to make the cans using this method? What is the cost per can?

SE **e.** Design another method for using the three sheets to produce these cans. Is it more or less efficient than the current one? Explain.

Solutions:

a.



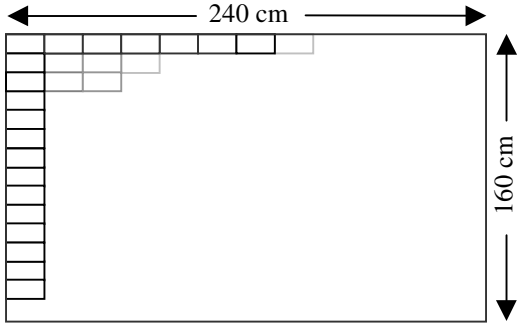
$\frac{240}{6} = 40$

$\frac{160}{6} = 26.\overline{6}$

$26 \times 40 = 1\,040$ ends

This is enough for 520 cans.

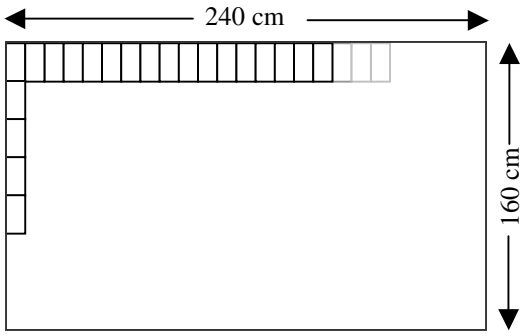
b. Since the cylindrical can has a circumference of 6π cm and a height of 11 cm, each side can be made using rectangles that are 6π cm $\times$ 11 cm. There are two ways that these rectangles can be placed on the sheet of tin. The first way is with the longer sides of the rectangles along the longer side of the sheet, as shown below.



With this method, the following number of sides can be made.

$$\left.\begin{array}{l}\dfrac{160}{11} \doteq 14.5 \\ \dfrac{240}{6\pi} \doteq 12.7\end{array}\right\} \quad \begin{aligned} 14 \times 12 &= 168 \text{ sides/sheet} \\ \times 2 &\text{ sheets} \\ &= 336 \text{ sides can be made}\end{aligned}$$

The other way is to put the longer sides of the rectangles along the shorter side of the sheet, as shown below.



$$\left.\begin{array}{l}\dfrac{160}{6\pi} \doteq 8.5 \\ \dfrac{240}{11} \doteq 21.8\end{array}\right\} 8 \times 21 \times 2 = 336$$

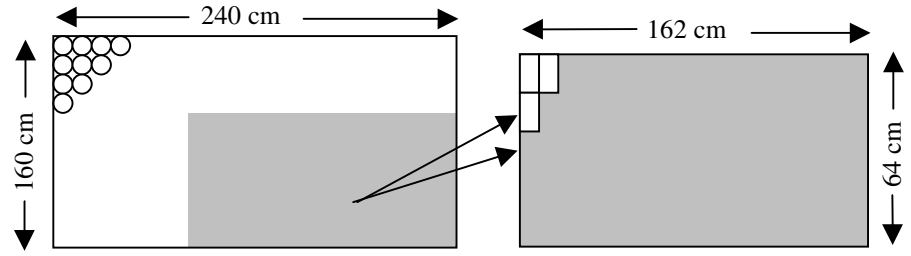
With this method, 336 sides can also be made.

c. There are enough ends to make 520 cans but only enough sides to make 336; therefore, 336 cans can be made from three sheets.

d. $C = \$3.20 \times 3 + 3 \times 336 \times \$0.008$
$\doteq \$9.60 + \$8.06$
$\doteq \$17.66$ to make 336 cans
The cost per can is approximately \$0.052.

SE **e.** Since a significant amount of the sheet used for the ends is not used in the above method (i.e., only 672 ends are needed for the 336 cans), part of that sheet could be used to make more sides.

One possible method is shown below.



$\left.\begin{array}{l}\frac{240}{6} = 40 \\ \frac{160-64}{6} = 16\end{array}\right\}$ $40 \times 16 = 640$ ends

$\left.\begin{array}{l}\frac{240-162}{6} = 13 \\ \frac{64}{6} \doteq 10.7\end{array}\right\}$ 130 ends

$\left.\begin{array}{l}\frac{162}{6\pi} \doteq 8.5 \\ \frac{64}{11} \doteq 5.8\end{array}\right\}$ $8 \times 5 = 40$ sides **or** $\left.\begin{array}{l}\frac{162}{11} \doteq 14.7 \\ \frac{64}{6\pi} \doteq 3.4\end{array}\right\}$ $14 \times 3 = 42$ sides

From the unused part of the sheet, you can make an additional 42 sides and an additional 98 ends. This will make a total of 378 sides and a total of 770 ends.

This means a total of 378 cans can be made if you use this method.

The cost for this method is as follows:

Cost for sheets	$\$3.20 \times 3 = \$9.60$
Cost for seals	$\$0.008 \times 3 \times 378 \doteq \$9.07$
Total cost	\$18.67
Cost per can	$\frac{\$18.67}{378} \doteq \$0.049$

The cost is about the same for this method. However, since this method costs slightly less, over time, the difference in cost may prove significant.

The following example could be used as a project.

2. A window cleaner has been asked by the management of a large office building to submit a quotation for cleaning the building's windows. The office building has

- 24 floors
- 14 windows per floor, on each side of the building
- 4 sides

To clean the windows, the cleaner starts at the top of one column of windows and works his way to the bottom of that column, cleaning each window on the way down. He then goes back to the top, moves to the second column, and repeats the process. He does this for each side of the building.

From experience, the cleaner knows that it takes

- 120 seconds to clean one window
- 30 seconds to transfer between floors on the same column
- 120 seconds to go back to the top of the building once he has reached the bottom
- 60 seconds to transfer from one column to the next on the same side of the building
- 120 seconds to transfer from one side of the building to the next

a. How many hours will it take to wash all the windows in the building?

b. For every three hours that the window cleaner works, he takes a half-hour break. How many hours will it be before he finishes the entire building?

c. The window washer charges $25 per hour (including the half-hour breaks). He has a base charge of $120, and he wishes to add an additional 10% to the cost of this project so that he can reinvest in his business. Determine the quotation he should give the management of this building. All amounts include GST.

Solutions:

a. Determine the time required to complete one side of the building.

First, determine the time required to complete one column of windows.

24 windows @ 120 seconds	= 2 880 seconds
24 floor transfers @ 30 seconds	= 720 seconds
Total	= 3 600 seconds or 1 hour

Since there are 14 columns of windows per side, it will take 14 hours to wash the windows on one side.

Add in the time required to transfer from one column to the next.

13 transfers between columns @ 60 seconds = 780 seconds
(13 min or $0.21\overline{6}$ h)

Add in the time required to go back to the top.

14 climbs @ 120 seconds = 1 680 seconds
(28 min or $0.4\overline{6}$ h)

Total time:	14	h
	$0.21\overline{6}$	h
	$0.4\overline{6}$	h
Time for one side:	$14.68\overline{3}$	h

Determine the time required to complete all four sides.

$14.68\overline{3}$ h $\times$ 4 sides = $58.7\overline{3}$ h
120 seconds to transfer between sides $\times$ 3 = 360 seconds (6 min or 0.1 h)

Therefore, it will take $58.8\overline{3}$ hours to wash all the windows in the building.

b. $\frac{58.8\overline{3}}{3} = 19.6\overline{1}$

Therefore, the cleaner will take 19 breaks as the job will be completed before the time required for the 20th break. 19 breaks @ 0.5 h each is 9.5 h extra.

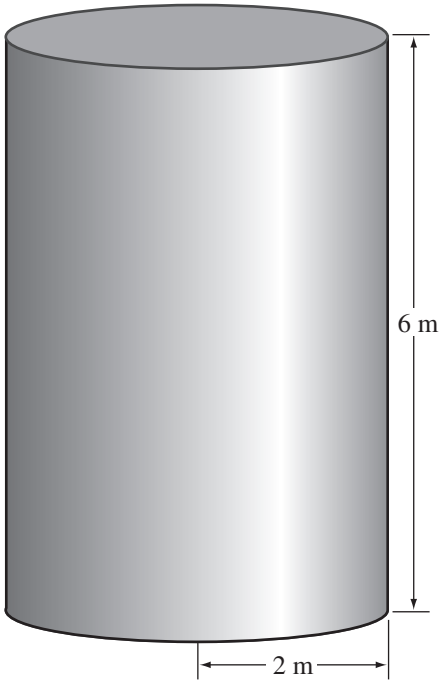
The washer will need $58.8\overline{3} + 9.5 = 68.\overline{3}$ h to complete the project.

c. $\$120 + \$25 \times 68.\overline{3} \doteq \$1\,828.33$
$\$1\,828.33 \times 1.10 \doteq \$2\,011.17$

The washer should quote the building manager \$2 011.17.

Use the following information to answer the next question.

A cylindrical storage tank that measures 6 m tall and has a radius of 2 m is going to be covered with sheet metal.



3. If sheet metal costs $\$2.08/m^2$, then the cost to cover the entire cylinder, including the top and bottom, will be

A. \$75.40
B. \$156.83
C. \$182.97
D. \$209.10

Solution:

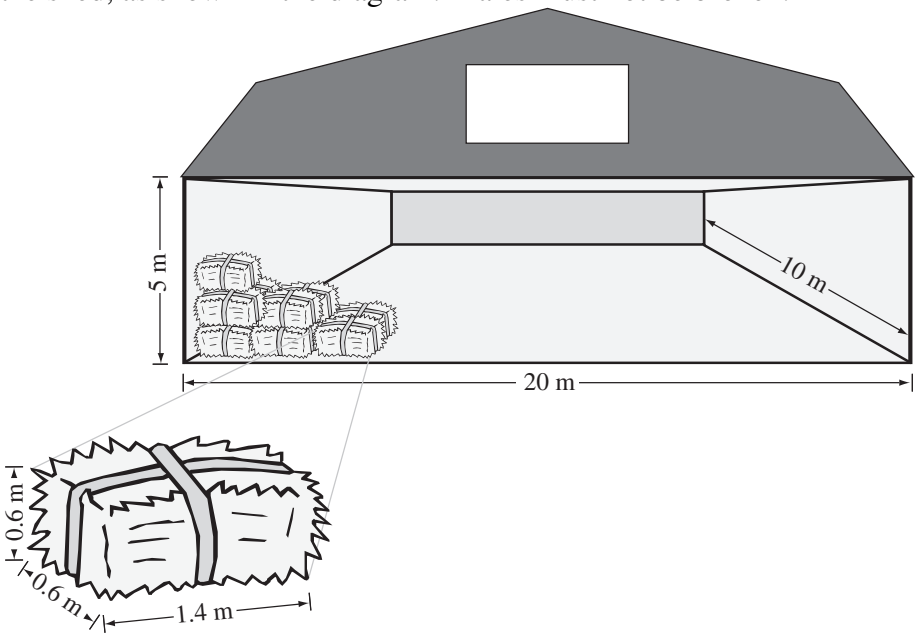
Surface Area of a Cylinder = $2\pi r^2 + 2\pi rh$

$$SA = 2\pi (2)^2 + 2\pi(2)(6)$$
$$SA = 32\pi \text{ m}^2$$

Cost of sheet metal = 32π (\$2.08)
= \$209.10

Use the following information to answer the next question.

The usable space in a farm storage shed has the dimensions $20\text{ m} \times 10\text{ m} \times 5\text{ m}$. The straw bales to be stored in the shed each measure $1.4\text{ m} \times 0.6\text{ m} \times 0.6\text{ m}$. The bales are to be stacked in such a way that the long side of the bale runs parallel to the long side of the shed, as shown in the diagram. Bales must not be broken.



4. The maximum number of bales that can be stored in this shed is

A. 1 120
B. 1 792
C. 1 982
D. 1 984

Solution:

$$\frac{20}{1.4} = 14.285...$$

$$\frac{10}{0.6} = 16.66...$$

$$\frac{5}{0.6} = 8.33...$$

One can stack 14 bales across, 16 bales back, and 8 bales up. The maximum number of bales that can be stored in this shed is 1 792.

Use the following information to answer the next question.

The surface area to be repainted on an older model car is $17\ m^2$. The paint costs $42.50 per can, including tax, and each can will cover $2.75\ m^2$. Paint must be purchased in whole cans.

5. The total cost to purchase the number of cans of paint required is

A. $297.50

B. $263.50

C. $262.73

D. $240.83

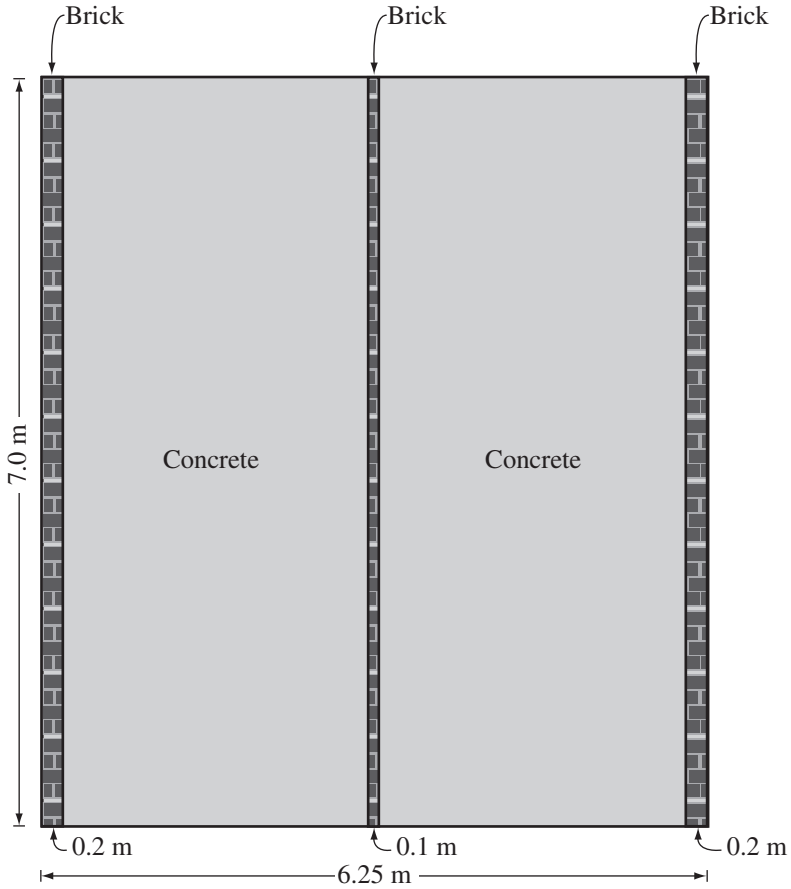
Solution:

$\frac{17\ m^2}{2.75\ m^2/can} = 6.18\ldots$ cans of paint are required, so purchase 7 cans of paint.

Each can costs $42.50, so the total cost to purchase enough paint is $297.50.

Use the following information to answer the next question.

The Hacketts are planning to pour a concrete driveway. The driveway will be 6.25 m by 7.0 m by 0.10 m deep, and the concrete pads will be bordered with brick, as shown in the diagram below.



To determine the cost of the concrete, the Hacketts must first calculate its **volume**.

6. If concrete costs \$142.25/m^3, the total cost of the concrete portion of the driveway will be

A. \$572.56

B. \$531.66

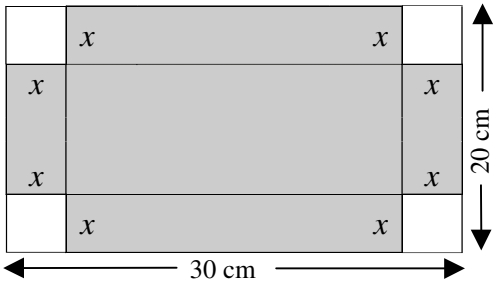
C. \$286.28

D. \$124.77

Solution:

Volume = (6.25 – 0.2 – 0.2 – 0.1)(7.0)(0.10)
Volume = 4.025 m^3
Total cost = 4.025 m^3 × \$142.25/m^3
= \$572.56

7. A rectangular container is constructed out of a sheet of tin that measures 30 cm × 20 cm. Squares of equivalent sizes are removed from each corner, and then the ends are folded up.



a. Determine an expression for the
- width of the container
- length of the container
- height of the container

b. The formula $V = x(20 - 2x)(30 - 2x)$ can be used to calculate the volume of the container. How do your expressions from part a relate to this formula?

c. Determine an appropriate window setting to graph this function given the context of this problem.

d. Graph the function in this viewing window and determine the value for x that will give the maximum volume. State the maximum volume.

SE **e.** Determine an expression for the surface area of the box.

SE **f.** The person using the container wants to maximize its volume while keeping the surface area to a relative minimum. Discuss the values for x that you would choose in order to meet these conditions.

Solutions:

a.
- width of the container $30 - 2x$
- length of the container $20 - 2x$
- height of the container x

b. The expressions from part a are multiplied together to determine the volume, V.

c. The values for x relate to the length of the sides of the rectangle. Therefore, 0 cm to 10 cm would be a reasonable choice because the length removed must be smaller than 20 cm to result in a container.

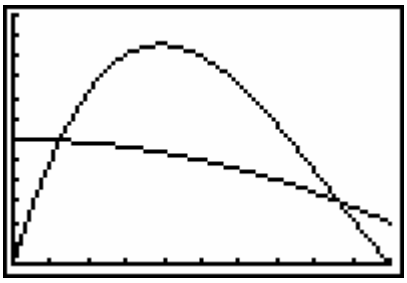
The volume will be greater than 0 cm^3. A maximum value of 1 200 seems reasonable. The viewing window should be x: [0, 10, 1] and y: [0, 1 200, 100].

d.

According to the graph, when $x \doteq 3.9$, the maximum volume is approximately $1\ 056\ \text{cm}^3$.

SE **e.** $A = 2x(20 - 2x) + 2x(30 - 2x) + (20 - 2x)(30 - 2x)$

SE **f.** If the function for surface area is graphed on the same grid as the volume function, then there is no minimum surface area.

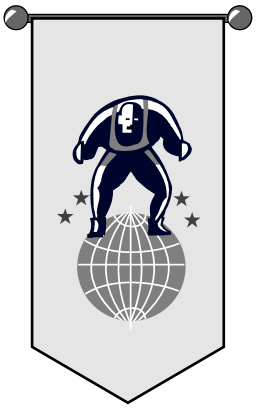


Since the two functions intersect at $x \doteq 1.23$ and $x \doteq 8.62$, and since the maximum volume is between these two values, any value for x that is between 1.23 cm and 8.62 cm would work well. Values less than 1.23 cm result in the volume being too small, as do values greater than 8.62 cm.

Note: This question can be solved many ways. Some alternate strategies include setting up a table and using regression, or setting up a spreadsheet.

Use the following information to answer the next question.

A student has been given a budget of $120 to make a banner acknowledging the victory of the school's wrestling team in the district playoffs.



The combined cost of the material and the dowel is $86.59, and the cost of the logo is $20.00.

Numerical Response

8. If lettering, priced at $0.23 per letter, is also to be placed on the banner, then the maximum number of letters that the student can buy is _____ letters.

Solution:

	$120.00	
–	86.59	Cost of material and dowel
–	20.00	Cost of logo
	$13.41	Amount remaining for lettering

$$\frac{\$13.41}{\$0.23} = 58.304\ldots$$

Therefore, the student can buy a maximum of 58 letters.