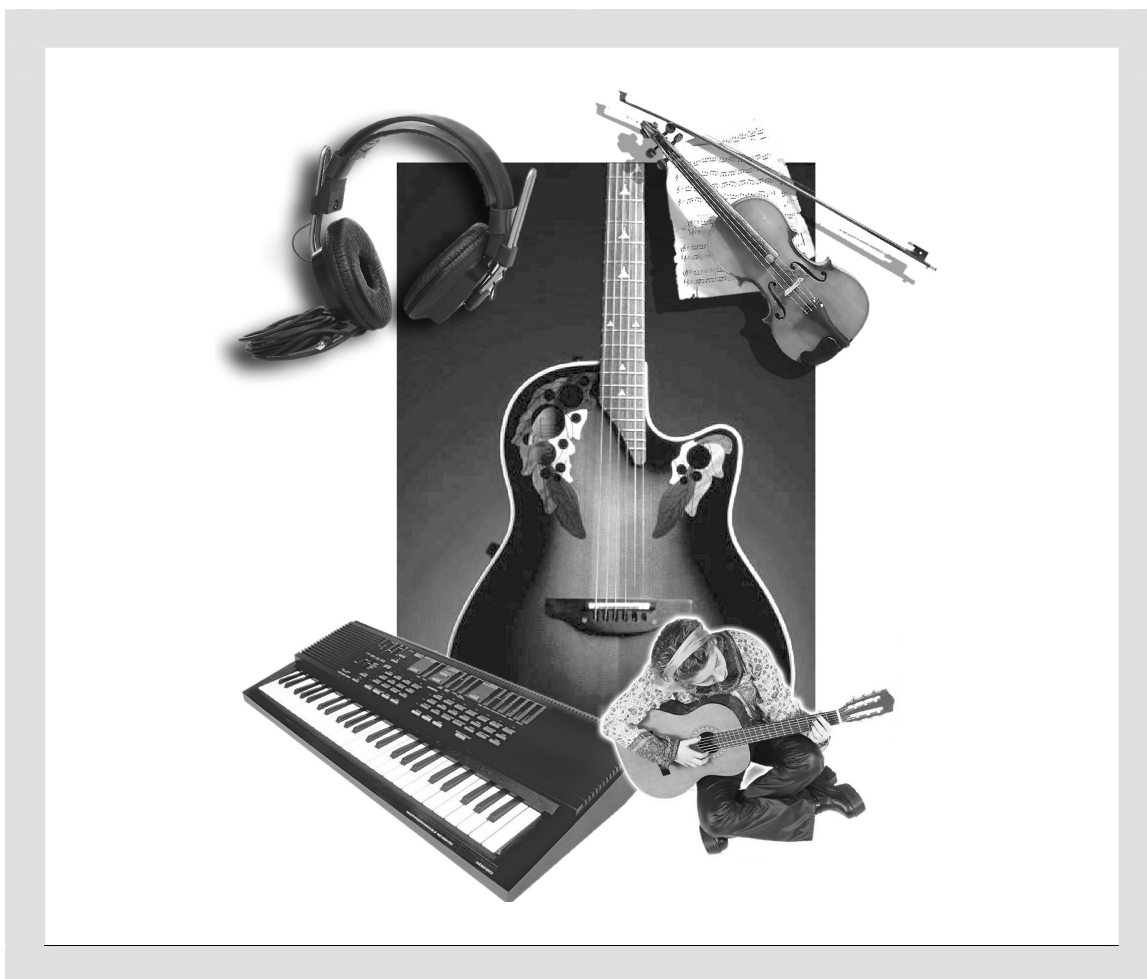


Pure Mathematics 30

**Teacher Notes:
Mathematics and Music**



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Pure Mathematics 30

Mathematics and Music—Teacher Notes

Introduction

This project has students work with various mathematical concepts and equations connected to several aspects of music theory involving instruments such as the piano, violin, and guitar. Students will need to use their knowledge from the Exponents, Logarithms, and Geometric Series Unit, the Trigonometry Unit, and the Permutations and Combinations Unit. This project is designed to be completed in three to five hours of student time. The use of this project is optional; however, if you do choose to use it, you may include it as a part of your assessment. Sample solutions for the project questions can be found on the Alberta Learning extranet <https://phoenix.edc.gov.ab.ca>. A hard copy will be mailed to your school in late February. The general scoring guide for the project is the same as the one issued in September 2000.

The first written-response question, which is worth 10%, on the Pure Mathematics 30 June 2007 diploma examination will be related to this project. Students who do not complete the project but who have completed the course will have the knowledge to answer the written-response question; however, students who have completed the project will have had experience with the related mathematical skills in another context.

Specific Notes

Teachers may wish to

- review with students specific instructions on how to set up lists, use appropriate window settings, and develop an exponential regression equation on a graphing calculator (Parts A and B)
- discuss the relationship between period and frequency and how these values relate to the parameters of $y = a \sin(bt)$ (Part B)
- discuss the difference between discrete and continuous data and its connection to exponential equations and geometric sequence equations (Part A)
- indicate to students that there is sufficient space between frets to have two fingers placed on strings between the 1st and 2nd frets

Program of Studies

The project relates to mathematics learned in the following units of Pure Mathematics 30.

Exponents, Logarithms, and Geometric Series

- Specific Outcomes**
- 2.1: Derive and apply expressions to represent general terms and sums for geometric growth and to solve problems. [CN, R, T]
 - 2.2: Connect geometric sequences to exponential functions over the natural numbers. [E, R, V]
 - 2.4: Use the laws of exponents and logarithms to
 - solve logarithmic equations
 - simplify logarithmic expressions [R]
 - 2.6: Model, graph, and apply exponential functions to solve problems. [PS, T, V]
 - 2.7: Change functions from exponential form to logarithmic form and vice versa. [CN]
 - 2.8: Use logarithms to model practical problems. [CN, PS, V]

Trigonometry

- Specific Outcomes**
- 3.8: Draw (using technology), sketch, and analyze the graphs of sine, cosine, and tangent functions for
 - amplitude, if defined
 - period
 - domain and range
 - asymptotes, if any
 - behaviour under transformations [CN, T, V]
 - 3.10: Use sine and cosine functions to model and solve problems. [PS, R, V]

Permutations and Combinations

- Specific Outcomes**
- 5.1: Use the fundamental counting principle to determine the number of different ways to perform multistep operations. [PS, R]
 - 5.2: Determine the number of linear permutations of n objects taken r at a time, and use this to solve problems. [PS, R, V]
 - 5.3: Determine the number of combinations of n distinguishable objects take r at a time, and use this to solve problems. [PS, R, V]
 - 5.7: Solve probability problems using either permutations and combinations, or the fundamental counting principle. [E, PS, R]

ICT Program of Studies

C.1—Students will access, use, and communicate information from a variety of technologies.

Specific Outcome 4.2: Select information from appropriate sources, including primary and secondary sources.

C.6—Students will use technology to investigate and/or solve problems.

Specific Outcomes 4.1: Investigate and solve problems of prediction, calculation, and inference.

4.2: Investigate and solve problems of organization and manipulation of information.

4.3: Manipulate data by using charting and graphing technologies in order to test inferences and probabilities.

P.2—Students will organize and manipulate data.

Specific Outcome 4.1: Manipulate and present data through the selection of appropriate tools, such as scientific instrumentation, calculators, databases, and/or spreadsheets.

Mathematical Processes

The seven mathematical processes identified in the *Program of Studies* are addressed in this project in the following manner.

Communication	Explain the choice of model (exponential or geometric sequence) that describes the ratio between the successive pitches of a piano. Describe the relationship between frequency and period. Explain why ${}_5C_4$ supports a choice process.
Connections	Understand the connections between exponential and geometric sequence equations and the frequency progression of pitches on the piano keyboard. Understand the connections between the features of graphs representing pure sound waves and the parameters of their corresponding equations. Connect the concepts of permutations, combinations, and probability to the selection process of playing a guitar chord.
Estimation and Mental Mathematics	Check reasonableness of calculator solutions and relate these to actual values within the context of the problem.
Problem Solving	Develop graphs and equations related to geometrical musical patterns and sound waves. Rewrite an exponential equation in terms of the variable exponent. Apply concepts related to permutations, combinations, and probability in a music context of chord selection.
Reasoning	Decide which mathematical model best describes the frequency progression of pitches on a piano. Compare two values of r (ratio between successive half-notes). Provide support to the relationship between frequency and period of a sound wave. Decide on a method to describe the choices for playing a D-chord on the guitar.
Technology	Use a graphing calculator to enter lists (and graph these), develop exponential regression equations, use appropriate window settings, graph and analyze trigonometric functions. Use permutation and combination calculator key entries.
Visualization	Visualize the shape of an exponential growth curve and of a trigonometric function.