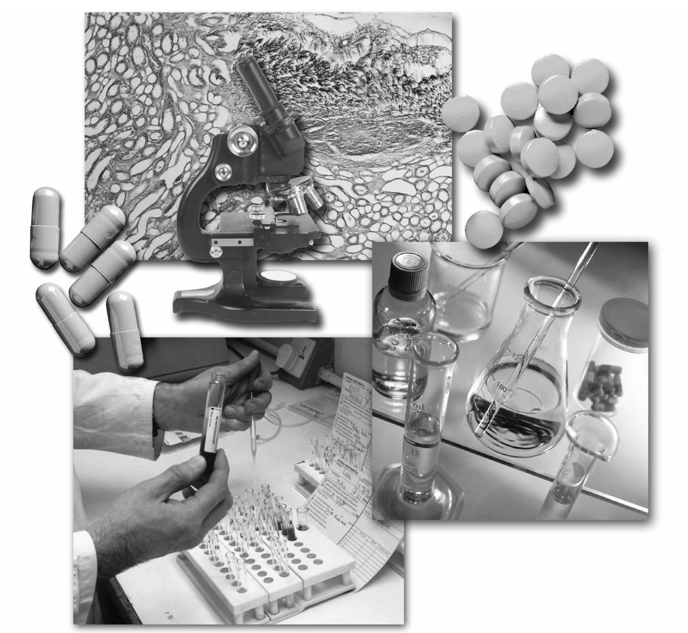
Pure Mathematics 30

Student Project: Applications of Exponential Functions



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Alberta
EDUCATION

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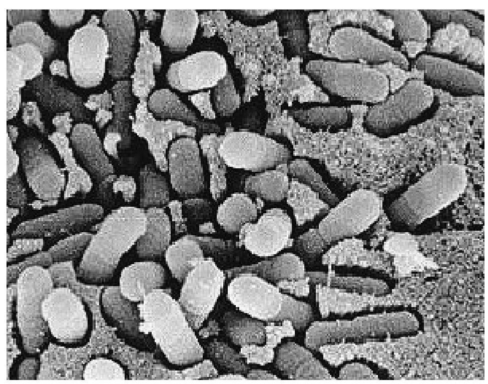
Project: Applications of Exponential Functions

Introduction

In his book *The Andromeda Strain* (1969), Michael Crichton warns that the uncontrolled growth of a single *E. coli* bacterium is a frightening phenomenon in that the exponential growth of a bacterial population, in ideal conditions, could produce a supercolony.

E. coli bacteria reproduce by simple cell division, which is also called “binary fission.” Under ideal conditions, a population of *E. coli* bacteria can double every 20 minutes. *E. coli* bacteria that live in the large intestine of humans are harmless, but when they are ingested, they can cause diarrhea, which leads to severe dehydration and even death. These harmful bacteria have been found in polluted water and in contaminated or undercooked food.

Escherichia coli



This project will explore the power of the exponential growth of bacteria, the effectiveness of antibiotics in treating bacterial infections, and other real-life applications of exponential functions.

Part A

1. Assume that a population of *E. coli* bacteria doubles every 20 minutes. Construct a table that shows the growth of a single *E. coli* bacterium for a 2-h period.

2. Write an equation for the number of *E. coli* bacteria, N, as a function of time, t.

3. Use your computer and/or calculator to model this exponential growth in the following ways. Construct three lists.

- In the first list (L_1), enter the numbers 0 to 8 to represent the time period where
  - 0 represents the initial stage at 0 min
  - 1 represents the time at 20 min
  - 2 represents the time at 40 min, and so on
- The second list (L_2) represents the number of bacteria present in each time period, so
  - 1 should be entered in the first space
  - 2 in the second space
  - 4 in the third space, and so on
- In the third list (L_3), enter the base 2 logarithm of list 2 (i.e., $\log_2 L_2$).

Graph L_2 as a function of L_1. Use the exponential regression function on your computer or calculator to find the equation of the graph. Compare this equation with the bacteria growth equation found in question 2. Do they differ? Explain why or why not.

4. *E. coli* is prevalent because of its rapid growth rate and because of its ability to mutate and produce variations that can survive in different environmental conditions. It is estimated that after a population of *E. coli* has divided 30 times, approximately 1.5% of the cells will be mutants. Using your L_2-as-a-function-of-L_1 graph, estimate the number of bacteria present after 30 doubling periods. How many of these bacteria, to the nearest million, would be mutants?

5. Graph L_3 as a function of L_1. What type of graph is produced? Determine the equation of this graph. Explain how this equation relates to the equation of the L_2-as-a-function-of-L_1 graph.

Part B

A particular colony of bacteria has an initial population of 2 000 cells and increases at a rate of 40%/h. A particular antibiotic, antibiotic *A*, is taken every 3 h and has an effectiveness factor of 75%; that is, after a person takes a dose of the antibiotic, 75% of the existing bacteria are immediately killed.

1. Given that the first dose of antibiotic *A* is taken at $t = 0$ h and that only the surviving bacteria increase for the 3-h period, the number of bacteria surviving at the end of 3 h can be determined by

$$\begin{aligned} 2\,000(1 - 0.75) &= 2\,000(0.25) \\ &= 500 \text{ survivors after the dose is taken} \end{aligned}$$

$$\text{then, } 500(1.40)^3 \doteq 1\,372 \text{ bacteria after 3 h at the instant before the second dose is taken}$$

- For antibiotic *A*, determine the bacterial population after 6 h (before the third dose is taken).

- For antibiotic *A*, determine the bacterial population after 9 h (before the fourth dose is taken).

- Complete the table below, and use the pattern established to express the number of bacteria, *N*, that are present before a dose is taken, as an exponential function of the time, *t*, where *t* is an exact multiple of 3.

This table shows how the bacterial population changes when it is treated with antibiotic *A*.

	Population of Bacteria				
Time (h)	0	3	6	9	*t*
Number of bacteria (*N*)	2 000	1 372			

- A patient is considered "cured" if the bacterial population is less than 50. How many treatment cycles does it take for patients who have an initial bacterial population of 2 000 to be cured when they are treated with antibiotic *A*?

2. Another antibiotic, antibiotic *B*, is taken every 6 hours and has an effectiveness factor of 90%. Determine the bacterial population after three days (72 h) in a patient who has an initial bacterial population of 2 000 and who is being treated with antibiotic *B*.

3. A third antibiotic, antibiotic *C*, has an effectiveness factor of 99.5%. Explain why antibiotic *C* **cannot** cure patients if it is administered every 24 hours.

Part C

Quality control is an important aspect in the manufacturing of antibiotics. A particular manufacturing operation produces batches of antibiotic *A* with effectiveness factors that are normally distributed with a mean of 75% and a standard deviation of 2%.

1. The operation automatically rejects any batch whose effectiveness factor is more than 1.5 standard deviations below the mean.

   • Determine the minimum effectiveness factor of an accepted batch.

   • Determine the probability that a randomly selected batch will be rejected.

2. A particular colony of bacteria has an initial population of 2 000 and increases at a rate of 40%/h. This bacterial population is treated every 3 h with a batch of antibiotic *A* that has the minimum effectiveness factor. Determine the number of bacterial cells after 3 h (before the second dose is taken), and after 6 h (before the third dose is taken).

Use the following information to answer the next question.

A second manufacturing operation produces batches of antibiotic *A* with effectiveness factors that are normally distributed with a mean of 75% and a standard deviation of 5%. The operation also rejects any batch whose effectiveness factor is more than 1.5 standard deviations below the mean; therefore, the minimum effectiveness factor of an accepted batch is 67.5%.

A patient with an initial population of 2 000 bacteria that have the same rate of increase of 40%/h is treated every 3 h with a batch of antibiotic *A*. The batch has a minimum effectiveness factor of 67.5%. The table below shows the number of bacteria at 0 h, 3 h, and 6 h, just before the first, second, and third doses are taken.

Time (h)	0	3	6
Number of bacteria (*N*)	2 000	1 783	1 588

3. Explain why antibiotic manufacturers must ensure that the standard deviation of the effectiveness factors be kept as low as possible.

Part D

Part A and ***part B*** of this project present examples of areas of real life that can be modelled by an exponential growth pattern.

1. The medical community has expressed concern about mutated bacteria, sometimes called "superbugs," that are resistant to available antibiotics. Research this medical problem, and then design a poster **or** a pamphlet to inform the public about the proper use of antibiotics. The following list of web sites may help in your research.

www.cdc.gov/drugresistance/community/

www.bbc.co.uk/education/asguru/generalstudies/sciencetechnology/18antibiotics/index.shtml

Note: Web site addresses sometimes change. If the web sites above are not available, use a search engine and type in key words such as "superbugs" or "antibiotic resistance."

OR

2. Investigate one other example of an exponential growth pattern. Your investigation should include a graph representing the growth, all pertinent equations and calculations, and a discussion of the real-life applications of the pattern.