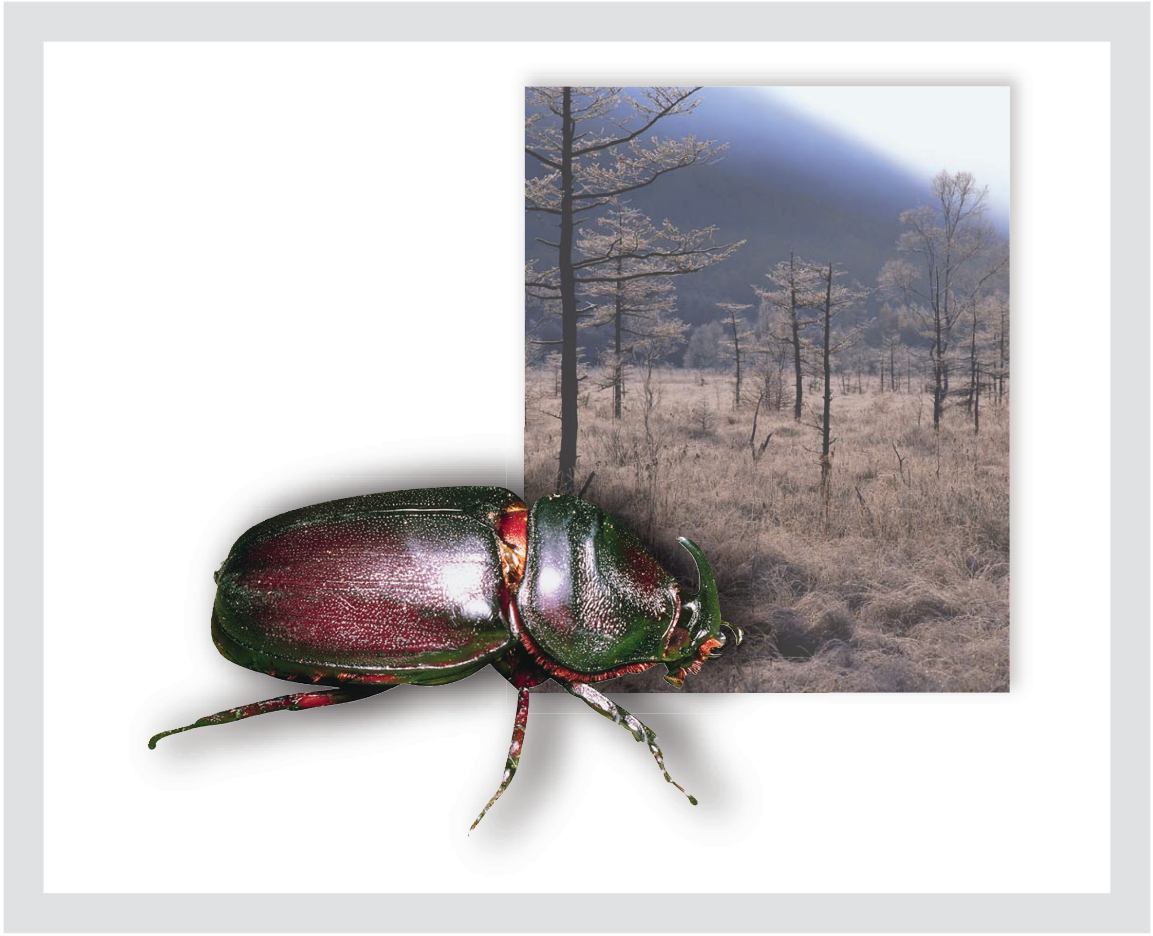
Applied Mathematics 30

Teacher Notes: Mountain Pine Beetles



February 2008

Alberta Education

Applied Mathematics 30

Mountain Pine Beetles—Teacher Notes

Introduction

This project relates to some of the mathematics involved in describing the spread of mountain pine beetles in Western Canada since 1993 and allows students to use their knowledge from the Finance and Spreadsheets unit; the Cyclic, Recursive, and Fractal Patterns unit; and the Vectors unit. It is designed to be completed in three to five hours of student time. The use of this project is optional; however, if you choose to use it, you may include it as part of your assessment. A hard copy will be mailed to your school in January 2008. Sample solutions can be found on the Alberta Education extranet at https://phoenix.edc.gov.ab.ca.

One of the written-response questions worth 10% on the Applied Mathematics 30 June 2008 Diploma Examination will be related to this project. Students who do not complete the project but who have completed the course will have the knowledge to answer the written-response question; however, students who complete the project will have gained experience with the related mathematical skills.

Specific Notes

Teachers may wish to

- refer to www.for.gov.bc.ca/hfp/mountain_pine_beetle for up-to-date information
- discuss the following terms with their students: fourfold, infestation level, salvaged, km/a, hectares, stand of forest
- tell students that the tables in Part A were the most up-to-date versions at the time of printing. As the data is updated and released to the public, other interesting trends and discussions may arise
- discuss how to input the data from the tables in Part A, Question 1 into a graphing calculator or spreadsheet. The sample solutions let 1999 be year 0, 2000 be year 1, etc., in each table; however, some students may choose to let 2000 be year 0, 2001 be year 1, etc., in the last two tables that have no data for 1999
- remind students that the a values in the exponential regression equations are the y-intercepts of the exponential curves and are only approximations of the initial values of the quantities under discussion
- explain why the b value in the exponential regression equation for infested area over time is greater than 2
- remind students to use appropriate window settings when graphing with calculators in Part A
- remind students that extrapolated values found from exponential regression equations should be treated with great caution

- remind students to save each version of their spreadsheet in Part B, Questions 1, 2, 3, and 4
- instruct students to round all values in their spreadsheets to whole numbers
- discuss with students the difference between the spreadsheet function ROUND and the decimal places setting for numbers in the format cells menu. The sample solution makes use of the ROUND function, which means that only the rounded value is used in future cell calculations. If the number of decimal places in the format cells menu is set to zero, then the unrounded value is actually used in future cell calculations. Be aware, however, that the ROUND function is beyond the scope of Applied Mathematics 30 and would not be an expected outcome for diploma examination purposes
- have students explore what is affected in the spreadsheet when both the infestation rate and the salvage rate are increased
- explain to students that the calculations in Part C assume that Earth is essentially flat and that effects from the curvature of Earth are neglected

Background Information to Part A, Question 3 for Teachers

In population biology, growth can be analyzed according to one of the following assumptions:

1. Population growth rate is proportional only to the number of organisms present (no-limit growth).

2. Population growth rate is jointly proportional to the number of organisms present and the remaining carrying capacity of the environment for that species (growth subject to limits).

Under the first assumption, the growth equation is $\frac{dN}{dt} = kN$, the solution of which is an exponential function, called a J-curve. The expression of this function can be written in one of two forms: either $N = N_0 e^{kt}$ (calculus) or $N = N_0 b^t$ (regression equations from calculator).

Under the second assumption, the growth equation is $\frac{dN}{dt} = kN(L - N)$, where L is the maximum carrying capacity. The solution of this equation is a logistic function, called an S-curve. The solution is either $N = \frac{L}{1 + \left(\frac{L}{N_0} - 1\right)e^{-kLt}}$ (calculus) or $N = \frac{L}{(1 + ae^{-bt})}$ (logistic regression from calculator).

The first assumption gives far simpler functions and is used whenever the ultimate carrying capacity is not important to know. For the beetle problem, the carrying capacity is 100% of the trees, and when more than 50% of the trees are infested, the more important quantity is the number of trees left for beetles to eat, and the logistic (S-curve) model works best. When less than 50% are infested, the important quantity is the number of trees infected, and the exponential model works best.

Further biological analyses can be found in any approved Biology 30 textbook.

Program of Studies

The project relates to mathematics contained in the following units of Applied Mathematics 30.

Finance and Spreadsheets

Specific Outcome 3.1: Design a financial spreadsheet template to allow users to input their own variables.
[C, PS, T]

Cyclic, Recursive, and Fractal Patterns

Specific Outcome 4.4: Use technology to generate and graph sequences that model real-life phenomena.
[PS, T, V]

Vectors

Specific Outcomes 5.1: Use appropriate terminology to describe:
- vector quantities
- scalar quantities

[C, CN]

5.4: Model and solve problems in 2-D and simple 3-D, using vector diagrams and technology.
[CN, PS, T, V]

Mathematical Processes

The seven mathematical processes identified in the Program of Studies are addressed in this project in the following manner.

Communication	Explain the meaning of the values of a and b in the exponential regression equation. Explain how to graphically solve for x in the exponential equation.
Connections	Relate mathematics to real-life situations, and connect different units within Applied Mathematics 30.
Estimation and Mental Mathematics	Check the reasonableness of calculator and spreadsheet solutions.
Problem Solving	Determine parameters for modelling various strategies to control the beetle population.
Reasoning	Predict which strategy will be most successful in combatting the effects of mountain pine beetles.
Technology	Use graphing techniques to estimate solutions. Use a spreadsheet to explore the effect of a change in infestation rate or salvage rate.
Visualization	Visualize the growth of beetle-infested forests, and use vector notation to describe the speed and the direction of this spread.

ICT Program of Studies

C.1—Students will access, use, and communicate information from a variety of technologies.

Specific Outcomes
4.2: Select information from appropriate sources, including primary and secondary sources.

C.3—Students will critically assess information accessed through the use of a variety of technologies.

Specific Outcomes
4.1: Assess the authority, reliability, and validity of electronically accessed information.

C.6—Students will use technology to investigate and/or solve problems.

Specific Outcomes
4.1: Investigate and solve problems of prediction, calculation, and inference.
4.2: Investigate and solve problems of organization and manipulation of information.
4.3: Manipulate data by using charting and graphing technologies in order to test inferences and probabilities.
4.4: Generate new understandings of problematic situations by using some form of technology to facilitate the process.

F.1—Students will demonstrate an understanding of the nature of technology.

Specific Outcome
4.2: Solve mathematical and scientific problems by selecting appropriate technology to perform calculations and experiments.

F.2—Students will understand the role of technology as it applies to self, work, and society.

Specific Outcome
4.7: Use current, reliable information sources from around the world.

P.2—Students will organize and manipulate data.

Specific Outcome
4.1: Manipulate and present data through the selection of appropriate tools, such as scientific instrumentation, calculators, databases, and/or spreadsheets.