

Pure Mathematics 30

**Teacher Notes:
Arch Design**



February 2008

Copyright 2008, the Crown in Right of Alberta, as represented by the Minister of Education, Alberta Education, Learner Assessment, 44 Capital Boulevard, 10044 108 Street NW, Edmonton, Alberta T5J 5E6, and its licensors. All rights reserved. Additional copies may be downloaded from the Alberta Education web site at www.education.alberta.ca.

Special permission is granted to **Alberta educators only** to reproduce, for educational purposes and on a non-profit basis, parts of this document that do **not** contain excerpted material.

Excerpted material in this document **shall not** be reproduced without the written permission of the original publisher (see credits, where applicable).

Pure Mathematics 30

Project: Arch Design—Teacher Notes

Introduction

This project portrays some of the various arch shapes used in the architectural design of structures. Students have a practical opportunity to see how their understanding of functions and conic relations can be applied to designing arches in various types of buildings and structures. Sample solutions for the project questions can be found on the Alberta Education extranet at <https://phoenix.edc.gov.ab.ca>. A hard copy will be mailed to your school in late January. The general scoring guide for the project is the same as the one issued in September 2000.

The first written-response question, which is worth 10%, on the Pure Mathematics 30 June 2008 diploma examination will be related to this project. Students who do not complete the project but who have completed the course will have the knowledge to answer the written-response question; however, students who do complete the project will gain experience with the related mathematical skills.

Specific Notes:

Teachers may wish to

- have students place the origin half way between point A and point B on the modelled graphs. Students may also want to use a grid to develop the parameters and equations of the conic curves (part A, question 1).
- remind students that the centre of the hyperbola must be at $(0, 30)$ in order for b to equal 10 in the equation $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = -1$. Students also need to be aware that substituting the coordinates of another point as x and y in the equation will enable them to find the value of a in this equation (part A, question 1).
- remind students that they only need to graph the upper semi-ellipse (i.e., $y = \sqrt{400 - 0.16x^2}$) in part A, question 2
- remind students that “20 m from either end” refers to the points $(30, y_1)$ and $(-30, y_2)$ (part A, question 3)
- review the shape of the toolbox function $f(x) = |x|$ (part B, bullet 1)
- consider the arches as the result of a sequence of transformations of $f(x) = |x|$, rather than as a single transformation $y = |h(x)|$ of the linear function $h(x) = mx + c$ (part B, bullets 2 and 3)
- remind themselves that the general absolute value transformation $y = |f(x)|$, where $f(x)$ is any function other than x , is beyond the scope of the Program of Studies (part B, bullet 3)
- remind students that websites and their addresses were accurate as of November 27, 2007, but no guarantee can be given that these addresses are accurate at any future date

Program of Studies

The project relates to mathematics learned in the following units of Pure Mathematics 30.

Transformations

- Specific Outcomes**
- 1.1: Describe how various translations of functions affect graphs and their related equations.
- $y = f(x - h)$
 - $y - k = f(x)$
- [C, T, V]
- 1.2: Describe how various stretches of functions (compressions and expansions) affect graphs and their related equations.
- $y = af(x)$
 - $y = f(kx)$
- [C, T, V]
- 1.5: Describe and perform single transformations and combinations of transformations on functions and relations. [C, T, V]

Trigonometry

- Specific Outcomes**
- 3.8: Draw (using technology), sketch, and analyze the graphs of sine, cosine, and tangent functions for:
- amplitude, if defined
 - period
 - domain and range
 - asymptotes, if any
 - behaviour under transformations.
- [CN, T, V]
- 3.10: Use sine and cosine functions to model and solve problems. [PS, R, V]

Conics

- Specific Outcomes**
- 4.2: Classify conic sections according to a given equation in general form or standard form. [CN, T, V]
- 4.3: Convert a given equation of a conic section from general form to standard form and vice versa. [R, T]

ICT Program of Studies

C.1—Students will access, use, and communicate information from a variety of technologies.

Specific Outcome 4.2: Select information from appropriate sources, including primary and secondary sources.

C.6—Students will use technology to investigate and/or solve problems.

Specific Outcomes 4.1: Investigate and solve problems of prediction, calculation, and inference.

4.2: Investigate and solve problems of organization and manipulation of information.

Mathematical Processes

The seven mathematical processes identified in the Program of Studies are addressed in this project in the following manner.

Communication	Explain the similarities and differences of two conic sections over a specific domain and range, and connect the graphical parameters of an equation/function to a real-life context.
Connections	Understand the connections between the features of the graphs of an absolute value function and several conic relations and the parameters of their corresponding equations. Understand how transformations affect the parameters of an equation.
Estimation and Mental Mathematics	Compare the values of a graph/equation to the specifications required for an arch design.
Problem Solving	Develop graphs and equations for an arch design from real-life dimensions.
Reasoning	Decide which form of an equation best represents the graph of a conic curve, and recognize the similarities and differences between these curves.
Technology	Use an appropriate window on a graphing calculator to generate graphs of conic curves. Use features on the graphing calculator to find values of points on each curve.
Visualization	Visualize the graphs of several arch designs and check their reasonableness with a graphing calculator.