

June 2001

Pure Mathematics 30

Grade 12 Diploma Examination

Description

Time: This examination was developed to be completed in 2.5 h; however, you may take an additional 0.5 h to complete the examination.

This is a **closed-book** examination consisting of

- 33 multiple-choice and 6 numerical-response questions, of equal value, worth 65% of the examination
- 3 written-response questions worth 35% of the examination

A tear-out formula sheet and two z-score pages are included in this booklet.

All graphs on this examination are computer-generated.

Note: *The perforated pages at the back of this booklet may be torn out and used for your rough work. No marks will be given for work done on the tear-out pages.*

Instructions

- You are expected to provide a graphing calculator approved by Alberta Learning.
- You are expected to have cleared your calculator of all information that is stored in the programmable or parametric memory.
- Use only an HB pencil for the machine-scored answer sheet.
- Fill in the information required on the answer sheet and the examination booklet as directed by the presiding examiner.
- Read each question carefully.
- Consider all numbers used in the questions to be **exact** numbers and not the result of a measurement.
- If you wish to change an answer, erase **all** traces of your first answer.
- Do not fold the answer sheet.
- The presiding examiner will collect your answer sheet and examination booklet and send them to Alberta Learning.
- Now turn this page and read the detailed instructions for answering machine-scored and written-response questions.

Correct-Order Question and Solution

When the following subjects are arranged in alphabetical order, the order is _____, _____, _____, and _____.

- 1 biology
- 2 physics
- 3 chemistry
- 4 mathematics

(Record **all four digits** of your answer in the numerical-response section on the answer sheet.)

Answer: 1342

Record 1342 on the answer sheet

1	3	4	2
•	•		
0	0	0	0
●	1	1	1
2	2	2	●
3	●	3	3
4	4	●	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

Written Response

- Write your answers in the examination booklet as neatly as possible.
- For full marks, your answers must address **all** aspects of the question.
- Descriptions and/or explanations of concepts must be correct and include pertinent ideas, diagrams, calculations, and formulas.
- Your answers must be presented in a well-organized manner using complete sentences and correct units.

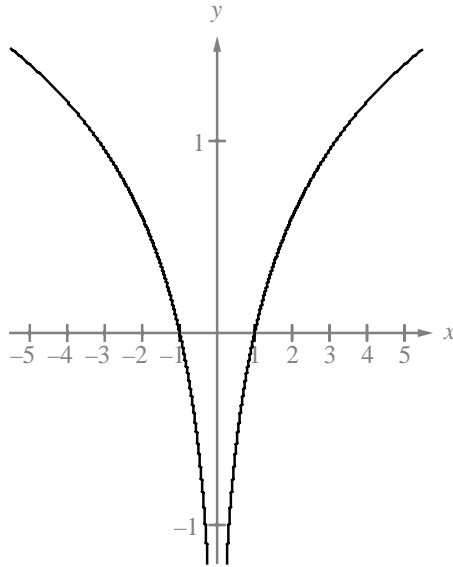
1. A clothing store is going out of business. The owner reduces the cost of each item by 10% of the current price at the start of each week. A jacket costs \$120.00 during the 1st week of the sale. If this jacket is still in the store during the 5th week of the sale, then the price of the jacket, to the nearest cent, will be
- A. \$70.00
 - B. \$70.86
 - C. \$78.73
 - D. \$80.00
2. The sum of the first 10 terms of the geometric sequence $-4, 6, -9, \dots$, to the nearest tenth, is
- A. 153.8
 - B. 90.7
 - C. -61.5
 - D. -453.3
3. If $\log_x\left(\frac{1}{64}\right) = -\frac{3}{2}$, then x is equal to
- A. 16
 - B. 8
 - C. $\frac{1}{8}$
 - D. $\frac{1}{16}$

4. If $\log_3 x = 15$, then $\log_3\left(\frac{1}{3}x\right)$ is equal to
- A. 14
 - B. 12
 - C. 5
 - D. -15
5. The equation $y = 4^{3x}$ can also be written as
- A. $y = \frac{\log_3 x}{4}$
 - B. $y = \frac{\log_4 x}{3}$
 - C. $x = \frac{\log_3 y}{4}$
 - D. $x = \frac{\log_4 y}{3}$
6. The number of cellular phone customers of a particular telephone company increased by an average of 40%/a from 1994 to 1998. If there were 963 300 customers on January 2, 1998, then the number of customers on January 2, 1994, to the nearest hundred, would have been
- A. 3 700 600
 - B. 688 100
 - C. 250 800
 - D. 172 100

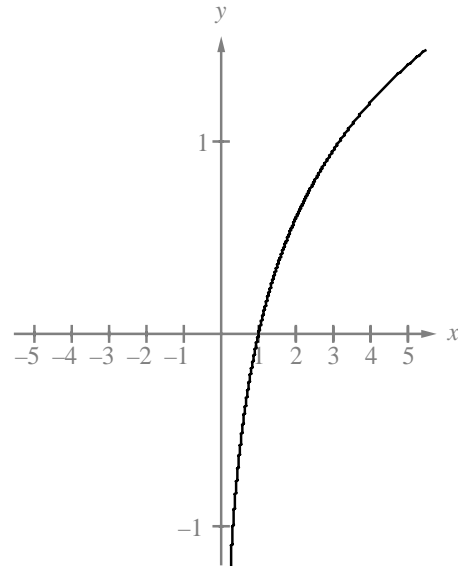
Use the following information to answer the next question.

A student used a graphing calculator to illustrate identities. The student assumed that $\log(x^2) = 2 \log x$ because $\log_a(M^n) = n \log_a M$. The student graphed $y = \log(x^2)$ and obtained the graph shown below on the left. The student then graphed $y = 2 \log x$ and obtained the graph shown below on the right.

Graph of $y = \log(x^2)$



Graph of $y = 2 \log x$



7. The student realized that the reason why the graphs are not identical is that
- A. $\log 0$ is not defined
 - B. where $x < 0$, $\log(x^2)$ is defined and $\log x$ is not defined
 - C. where $x < 0$, $\log x$ is defined and $\log(x^2)$ is not defined
 - D. the range of $y = \log(x^2)$ is different from the range of $y = 2 \log x$

Numerical Response

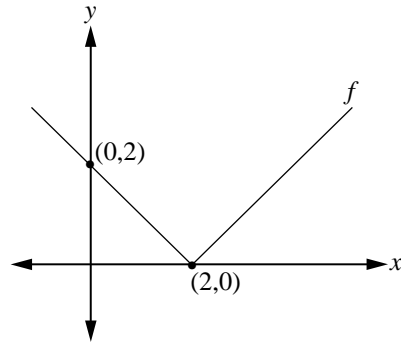
- 1.** Three sums obtained from a particular infinite geometric sequence are $S_1 = 10$, $S_2 = 15$, and $S_3 = \frac{35}{2}$. The sum of this entire infinite sequence is _____.

(Record your answer in the numerical-response section on the answer sheet.)

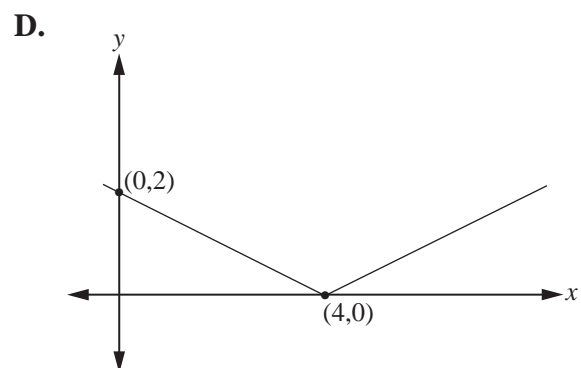
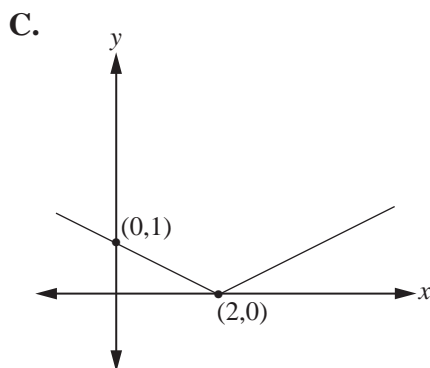
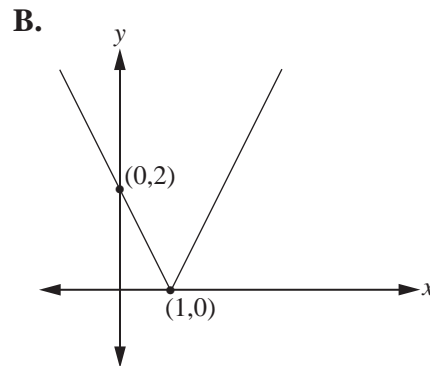
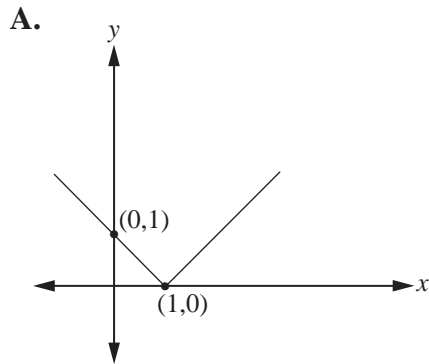
- 8.** A function $y = f(x)$ is graphed. If $g(x) = -f(x)$, then the graph of $y = g(x)$ is the same as the graph of
- A.** $y = f(x)$ reflected in the line $y = x$
 - B.** $y = f(x)$ reflected in the y -axis
 - C.** $y = f(x)$ reflected in the x -axis
 - D.** the reciprocal of $y = f(x)$
- 9.** The graph of a function f is a parabola opening upward, with its vertex on the x -axis. The graph of a new function g , where $g(x) = 2f(x)$, will have
- A.** the same domain and the same range as f
 - B.** the same domain but a different range than f
 - C.** a different domain but the same range as f
 - D.** a different domain and a different range than f

Use the following information to answer the next question.

The partial graph of $y = f(x)$ is shown below.

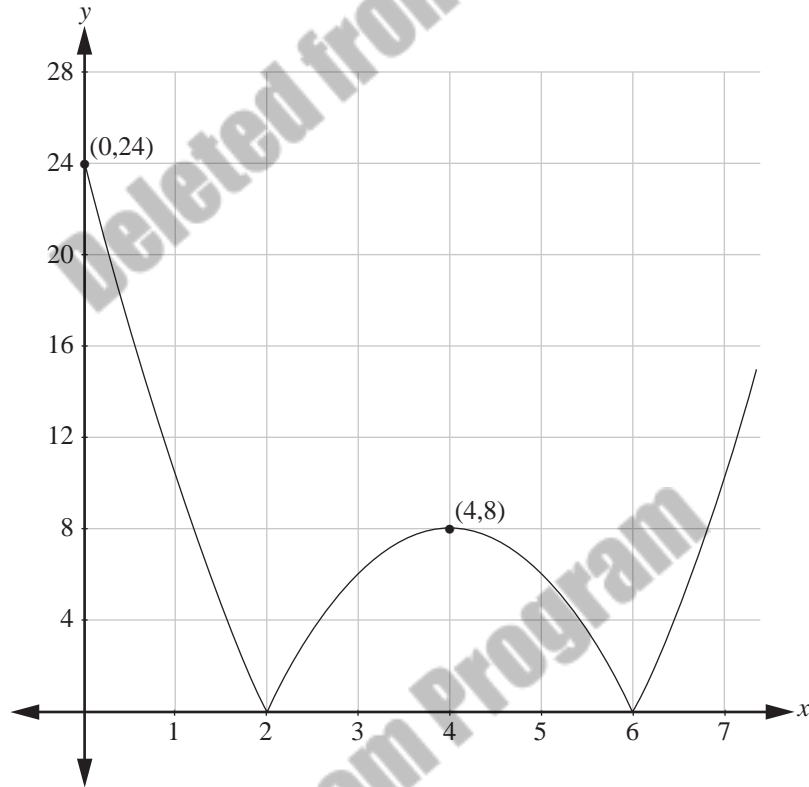


10. Which of the following partial graphs represents the transformed function $y = f\left(\frac{1}{2}x\right)$?



Use the following information to answer the next question.

The partial graph of $y = |f(x)|$, where $f(x) = ax^2 + bx + c$, $a < 0$, is shown below.



11. The equation of $y = f(x)$ is

- A. $f(x) = 2(x - 4)^2 + 8$
- B. $f(x) = 2(x + 4)^2 - 8$
- C. $f(x) = -2(x - 4)^2 + 8$
- D. $f(x) = -2(x + 4)^2 - 8$

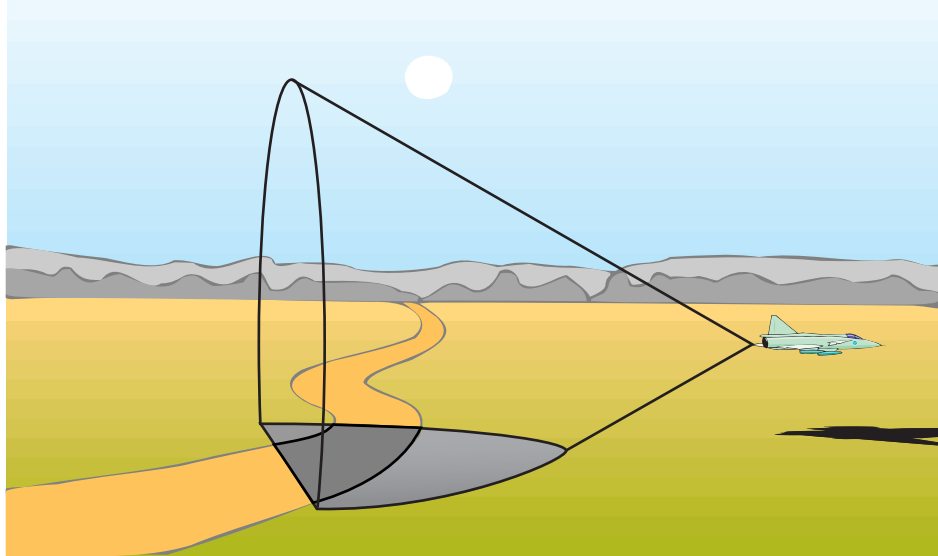
Numerical Response

2. As a result of the transformation of the graph of $y = x^3$ into the graph of $y - 4 = (x - 3)^3$, point $(3, 27)$ becomes point $(6, y)$. The value of y is _____.

(Record your answer in the numerical-response section on the answer sheet.)

Use the following information to answer the next question.

The shock wave associated with the sonic boom created by a jet flying faster than the speed of sound has the shape of a cone.

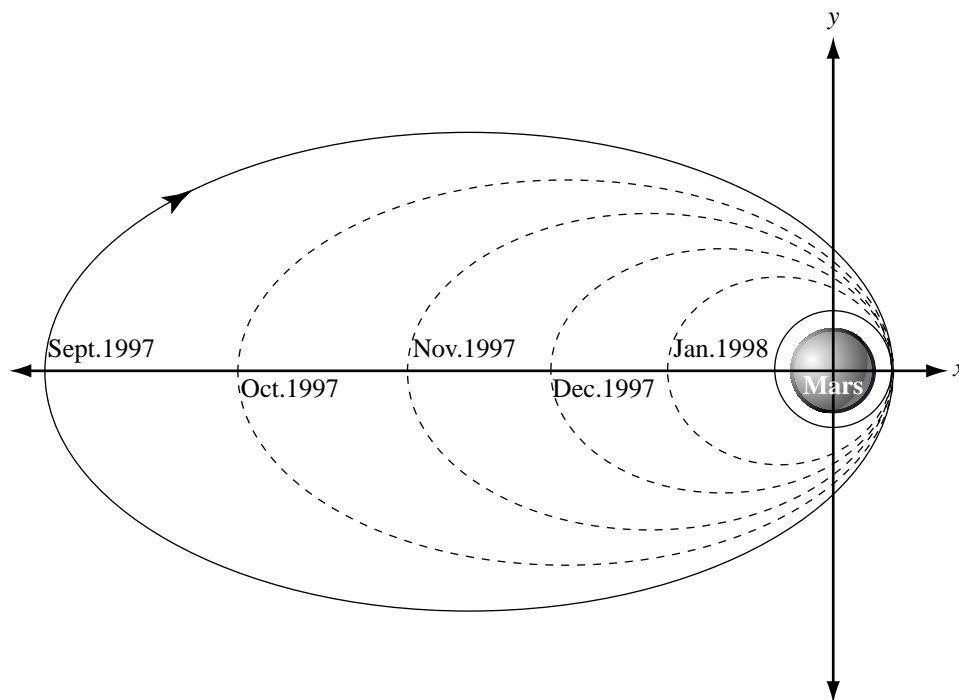


12. If a jet is flying parallel to the ground, then the intersection of its shock wave with the ground forms one branch of a hyperbola because the ground is
- A. perpendicular to the axis of symmetry of the cone
 - B. parallel to the axis of symmetry of the cone
 - C. perpendicular to the generator of the cone
 - D. parallel to the generator of the cone
-

13. When the equation $\frac{(x - 3)^2}{4} + \frac{(y + 2)^2}{9} = 1$ is converted to general form, it becomes
- A. $4x^2 + 9y^2 - 6x + 4y + 36 = 0$
 - B. $4x^2 + 9y^2 - 24x + 36y + 36 = 0$
 - C. $9x^2 + 4y^2 - 54x + 16y + 61 = 0$
 - D. $9x^2 + 4y^2 - 27x + 8y + 61 = 0$

Use the following information to answer the next question.

The Mars Global Surveyor is a spacecraft that began orbiting Mars in September 1997. In order to save fuel, the Surveyor orbited the planet in successively smaller elliptical paths. The paths are shown below on the coordinate plane.



For each elliptical orbit in the diagram above, a student determined a new equation by using the standard quadratic equation $\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$.

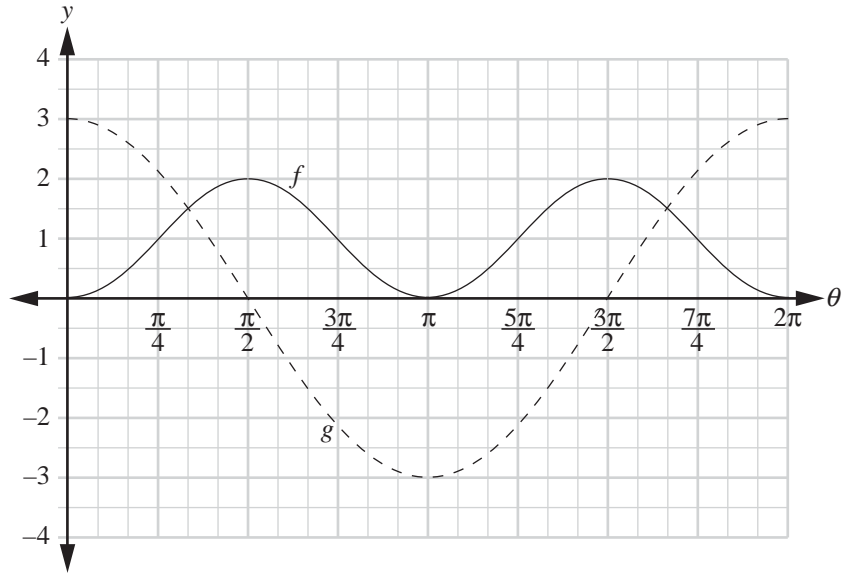
The student calculated each new equation, in order, from September 1997 to January 1998.

14. As the student determined each new equation, she noticed that in addition to a change in the value of a and a change in the value of b , the value of
- A. h increased
 - B. h decreased
 - C. k increased
 - D. k decreased

15. When a circle represented by the equation $x^2 + 10x + y^2 - 8y = 11$ is translated 7 units to the right, the new circle has its centre at
- A. $(12, -4)$
 - B. $(-12, 4)$
 - C. $(-2, -4)$
 - D. $(2, 4)$
16. To create an identity (a statement that is true for all x in the domain) for the equation $\cos^2 x(1 + \cot^2 x) = A$, the value of A would need to be
- A. $\sin^2 x$
 - B. $\cos^2 x$
 - C. $\cot^2 x$
 - D. $\sec^2 x$
17. If the terminal arm of angle θ , in standard position, passes through point $(-b, 2b)$, where $b > 0$, then the exact values of $\sin \theta$, $\cos \theta$, and $\tan \theta$ are, respectively,
- A. $\frac{-2}{\sqrt{5}}$, $\frac{1}{\sqrt{5}}$, and 2
 - B. $\frac{2}{\sqrt{5}}$, $\frac{-1}{\sqrt{5}}$, and -2
 - C. $\frac{-1}{\sqrt{5}}$, $\frac{2}{\sqrt{5}}$, and -2
 - D. $\frac{1}{\sqrt{5}}$, $\frac{-2}{\sqrt{5}}$, and 2

Use the following information to answer the next question.

The partial graph of $f(\theta) = 2 \sin^2 \theta$ and the partial graph of $g(\theta) = 3 \cos \theta$ are shown below.



18. Within the domain $0 < \theta < 2\pi$, the interval where $f(\theta) > g(\theta)$ is
- A. $0 < \theta < 2\pi$
 - B. $\frac{\pi}{2} < \theta < \frac{3\pi}{2}$
 - C. $\frac{\pi}{3} < \theta < \pi$
 - D. $\frac{\pi}{3} < \theta < \frac{5\pi}{3}$

Use the following information to answer the next question.

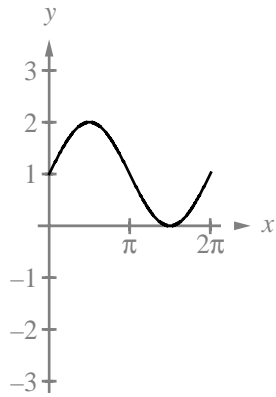
In attempting to prove an identity, a student performed the following steps.

$$\text{Step 1} \quad (\sec \theta + 1)^2 = (\sec \theta)^2 + 1^2$$

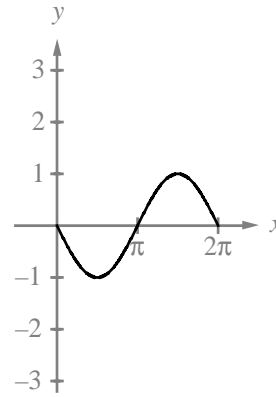
19. The student's **first** error was made in step
- A. 1
 - B. 2
 - C. 3
 - D. 4
-
20. In one minute, the second hand of a clock completes one revolution around the clock face. In $1\frac{1}{2}$ minutes, the second hand of a clock completes an angle of
- A. $\frac{3\pi}{2}$
 - B. 3π
 - C. 6π
 - D. 180π

21. If the equation $a \sin x + d = 0$ has no solution within the interval $0 \leq x \leq 2\pi$, then the graph of $y = a \sin x + d$ could be

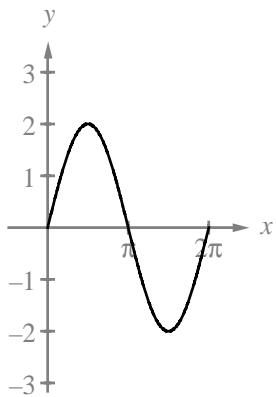
A.



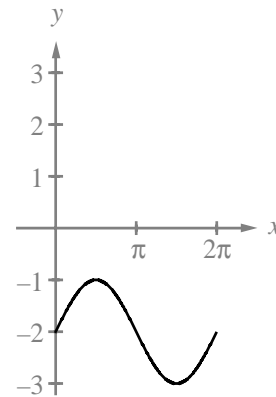
B.



C.



D.



22. The three solutions of the equation $f(\theta) = 0$ are 0° , 180° , and 360° . Therefore, the three solutions of the equation $f(\theta - 30^\circ) = 0$ are

- A. 0° , 60° , and 120°
- B. -30° , 150° , and 330°
- C. 30° , 210° , and 390°
- D. 0° , 540° , and 1080°

Numerical Response

3. The flight pattern of a certain bee can be represented by the function $f(x) = \cos^2 x - \sin x + 4$. The range of the graph that represents this flight pattern is $a \leq y \leq b$. The value of b , correct to the nearest hundredth, is _____.

(Record your answer in the numerical-response section on the answer sheet.)

Use the following information to answer the next question.

The following instructions were given on a survey.

Place an **X** in the box beside the activities that interest you when you are on vacation. You may place an **X** in as many boxes as you like, or you may leave all boxes blank.

- sightseeing
- theatre
- hiking
- skiing
- museums
- golfing
- shopping

Before the results of all the completed surveys can be tabulated, the number of different possible combinations that can be selected must be determined.

23. What is the number of different possible combinations?
- A. 28
 - B. 128
 - C. 5 040
 - D. 13 700

24. A school committee consists of 1 vice-principal, 2 teachers and 3 students. The number of different committees that can be selected from 2 vice-principals, 5 teachers, and 9 students is
- A. 20 160
 - B. 8 008
 - C. 1 680
 - D. 90
25. Tim and Rebecca are the first and second students in a line of 7 students waiting to buy tickets for a concert. The number of different orders in which the remainder of the students can line up behind them is
- A. $5!$
 - B. $7!$
 - C. $5! \times 2!$
 - D. $\frac{7!}{2!}$
26. At one time, a standard licence plate consisted of any 2 consonants followed by any 4 digits. Later, the standard licence plate was changed to consist of any 3 consonants followed by any 3 digits. Given that all 20 consonants can be used, and that any consonant and any digit can be repeated, how many **more** standard licence plates were available after this change?
- A. 3 009 600
 - B. 4 000 000
 - C. 8 000 000
 - D. 12 000 000

Numerical Response

4. In a particular town, all of the streets run north–south or east–west. A student lives 5 blocks west and 3 blocks south of a school. The number of different routes, 8 blocks in length, that the student can take to get to the school is _____.

(Record your answer in the numerical-response section on the answer sheet.)

27. An example of dependent events is
- A. tossing three coins, one at a time
 - B. throwing a dart at a dart board, removing it, and throwing it again
 - C. drawing a card from a deck, replacing it, and drawing a second card
 - D. drawing a card from a deck, not replacing it, and drawing a second card
28. A child has 2 quarters, 2 dimes, and 3 nickels in his pocket, but he does not understand the value of any of the coins. He puts 35¢ worth of candy on the counter at a store and randomly selects two coins from his pocket. The probability that the two coins he selects will have a total value at least as high as the value of the candy is
- A. $\frac{2}{9}$
 - B. $\frac{5}{21}$
 - C. $\frac{12}{49}$
 - D. $\frac{2}{7}$

Use the following information to answer the next question.

Including Peter and Deanna, a particular school council has 10 members. The probabilities of 3 possible committees each containing 4 members from this council are shown below.

Committee 1 (Peter and Deanna are both chosen)

$$\frac{{}_2C_2 \times {}_8C_2}{{}_{10}C_4} = \frac{2}{15}$$

Committee 2 (either Peter or Deanna is chosen)

$$\frac{{}_2C_1 \times {}_8C_3}{{}_{10}C_4} = \frac{8}{15}$$

Committee 3 (neither Peter nor Deanna is chosen)

$$\frac{{}_8C_4}{{}_{10}C_4} = \frac{1}{3}$$

29. The probability that both Peter and Deanna are **not** chosen is
- A. (Probability of Committee 1) \times (Probability of Committee 2)
 - B. $1 -$ (Probability of Committee 3)
 - C. $1 -$ (Probability of Committee 1)
 - D. Probability of Committee 3
-
30. A utility company has 11 power plants that generate electricity. The probability that any 1 of the 11 power plants will not be functioning at any given time is 0.10. The probability that exactly 3 of the 11 power plants will not be working at any one time, to the nearest thousandth, is
- A. 0.300
 - B. 0.270
 - C. 0.165
 - D. 0.071

Use the following information to answer the next question.

On the table below, a school counsellor has recorded the percentage of each school day, for 8 days, that he spent talking with students.

Day	1	2	3	4	5	6	7	8
%	90	80	75	68	88	92	85	82

Numerical Response

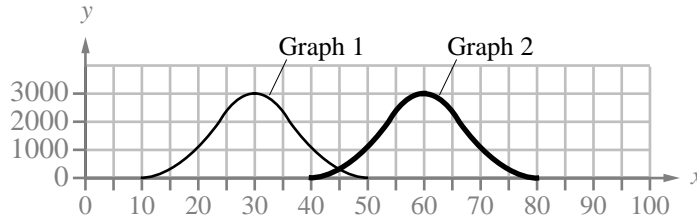
- 5.** The value of the standard deviation for the percentages, correct to the nearest hundredth, is _____.

(Record your answer in the numerical-response section on the answer sheet.)

- 31.** In a standard normal distribution, the probability that a z -score falls within 0.7 standard deviations of the mean ($-0.7 < z < 0.7$) is
- A. 0.5160
 - B. 0.4840
 - C. 0.2580
 - D. 0.2420

Use the following information to answer the next question.

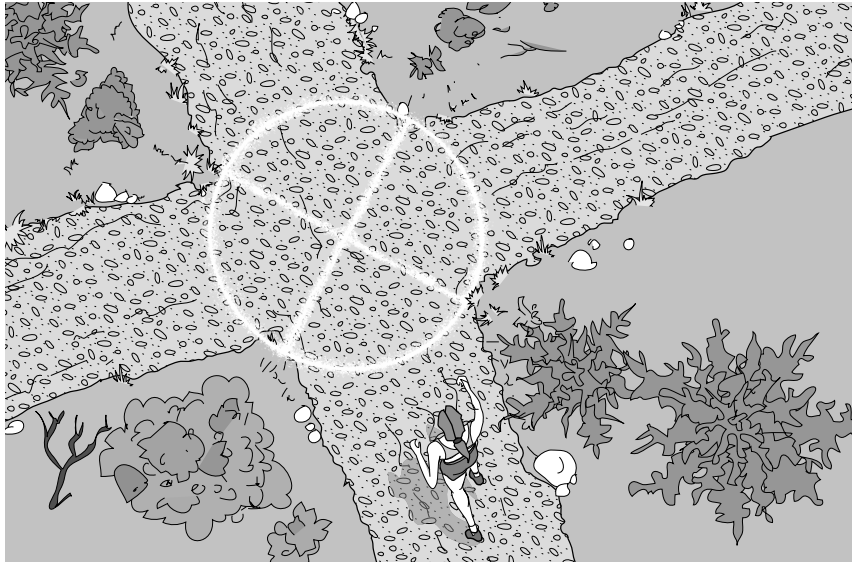
A student graphed a set of scores, as shown in graph 1 below. He then added 30 to each score, and graphed the new set of scores, as shown in graph 2 below.



32. When comparing graph 2 to graph 1, the student found that the
- A. mean and the standard deviation had increased
 - B. mean and the standard deviation had stayed the same
 - C. mean had increased but the standard deviation had stayed the same
 - D. mean had stayed the same but the standard deviation had increased
-
33. A teacher recorded the height of each six-year-old girl in her school. She determined that their heights were normally distributed with a mean of 95 cm and a standard deviation of 10 cm. If Samantha's height is 1.20 standard deviations below the mean, then her height, to the nearest tenth of a centimetre, is
- A. 83.0 cm
 - B. 93.8 cm
 - C. 94.9 cm
 - D. 107.0 cm

Use the following information to answer the next question.

A local running club sponsors an unusual cross-country race. At certain points, a special symbol \otimes is marked on the trail to indicate that a runner must choose one of three possible directions. Only one of the directions will lead a runner down the correct path.



Numerical Response

6. If there are 3 such symbols on the route, then the probability, to the nearest hundredth, that a runner chooses the correct path on the first try at each symbol is _____.

(Record your answer in the numerical-response section on the answer sheet.)

The written-response questions follow on the next page.

1.

Written Response—10%

Use the following information to answer this question.

If each team in a basketball league must play each of the other teams twice, then the total number of games, g , that can be played is given by the function $g(n) = n^2 - n$, where n is the number of teams in the league and $n \in N, n \geq 2$.

- Determine the first 3 terms of the sequence defined by $g(n) = n^2 - n$, $n \in N, n \geq 2$.

- In this league, where each team must play each of the other teams twice, 72 games have been scheduled. Determine the number of teams in this league. Justify your answer.

- Explain why $2 \times {}_n C_2$ models the number of games that must be scheduled so that n basketball teams play each other exactly two times.

- The function $g(n) = n^2 - n$ can be determined using combinations. Verify algebraically that $2 \times {}_n C_2$ equals $n^2 - n$.

Written-response question 2 begins on the next page.

Written Response—10%

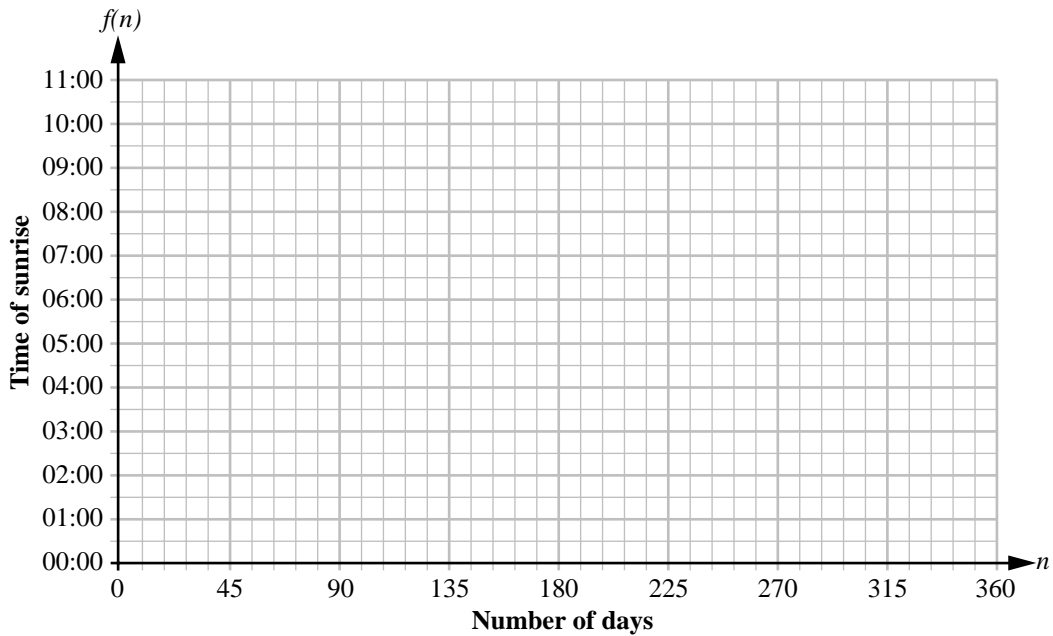
2. The function $f(n) = 2.32 \sin[0.0172(n - 285.366)] + 6.54$ approximates the time of **sunrise** in Saskatoon as a function of day number, n . In this function, n is the number of days after December 11, $f(n)$ is measured in hours after midnight, and sine is computed in radians.

- Determine the time of sunrise in Saskatoon on January 31, which is day 51, to the nearest hundredth of an hour.

- Another function, $g(n) = -2.39 \sin[0.0172(n - 285.366)] + 18.64$, gives the time of **sunset** for any day, n , after December 11, in Saskatoon. The value 0.0172 is the same for both functions, f (sunrise) and g (sunset). Explain what this value represents in terms of its sinusoidal graph.

- With your graphing calculator in radian mode, create a graph for the function f that approximates the time of **sunrise** in Saskatoon. Predict the approximate time of the earliest sunrise.

Sketch your graph on the grid below.



The approximate time of the earliest sunrise, to the nearest tenth of an hour, is _____ h.

- If the function g were to be written in terms of cosine, $y = a \cos b(n - c) + d$, then only **one** of the four parameters, a , b , c , or d , would change. Which parameter would change? Justify your answer.

Written-response question 3 begins on the next page.

3.

Written Response—15%

Use the following information to answer this question.

A student places a spherical jawbreaker in a cup of water. As the surface of the jawbreaker dissolves, the jawbreaker maintains a spherical shape. The volume of the jawbreaker, V , as a function of time, t , is shown below.

Time t (min)	Volume V (mm^3)
0	905.000
1	814.500
2	733.050
3	659.745
4	

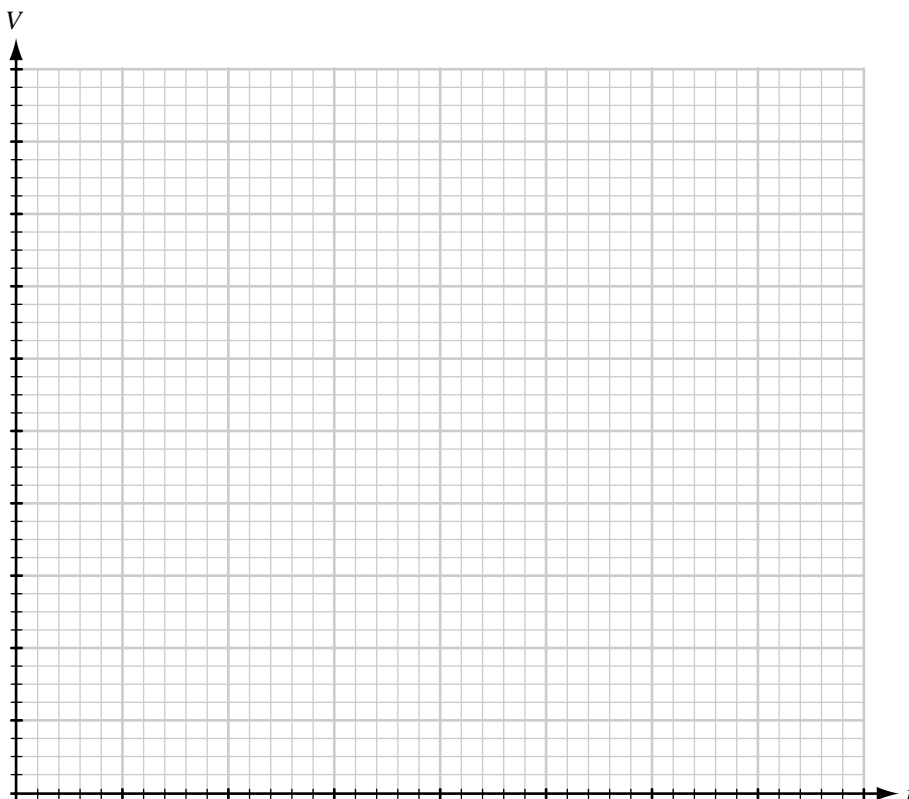
- If it is assumed that this geometric pattern continues, what is the volume of the jawbreaker, to the nearest thousandth of a mm^3 , when $t = 4$ min?
- Write an equation for volume, V , as a function of time, t , in the form $V = a(b^t)$.
- Using your equation from the previous bullet, determine, to the nearest tenth of a minute, the time it takes the jawbreaker to reach 100 mm^3 .

- Create a graph of the equation that you found for volume in the second bullet. State an appropriate viewing window for the first 40 minutes of the dissolving time. Use the form

$$[x_{\min}, x_{\max}, x_{\text{scl}}] [y_{\min}, y_{\max}, y_{\text{scl}}]$$

$$x: [\quad , \quad , \quad] \quad y: [\quad , \quad , \quad]$$

On the grid below, sketch your graph and indicate the point where the volume equals 100 mm^3 .



Written-response question 3 continues on the next page.

- A particular machine produces many different coloured jawbreakers. The probability that a black jawbreaker is produced is 0.12. If the jawbreakers are packaged in boxes of 60, then what is the probability, to the nearest hundredth, that a box will contain at least 5 black jawbreakers?

***You have now completed the examination.
If you have time, you may wish to check your answers.***

Pure Mathematics 30 Formula Sheet

The following information may be useful in writing this examination.

For $ax^2 + bx + c = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**Exponents, Logarithms,
and Geometric Series**

$$\log_a(M \times N) = \log_a M + \log_a N$$

$$\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$$

$$\log_a M^n = n \log_a M$$

$$\log_b c = \frac{\log_a c}{\log_a b}$$

$$t_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}, r \neq 1$$

$$S_n = \frac{rt_n - a}{r - 1}, r \neq 1$$

$$S = \frac{a}{1 - r}, |r| < 1$$

Conics*General Form*

$$Ax^2 + Cy^2 + Dx + Ey + F = 0$$

Standard Form

$$\frac{(x - h)^2}{a^2} \pm \frac{(y - k)^2}{b^2} = \pm 1$$

$$y - k = a(x - h)^2$$

$$x - h = a(y - k)^2$$

Permutations and Combinations

$${}_n P_r = \frac{n!}{(n - r)!}$$

$${}_n C_r = \frac{n!}{(n - r)! r!}$$

In the expansion of $(x + y)^n$, the general

term is $t_{k+1} = {}_n C_k x^{n-k} y^k$.

Graphing Calculator Window Format

$$x: [x_{\min}, x_{\max}, x_{\text{scl}}]$$

$$y: [y_{\min}, y_{\max}, y_{\text{scl}}]$$

Statistics

$$\mu = np \quad \sigma = \sqrt{np(1 - p)}$$

$$z = \frac{x - \mu}{\sigma}$$

If $np \geq 5$ and $n(1 - p) \geq 5$, then the binomial distribution is a large sample.

Probability

$$P(A \text{ and } B) = P(A) \times P(B/A)$$

$$P(k) = {}_n C_k p^k (1 - p)^{n-k}$$

Trigonometry

$$a = r\theta$$

$$\csc x = \frac{1}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\cot x = \frac{\cos x}{\sin x}$$

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

$$\sin(A + B) = \sin A \cos B + \sin B \cos A$$

$$\sin(A - B) = \sin A \cos B - \sin B \cos A$$

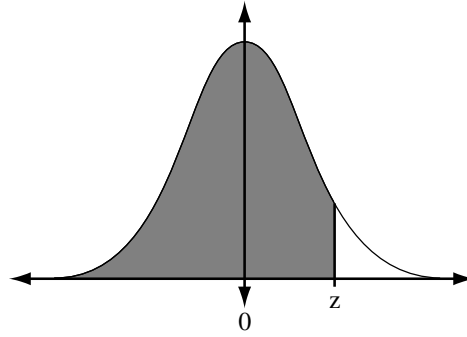
$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\sin(2A) = 2 \sin A \cos A$$

$$\cos(2A) = \cos^2 A - \sin^2 A$$

$$z = \frac{x - \mu}{\sigma}$$

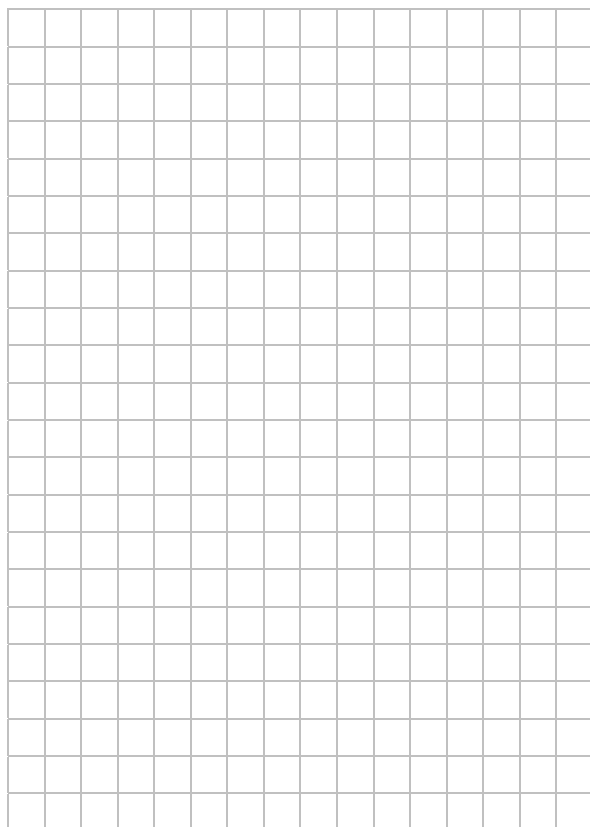
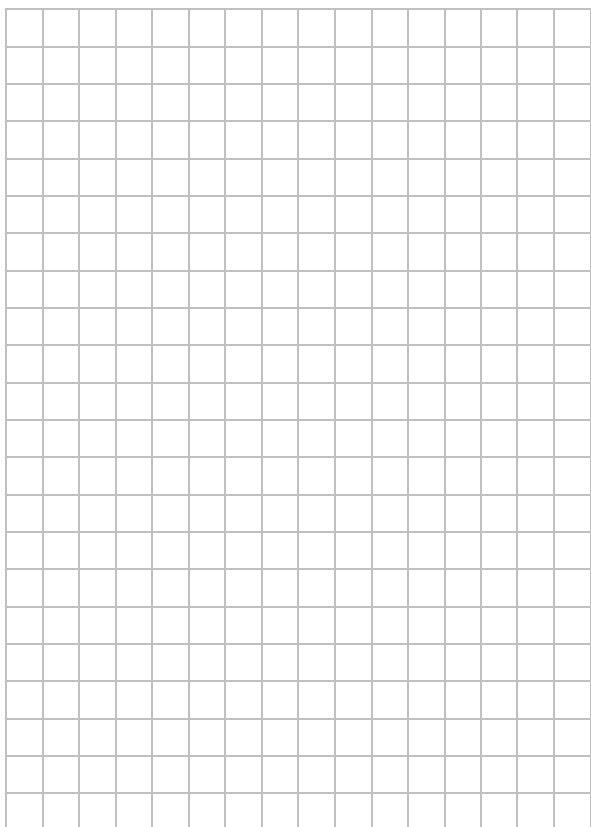
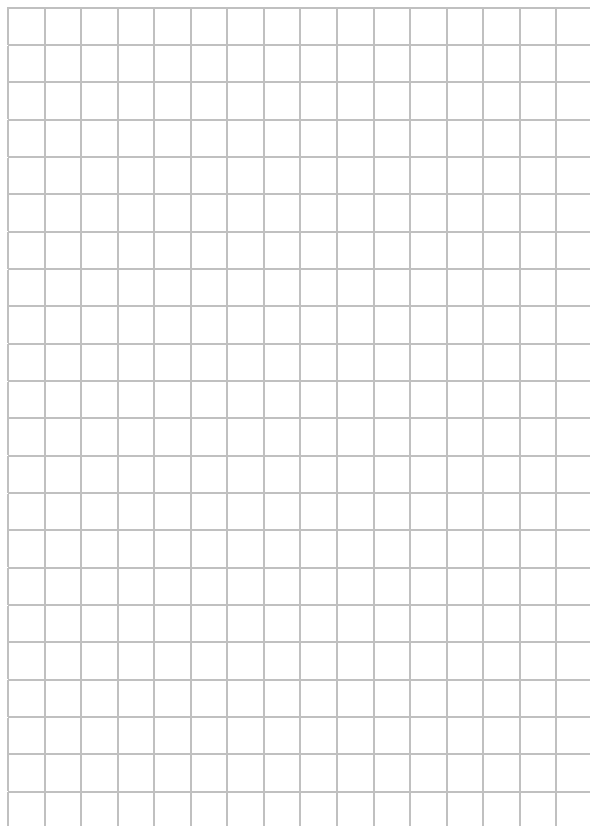
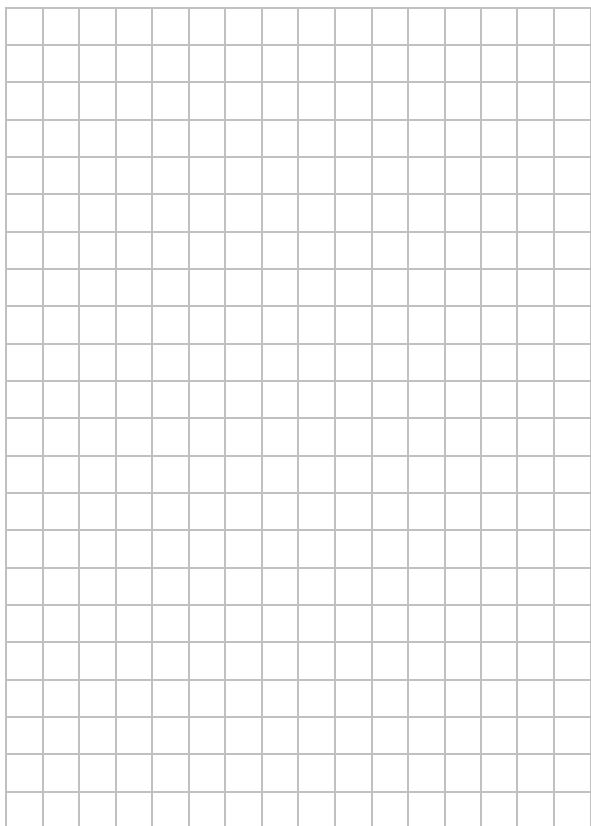


Areas under the Standard Normal Curve

Fold and tear along perforation.

<i>z</i>	0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01	0.00
-3.4	0.0002	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003
-3.3	0.0003	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0005	0.0005	0.0005
-3.2	0.0005	0.0005	0.0005	0.0006	0.0006	0.0006	0.0006	0.0006	0.0007	0.0007
-3.1	0.0007	0.0007	0.0008	0.0008	0.0008	0.0008	0.0009	0.0009	0.0009	0.0010
-3.0	0.0010	0.0010	0.0011	0.0011	0.0011	0.0012	0.0012	0.0013	0.0013	0.0013
-2.9	0.0014	0.0014	0.0015	0.0015	0.0016	0.0016	0.0017	0.0018	0.0018	0.0019
-2.8	0.0019	0.0020	0.0021	0.0021	0.0022	0.0023	0.0023	0.0024	0.0025	0.0026
-2.7	0.0026	0.0027	0.0028	0.0029	0.0030	0.0031	0.0032	0.0033	0.0034	0.0035
-2.6	0.0036	0.0037	0.0038	0.0039	0.0040	0.0041	0.0043	0.0044	0.0045	0.0047
-2.5	0.0048	0.0049	0.0051	0.0052	0.0054	0.0055	0.0057	0.0059	0.0060	0.0062
-2.4	0.0064	0.0066	0.0068	0.0069	0.0071	0.0073	0.0075	0.0078	0.0080	0.0082
-2.3	0.0084	0.0087	0.0089	0.0091	0.0094	0.0096	0.0099	0.0102	0.0104	0.0107
-2.2	0.0110	0.0113	0.0116	0.0119	0.0122	0.0125	0.0129	0.0132	0.0136	0.0139
-2.1	0.0143	0.0146	0.0150	0.0154	0.0158	0.0162	0.0166	0.0170	0.0174	0.0179
-2.0	0.0183	0.0188	0.0192	0.0197	0.0202	0.0207	0.0212	0.0217	0.0222	0.0228
-1.9	0.0233	0.0239	0.0244	0.0250	0.0256	0.0262	0.0268	0.0274	0.0281	0.0287
-1.8	0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351	0.0359
-1.7	0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436	0.0446
-1.6	0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	0.0526	0.0537	0.0548
-1.5	0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.0630	0.0643	0.0655	0.0668
-1.4	0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0808
-1.3	0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0968
-1.2	0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131	0.1151
-1.1	0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335	0.1357
-1.0	0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1587
-0.9	0.1611	0.1635	0.1660	0.1685	0.1711	0.1736	0.1762	0.1788	0.1814	0.1841
-0.8	0.1867	0.1894	0.1922	0.1949	0.1977	0.2005	0.2033	0.2061	0.2090	0.2119
-0.7	0.2148	0.2177	0.2206	0.2236	0.2266	0.2296	0.2327	0.2358	0.2389	0.2420
-0.6	0.2451	0.2483	0.2514	0.2546	0.2578	0.2611	0.2643	0.2676	0.2709	0.2743
-0.5	0.2776	0.2810	0.2843	0.2877	0.2912	0.2946	0.2981	0.3015	0.3050	0.3085
-0.4	0.3121	0.3156	0.3192	0.3228	0.3264	0.3300	0.3336	0.3372	0.3409	0.3446
-0.3	0.3483	0.3520	0.3557	0.3594	0.3632	0.3669	0.3707	0.3745	0.3783	0.3821
-0.2	0.3859	0.3897	0.3936	0.3974	0.4013	0.4052	0.4090	0.4129	0.4168	0.4207
-0.1	0.4247	0.4286	0.4325	0.4364	0.4404	0.4443	0.4483	0.4522	0.4562	0.4602
-0.0	0.4641	0.4681	0.4721	0.4761	0.4801	0.4840	0.4880	0.4920	0.4960	0.5000

Fold and tear along perforation.



No marks will be given for work done on this page.

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**Pure Mathematics 30
Diploma Examination
June, 2001**

**Multiple-Choice Key,
Numerical-Response Key
and
Sample Answers to
Written-Response
Questions**

PURE MATHEMATICS 30 - June, 2001

MULTIPLE-CHOICE KEY

- | | |
|-------|-------|
| 1. C | 18. D |
| 2. B | 19. A |
| 3. A | 20. B |
| 4. A | 21. D |
| 5. D | 22. C |
| 6. C | 23. B |
| 7. B | 24. C |
| 8. C | 25. A |
| 9. A | 26. B |
| 10. D | 27. D |
| 11. C | 28. B |
| 12. B | 29. D |
| 13. C | 30. D |
| 14. A | 31. A |
| 15. D | 32. C |
| 16. C | 33. A |
| 17. B | |

NUMERICAL-RESPONSE KEY

1. 20
2. 31
3. 5.25
4. 56
5. 7.55
6. 0.04

SAMPLE ANSWERS TO THE WRITTEN-RESPONSE SECTION

Note: The responses that follow represent **ONE** approach to each of the problems. During the diploma examination marking session, provision is made for considering the various approaches students may have used.

Sample Answer for Written Response 1—Total: 10%

1.

Written Response—10%

Use the following information to answer this question.

If each team in a basketball league must play each of the other teams twice, then the total number of games, g , that can be played is given by the function $g(n) = n^2 - n$, where n is the number of teams in the league and $n \in N, n \geq 2$.

- Determine the first 3 terms of the sequence defined by $g(n) = n^2 - n$, $n \in N, n \geq 2$.

A POSSIBLE SOLUTION

$$g(2) = 2, \quad g(3) = 6, \quad g(4) = 12$$

- In this league, where each team must play each of the other teams twice, 72 games have been scheduled. Determine the number of teams in this league. Justify your answer.

A POSSIBLE SOLUTION

9 teams

Graph as a system

$$\begin{aligned} y &= 72 \\ y &= x^2 - x \end{aligned}$$

and determine the intersection point $(9, 72)$, therefore, 9 teams.

Graphs as a function $y = x^2 - x - 72$ and calculate the zeros or root to get 9 teams.

let

$$\begin{aligned} n^2 - n &= 72 \\ n^2 - n - 72 &= 0 \\ (n - 9)(n + 8) &= 0 \\ n &= 9 \text{ or } -8 \end{aligned}$$

- Explain why $2 \times {}_n C_2$ models the number of games that must be scheduled so that n basketball teams play each other exactly two times.

A POSSIBLE SOLUTION

${}_n C_2$ represents all combinations of 2 different teams and therefore represents the number of games if each team plays every other team once. The $2 \times {}_n C_2$ simply doubles the number of games.

- The function $g(n) = n^2 - n$ can be determined using combinations. Verify algebraically that $2 \times {}_n C_2$ equals $n^2 - n$.

A POSSIBLE SOLUTION

$$\begin{aligned}
 2 \times {}_n C_2 &= 2 \times \frac{n!}{(n-2)!2!} \\
 &= 2 \times \frac{n(n-1)}{2} \\
 &= n^2 - n
 \end{aligned}$$

Sample Answer for Written Response 2—Total: 10%

Written Response—10%

2. The function $f(n) = 2.32 \sin[0.0172(n - 285.366)] + 6.54$ approximates the time of **sunrise** in Saskatoon as a function of day number, n . In this function, n is the number of days after December 11, $f(n)$ is measured in hours after midnight, and sine is computed in radians.

- Determine the time of sunrise in Saskatoon on January 31, which is day 51, to the nearest hundredth of an hour.

A POSSIBLE SOLUTION

$$f(n) = 2.32 \sin(0.0172(51 - 285.366)) + 6.54$$
$$f(n) = 8.34 \text{ hours or } 8:20 \text{ am}$$

- Another function, $g(n) = -2.39 \sin[0.0172(n - 285.366)] + 18.64$, gives the time of **sunset** for any day, n , after December 11, in Saskatoon. The value 0.0172 is the same for both functions, f (sunrise) and g (sunset). Explain what this value represents in terms of its sinusoidal graph.

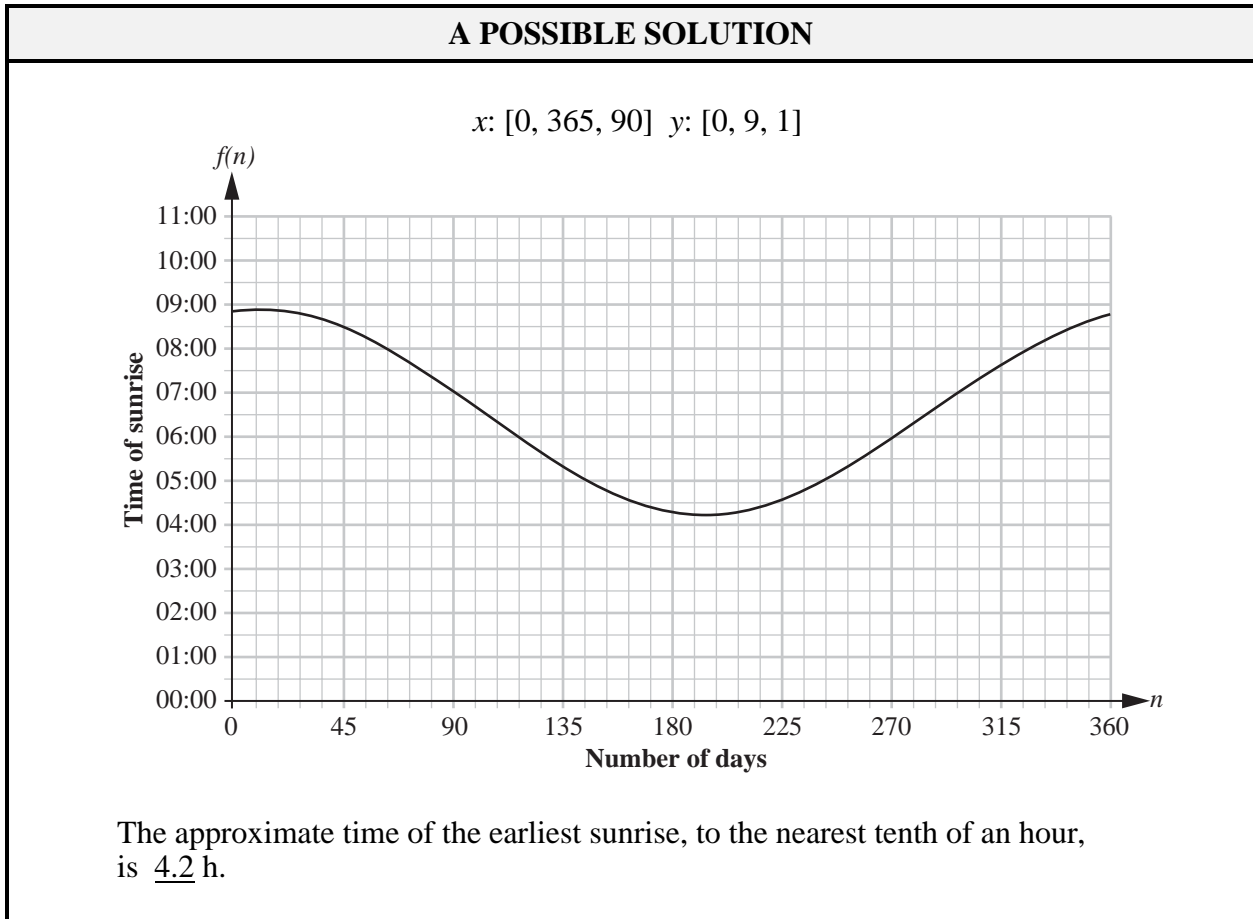
A POSSIBLE SOLUTION

0.0172 is used to determine the period **or** $P = \frac{2\pi}{0.0172} = 365$ days

- With your graphing calculator in radian mode, create a graph for the function f that approximates the time of **sunrise** in Saskatoon. Predict the approximate time of the earliest sunrise.

Sketch your graph on the grid below.

The approximate time of the earliest sunrise, to the nearest tenth of an hour, is _____ h.



- If the function g were to be written in terms of cosine, $y = a \cos b(n - c) + d$, then only **one** of the four parameters, a , b , c , or d , would change. Which parameter would change? Justify your answer.

A POSSIBLE SOLUTION

c would change. The phase shift for sine and cosine graphs is always different. Since the maximum and minimum values and period are the same, a , b , and d will not change

Sample Answer for Written Response 3—Total: 15%

3.

Written Response—15%

Use the following information to answer this question.

A student places a spherical jawbreaker in a cup of water. As the surface of the jawbreaker dissolves, the jawbreaker maintains a spherical shape. The volume of the jawbreaker, V , as a function of time, t , is shown below.

Time t (min)	Volume V (mm^3)
0	905.000
1	814.500
2	733.050
3	659.745
4	

- If it is assumed that this geometric pattern continues, what is the volume of the jawbreaker, to the nearest thousandth of a mm^3 , when $t = 4$ min?

A POSSIBLE SOLUTION

593.771

- Write an equation for volume, V , as a function of time, t , in the form $V = a(b^t)$.

A POSSIBLE SOLUTION

$$V = 905(0.9)^t, t \geq 0$$

- Using your equation from the previous bullet, determine, to the nearest tenth of a minute, the time it takes the jawbreaker to reach 100 mm^3 .

A POSSIBLE SOLUTION

$$V = 905(0.9)^t$$

$$100 = 905(0.9)^t$$

$$\frac{100}{905} = (0.9)^t$$

$$\log\left(\frac{100}{905}\right) = t \log(0.9)$$

$$\frac{\log\left(\frac{100}{905}\right)}{\log 0.9} = t$$

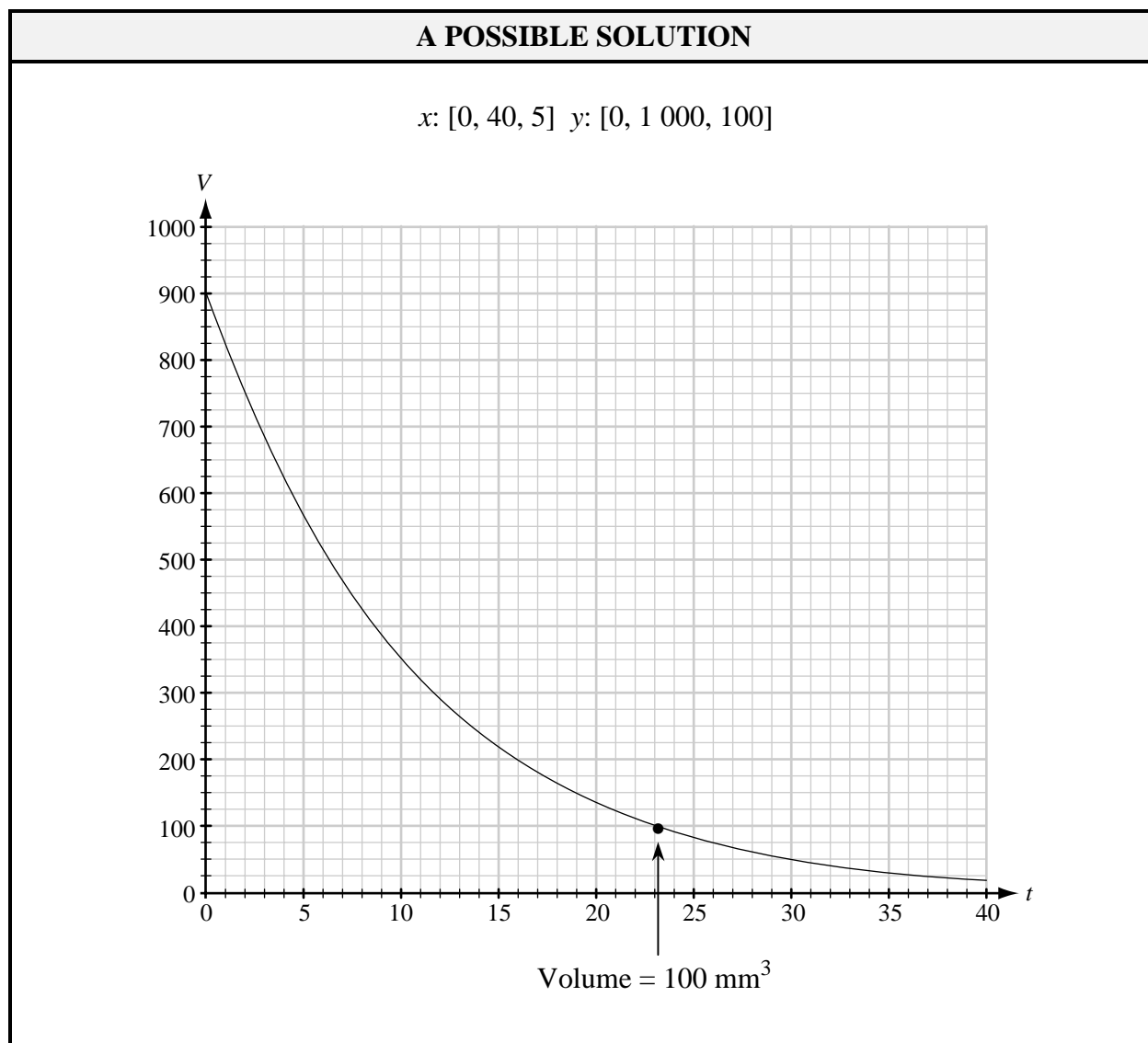
$$20.9 \text{ min} = t$$

- Create a graph of the equation that you found for volume in the second bullet. State an appropriate viewing window for the first 40 minutes of the dissolving time. Use the form

$$[x_{\min}, x_{\max}, x_{\text{scl}}] [y_{\min}, y_{\max}, y_{\text{scl}}]$$

$$x: [\quad , \quad , \quad] \quad y: [\quad , \quad , \quad]$$

On the grid below, sketch your graph and indicate the point where the volume equals 100 mm^3 .



- A particular machine produces many different coloured jawbreakers. The probability that a black jawbreaker is produced is 0.12. If the jawbreakers are packaged in boxes of 60, then what is the probability, to the nearest hundredth, that a box will contain at least 5 black jawbreakers?

A POSSIBLE SOLUTION

$$n(1-p) = 0.88 \times 60 = 52.8$$

$$n(p) = 0.12 \times 60 = 7.2 \quad \therefore \text{Large sample}$$

At least 5

$$\mu = 0.12 \times 60 = 7.2$$

$$\sigma = \sqrt{7.2(0.88)} = 2.57$$

$$z_1 = \frac{5 - 7.2}{2.57} \doteq -0.86$$

$$1 - 0.1922 = 0.808$$

$$\doteq 0.81$$

or

Normalcdf

$$(5, 1E99, 7.2, \sqrt{(7.2)(0.88)}) = 0.809$$

$$\doteq 0.81$$

or

$$\text{binomcdf}(60, 0.88, 55) = 0.861$$

$$\doteq 0.86$$

No marks will be given for work done on this page.