Pure Mathematics 30

Student Project: Skydiving

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Alberta Education

Pure Mathematics 30

Project—Skydiving

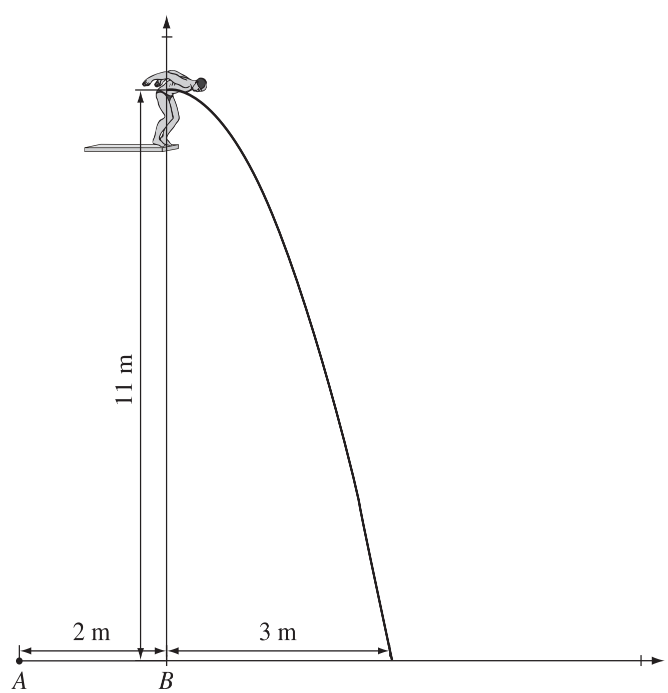
Introduction

In this project, you will be investigating the behaviour of falling objects, first without air resistance, and then with air resistance. Part A deals with the parabolic paths followed by free-falling objects, Part B deals with the use of exponential functions to describe the effects of air resistance on falling objects, and Part C deals with the relationship between parachute design factors and terminal velocity. In Part D, you will research another aspect of skydiving of your choice.

Part A

In Part A, the effects of air resistance are ignored, and if objects fall relatively short distances, the model shown below gives accurate results. For falls from greater heights, this model gives very inaccurate results.

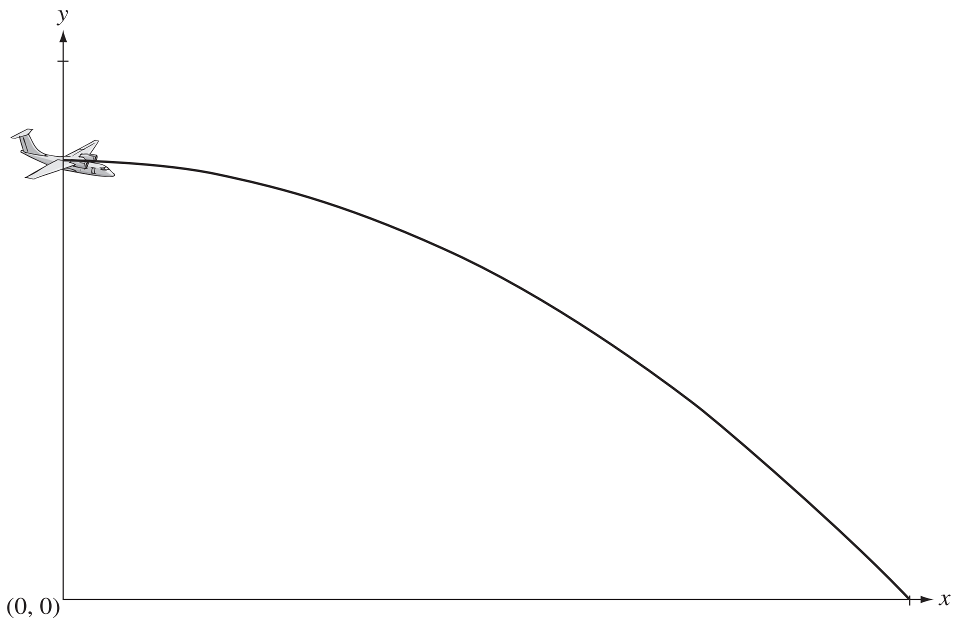
Olympic platform divers carry out a series of complex manoeuvres from a board that is 10 m above the surface of the water. One simple dive can be modelled by the parabolic path shown in the diagram below.



1. If the origin of the coordinate grid is at point *B*, determine the standard-form equation of the parabolic path.

2. If the origin of the coordinate grid is at point *A*, determine the general-form equation of the parabolic path.

Objects falling from moving aircraft will also follow parabolic paths, as shown in the diagram below. The equations for these paths have y as the height above the ground in metres and x as the horizontal distance along the ground in metres.



3. An aircraft is travelling at 250 m/s at an altitude of 12 250 m when a piece of ice falls off the wing and falls to the ground along a parabolic path. If there is no air resistance, the equation of the parabolic path, in standard form, is given by $y - 12\ 250 = -\left(\frac{4.9}{62\ 500}\right)x^2$. Determine algebraically the coordinates of the point where the ice hits the ground.

This prediction is very far from the observed behaviour, where the horizontal distance travelled is approximately 0.83 km before the forward motion stops and the ice falls vertically. Air resistance (see Part B) has to be taken into account for an accurate prediction.

Part B

Air resistance will reduce the net downward acceleration, with air resistance becoming greater at higher velocities. One model for air resistance has the features listed below.

- The vertical acceleration, a, is given by the function $a = g - kv$, where g is the acceleration due to gravity, v is the vertical velocity downward, and k is a constant dependent on the size and shape of the falling object.
- After some time, the object falls at a constant velocity called the terminal velocity, v_t. At this time the downward force due to gravity is exactly equal to the upward force due to air resistance, and the vertical acceleration is zero. At the terminal velocity $a = 0$, so $0 = g - kv_t$ or $v_t = \frac{g}{k}$.

1. For a human in free fall, $k = 0.15/\text{s}$, and for a parachutist, $k = 2.00/\text{s}$. If the acceleration due to gravity is $9.81\ \text{m/s}^2$, determine the terminal velocities, to the nearest tenth of a metre per second, of the human in free fall and of the parachutist.

2. Describe the effects that changing the value of k would have on the terminal velocity of the object.

The equation for vertical velocity, v, as a function of time, t, can be written in terms of the terminal velocity, v_t, and the base, b, of an exponential function. The equation in this simpler form is $v = v_t(1 - b^t)$, where v_t and b depend on the shape of the falling object.

3. Ice falling from the wing of an aircraft has a terminal velocity of 45 m/s. An equation describing the vertical velocity of the falling ice as a function of time is $v = 45\left(1 - (0.804)^t\right)$. Determine the vertical velocity, to the nearest tenth of a metre per second, of the falling ice after 12.0 s.

4. The equation describing the velocity of the ice falling from the wing as a function of time is $v = 45\left(1 - (0.804)^t\right)$. Determine algebraically the time required, to the nearest tenth of a second, for the ice to reach a velocity of 98% of its terminal velocity of 45 m/s.

Part C

Parachutes are designed to minimize the impact of a skydiver with the ground. Typical parachutes limit the skydiver's terminal velocity to between 4.50 m/s and 6.10 m/s, which is like jumping off a platform 1.00 m to 1.90 m above the ground.

1. If the acceleration due to gravity is 9.81 m/s^2, determine the range of values of k, to the nearest hundredth, corresponding to terminal velocities between 4.50 m/s and 6.10 m/s. Use the formula $v_t = \frac{g}{k}$ and express your answer as an inequality.

2. For a particular design of training parachute, the values of k are normally distributed with a mean of 2.13/s and a standard deviation of 0.04/s. Determine the probability, to the nearest hundredth, that the value of k is greater than or equal to 2.08/s.

3. It is found that 99% of a particular design of sport parachute have k-values greater than or equal to 1.650/s. If the mean value of k is 1.730/s, determine, to the nearest thousandth, the standard deviation of the distribution of k-values.

4. For beginner parachutists, the terminal velocity must be less than 5.00 m/s, and the parachutes used have a k-value that is normally distributed with a mean of 2.05/s and a standard deviation of 0.04/s. If the acceleration due to gravity is 9.81 m/s^2, determine the probability, to the nearest thousandth, that the terminal velocity is less than 5.00 m/s.

Part D

1. Parachute design has evolved over the years, enabling parachutists to make jumps from greater altitudes, and allowing them more control over how and where they land. Research different designs for parachutes, and compare two designs of parachutes in terms of how they solve the problems of high-altitude and precision jumping.

2. Ski-jumpers fly through the air in such a way that the air resistance is very small in the horizontal direction, but is large in the vertical direction. Explain how this allows the ski-jumper to reach a higher take-off velocity, to stay in the air for a longer time interval, and to jump a longer distance. Support your explanations with numerical examples.

You may wish to use the following websites for information:

www.seed.slb.com/en/scictr/lab/parachute/notes.htm
(building and testing parachutes in a school setting)

www.skydiving-parachuting-guide.com
(history and analysis of sport and military parachuting)

www.pcprg.com/rocketre.htm
(designing a parachute to recover model rockets)

www.edinformatics.com/inventions_inventors/parachute.htm
(history and analysis; contains many links to further research)

www.fuzilogik.com/index.php?option=com_content&task=view&id=559&Itemid=105
(ski-jumping, describes most of the key ideas)

www.smso.net/Ski-jumping
(contains further links to many of the Olympic and world championship venues)

www.vancouver2010.com/en/WinterGames/WinterGamesSports/SJ
(contains further links to technical sites governing the construction of ski hills and ski jumps)

Note: Website addresses sometimes change. If the websites listed above are not available, use a search engine and type in keywords such as those listed below:

"parachute design"

"parachute history"

"ski jumping"

"ski jumping techniques"