2018–2019 Written-Response Information

Mathematics 30-1

Alberta Government

This document was written primarily for:

Students	✓
Teachers	✓ of Mathematics 30–1
Administrators	✓
Parents	
General Audience	
Others	

Distribution: This document is posted on the Alberta Education website.

Contents

Please note that if you cannot access one of the direct website links referred to in this document, you can find diploma examination-related materials on the Alberta Education website.

Introduction

Starting in 2018–2019, the Mathematics 30-1 Diploma Examination will contain written-response questions. The purpose of this document is to provide information about these diploma exams. Examples of written-response questions, sample responses, and scoring rationales as they relate to the scoring guides are included. This document should be used in conjunction with the *Mathematics 30–1 Program of Studies* and the *Mathematics 30–1 Assessment Standards and Exemplars* documents, which contain details about the philosophy of the program and the assessment standards. For examples of machine-scored questions, please refer to the *Mathematics 30-1 Released Items*, which can be found on the Alberta Education website.

Teachers are encouraged to share the contents of this document with students.

If you have comments or questions regarding this document, please contact Delcy Rolheiser, Mathematics 30-1 Exam Manager, by email at Delcy.Rolheiser@gov.ab.ca, or by phone at (780) 415-6181 (dial 310-0000 to be connected toll free).

Intent of the Written-response Component

In 2016, it was announced that high school mathematics diploma examinations will integrate a written-response component that will require students to communicate their understanding of mathematical concepts and demonstrate their algebraic skills. Therefore, the written-response component is designed to complement the machine-scored portion of the diploma examination by allowing for greater coverage of the learning outcomes in the program of studies.

The written-response component also provides an opportunity to address the mathematical processes outlined in the *Mathematics 30-1 Program of Studies*. Of the seven mathematical processes, the written-response component will focus primarily on communication (C), problem solving (PS), connections (CN), reasoning (R), and visualization (V).

Each specific outcome in the *Mathematics 30-1 Program of Studies* lists the related mathematical processes for that outcome. If technology (T) is not listed as a process, students are expected to meet the outcome **without** the use of technology and must use an algebraic process to receive credit on a question involving the outcome.

2018–2019 Diploma Examination Specifications and Design

Each Mathematics 30–1 diploma examination is designed to reflect the content outlined in the *Mathematics 30–1 Program of Studies*. The percentage weightings shown below will not necessarily match the percentage of class time devoted to each topic. The diploma examination will be developed to be completed in 2.5 hours.

Specifications

The format and content of the Mathematics 30–1 diploma examinations in the 2018–2019 school year are as follows:

Question Format	*Number of Questions*	*Emphasis*
Machine-scored • Multiple Choice • Numerical Response	 24 8	75%
Written Response	3	25%

Note: The three written-response questions are equally weighted.

Topic	*Emphasis*
Relations and Functions	53%–58%
Trigonometry	27%–33%
Permutations, Combinations, and Binomial Theorem	14%–18%

Procedural, conceptual, and problem-solving cognitive levels are addressed throughout the examination. The approximate emphasis of each cognitive level is given below. An explanation of the cognitive levels can be found on page 40.

Cognitive Level	*Emphasis*
Procedural	30%
Conceptual	34%
Problem Solving	36%

Written-response Instructions

The following instructions will be included in the instructions pages of all mathematics diploma exam booklets.

- Write your responses in the test booklet as neatly as possible.
- For full marks, your responses must address **all** aspects of the question.
- All responses, including descriptions and/or explanations of concepts, must include pertinent ideas, calculations, formulas, and correct units.
- Your responses must be presented in a well-organized manner. For example, you may organize your responses in paragraphs or point form.

Written-response Question Design

The written-response component is designed to assess the degree to which students can draw on their mathematical experiences to solve problems, explain mathematical concepts, and demonstrate their algebraic skills. A written-response question may cover more than one specific outcome and will require students to make connections between concepts. Each written-response question will consist of two bullets and will address multiple cognitive levels. Students should be encouraged to try to solve the problems in both bullets as an attempt at a solution may be worth partial marks.

Students may be asked to solve, explain, or prove in a written-response question. Students are required to know the definitions and expectations of directing words such as **algebraically**, **compare**, **determine**, **evaluate**, **justify**, and **sketch**. A list of these directing words and their definitions can be found on page 39.

General Scoring Guides

The General Scoring Guides, developed in consultation with teachers and Alberta Education staff, describe the criteria and performance level at each score-point value. These General Scoring Guides will be used to develop specific scoring descriptions for each written-response question.

In scoring the written-response questions, markers will evaluate how well students

- understand the problem or the mathematical concept;
- correctly apply mathematical knowledge and skills;
- use problem-solving strategies and explain their solutions and procedures;
- communicate their solutions and mathematical ideas.

General Scoring Guide for a 2-mark bullet

Score	General Description
NR	No response is provided.
0	In the response, the student does not address the question or provides a solution that is invalid.
0.5	
1	In the response, the student demonstrates basic mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a partial solution.
1.5	
2	In the response, the student demonstrates complete mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a complete and correct solution.

General Scoring Guide for a 3-mark bullet

Score	General Description
NR	No response is provided.
0	In the response, the student does not address the question or provides a solution that is invalid.
0.5	
1	In the response, the student demonstrates minimal mathematical understanding of the problem by applying an appropriate strategy or some relevant mathematical knowledge to complete initial stages of a solution.
1.5	
2	In the response, the student demonstrates good mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a partial solution.
2.5	
3	In the response, the student demonstrates complete mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a complete and correct solution.

Specific Scoring Guides for each written-response question will provide detailed descriptions to clarify expectations of student performance at each benchmark score of 0, 1, 2 and 3. A student response that does not meet the performance level of a benchmark score may receive an augmented score of 0.5, 1.5, or 2.5. Descriptions of these augmented scores will be determined with teachers at each marking session and are not an exhaustive list. Each bullet will be scored separately and the scores will be combined for a total of 5 marks.

Written-response Question 1

Use the following information to answer written-reponse question 1.

Two rural Alberta towns experience changes in population due to the construction of a new highway. Equations modelling the population of each town, where P is population and t is the number of years after January 1, 2016, are shown below.

Town A $P_A = 4\,000(0.97)^t$

Town B $P_B = 2\,500(1.05)^t$

Written Response—5 marks

- **Describe** the significance of each numerical value in each equation listed above. **[2 marks]**

A POSSIBLE SOLUTION

The first numerical value in each equation represents the starting population of each town. This means that 4 000 is the starting population of Town A and 2 500 is the starting population of Town B.

The second numerical value in each equation (inside the brackets) represents the rate of population change for each town. This means that 0.97 represents a 3% annual decrease in the population of Town A and 1.05 represents a 5% annual increase in the population of Town B.

- **Algebraically determine** the number of years, to the nearest tenth, that it will take for the population of the two towns to be the same. **[3 marks]**

A POSSIBLE SOLUTION

$$P_A = P_B$$

$$4\,000(0.97)^t = 2\,500(1.05)^t$$

$$\left(\frac{4\,000}{2\,500}\right)(0.97)^t = (1.05)^t$$

$$1.6(0.97)^t = (1.05)^t$$

$$1.6 = \frac{(1.05)^t}{(0.97)^t}$$

$$1.6 = \left(\frac{1.05}{0.97}\right)^t$$

$$\log 1.6 = t\log\left(\frac{1.05}{0.97}\right)$$

$$t = \frac{\log 1.6}{\log\left(\frac{1.05}{0.97}\right)}$$

$$t = 5.930\,692\,186$$

$$t = 5.9 \text{ years}$$

The populations will be the same in 5.9 years.

Specific Scoring Guide for Written-response Question 1

Bullet 1:

Score	General Description	Specific Description
NR	No response is provided.	
0	In the response, the student does not address the question or provides a solution that is invalid.	The response does not contain a valid description for any of the numerical values in either equation.
0.5		*For example, the student could* • *correctly identify the significance of one numerical value in one equation.*
1	In the response, the student demonstrates basic mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a partial solution.	In the response, the student • correctly identifies the initial populations of both towns OR • correctly identifies the specific rates of population change for both towns OR • correctly identifies the initial population and specific rate of population change for one town.
1.5		*For example, the student could* • *correctly identify the significance of three numerical values in the two equations.*
2	In the response, the student demonstrates complete mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a complete and correct solution.	In the response, the student • correctly identifies the initial populations and specific rates of population change for both towns (the rate of increase must be explicitly differentiated from the rate of decrease).

Please note that the augmented score descriptions (i.e., the italicized statements) are determined at marking sessions. They are not an exhaustive list.

Bullet 2:

Score	General Description	Specific Description
NR	No response is provided.	
0	In the response, the student does not address the question or provides a solution that is invalid.	The response does not contain a valid equation that could be used to represent the problem or relevant algebraic steps that would lead to a solution to the problem. Note: An answer determined from a non-algebraic process will receive a score of 0.
0.5		*For example, the student could* • *create an equation that represents the problem but does not attempt to solve the equation.*
1	In the response, the student demonstrates minimal mathematical understanding of the problem by applying an appropriate strategy or some relevant mathematical knowledge to complete initial stages of a solution.	In the response, the student • creates an equation that represents the problem and completes an initial algebraic step but does not arrive at a solution or arrives at an incorrect solution.
1.5		*For example, the student could* • *create an equation that represents the problem and correctly complete some algebraic steps to arrive at a correct solution; however, the response does not contain the correct application of a logarithm.*
2	In the response, the student demonstrates good mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a partial solution.	In the response, the student • creates an equation that represents the problem and correctly completes some of the steps of an algebraic process. The response contains the application of a logarithm but the solution is incomplete or incorrect.
2.5		*For example, the student could* • *create an equation that represents the problem and correctly apply a logarithm; however, the solution is incorrect due to a misunderstanding of the mathematics.*
3	In the response, the student demonstrates complete mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a complete and correct solution.	In the response, the student • creates an equation that represents the problem and correctly completes all of the steps of an algebraic process to arrive at a correct solution (the answer may be incorrect due to a calculation error).

Sample Responses to Written-response Question 1

Note: The sample responses are intended to inform teachers and students of how the scoring guide is applied to specific questions and to illustrate the expectations for student performance.

Sample Response 1

Use the following information to answer written-reponse question 1.

Two rural Alberta towns experience changes in population due to the construction of a new highway. Equations modelling the population of each town, where P is population and t is the number of years after January 1, 2016, are shown below.

Town A $P_A = 4\,000(0.97)^t$

Town B $P_B = 2\,500(1.05)^t$

Written Response—5 marks

- **Describe** the significance of each numerical value in each equation listed above. **[2 marks]**

$P_A = 4000(0.97)^t$ → time passed

4000: original population.

0.97: growth #, (will be decreasing by 3% each unit of t: time.

$P_B = 2500(1.05)^t$ → time passed.

2500: original pop.

1.05: growth rate. will increase by 5% each unit of t: time

To go back after using an internal link, simultaneously press and hold Alt ← (in some browsers).

Intersect: 5.93 years.

- **Algebraically determine** the number of years, to the nearest tenth, that it will take for the population of the two towns to be the same. **[3 marks]**

$$4000\log(0.97)^t = 2500(\log 1.05)^t$$

$$4000t\log 0.97 = 2500t\log 1.05$$

$$P_A = P_B$$

$$\frac{4000(0.97)^t}{2500} = \frac{2500(1.05)^t}{2500}$$

$$\frac{1.6(0.97)^t}{0.97^t} = \frac{(1.05)^t}{(0.97)^t}$$

$$1.6 = (1.05)^t/(0.97)^t$$

$$1.6 = (1.05/0.97)^t$$

$$t = \log_{(1.05/0.97)} 1.6$$

$$\boxed{t = 5.93 \text{ years.}}$$

Score	Rationale
Bullet 1: 2 marks **Bullet 2:** 3 marks	In the first bullet, an accurate and detailed description of all of the numerical values in both equations is provided. The response clearly indicates that 0.97 represents a decrease of 3% for each unit of time (i.e., every year) and that 1.05 represents an increase of 5% for each unit of time. In the second bullet, an equation representing the problem is shown and an appropriate algebraic strategy, including the application of a logarithm, is used to determine the correct solution. Even though the final answer is rounded incorrectly, the error does not illustrate a misunderstanding of solving exponential equations, so the response receives full credit.

To go back after using an internal link, simultaneously press and hold Alt ← (in some browsers).

Sample Response 2

Use the following information to answer written-reponse question 1.

Two rural Alberta towns experience changes in population due to the construction of a new highway. Equations modelling the population of each town, where P is population and t is the number of years after January 1, 2016, are shown below.

Town A $P_A = 4\,000(0.97)^t$

Town B $P_B = 2\,500(1.05)^t$

Written Response—5 marks

- **Describe** the significance of each numerical value in each equation listed above. **[2 marks]**

$P_A = 4000\,(0.97)^{t}$ ← time

initial value change

$2500 \cdot (1.05)^{t}$

To go back after using an internal link, simultaneously press and hold Alt ← (in some browsers).

- **Algebraically determine** the number of years, to the nearest tenth, that it will take for the population of the two towns to be the same. **[3 marks]**

$$4000(0.97)^t = 2500(1.05)^t$$

$$\frac{4000}{2500}(0.97)^t = (1.05)^t$$

$$1.6(0.97)^t = (1.05)^t$$

$$1.6 = \frac{(1.05)^t}{(0.97)^t}$$

$$1.6 = \left(\frac{1.05}{0.97}\right)^t$$

$$1.6 = 1.0824^t$$

$$\log_{1.0824} 1.6 = t$$

$$t = 5.9 \text{ years}$$

Score	Rationale
Bullet 1: 1 mark **Bullet 2:** 3 marks	In the first bullet, the initial populations of the two towns, 4 000 and 2 500, are identified as initial values. However, the solution does not contain specific details regarding the significance of 0.97 and 1.05. In the second bullet, an equation that represents the problem is created and an appropriate algebraic strategy, including the application of a logarithm, is used to find a complete and correct solution.

To go back after using an internal link, simultaneously press and hold Alt ← (in some browsers).

Sample Response 3

Use the following information to answer written-reponse question 1.

Two rural Alberta towns experience changes in population due to the construction of a new highway. Equations modelling the population of each town, where P is population and t is the number of years after January 1, 2016, are shown below.

Town A $P_A = 4\,000(0.97)^t$

Town B $P_B = 2\,500(1.05)^t$

Written Response—5 marks

- **Describe** the significance of each numerical value in each equation listed above. **[2 marks]**

Town A

$P_A = 4000\,(0.97)^t$ ← years/length of time

Population after

initial Population

Rate of population decay

Town B

$P_B = 2500\,(1.05)^t$ ← time in years/length

final pop

initial population

Rate of growth

- **Algebraically determine** the number of years, to the nearest tenth, that it will take for the population of the two towns to be the same. **[3 marks]**

$P_A = 4000\,(0.97)^t$

$P_B = 2500\,(1.05)^t$

① 5.93 years they will both are a value of 3338.92 people

$\boxed{5.9 \text{ years}}$

$\boxed{3339 \text{ people living in each town}}$

Score	Rationale
Bullet 1: 1.5 marks **Bullet 2:** 0 marks	In the first bullet, the initial populations of both towns and the rates of population change are clearly indicated. However, even though the rates of change are described as growth and decay, specific details about the rates of change are not included. In the second bullet, there is no evidence that the solution was determined using an algebraic process, so no credit is awarded.

Sample Response 4

Use the following information to answer written-reponse question 1.

Two rural Alberta towns experience changes in population due to the construction of a new highway. Equations modelling the population of each town, where P is population and t is the number of years after January 1, 2016, are shown below.

Town A $P_A = 4\,000(0.97)^t$

Town B $P_B = 2\,500(1.05)^t$

Written Response—5 marks

- **Describe** the significance of each numerical value in each equation listed above. **[2 marks]**

$y = ab^{t/p}$

$P_A = 4000\,(0.97)^t$

Starting population

The town population decreases by 3% each year (t)

$P_B = 2500\,(1.05)^t$

Starting pop

Population increases by 5% each year

- **Algebraically determine** the number of years, to the nearest tenth, that it will take for the population of the two towns to be the same. **[3 marks]**

$c^x = y$

$\log_c y = x$

$P_A = P_B$

$$\frac{4000(0.97)^t}{2500} = \frac{2500(1.05)^t}{2500}$$

$$\frac{1.6(0.97)^t}{0.97^t} = \frac{1.05^t}{0.97^t}$$

$$1.6 = \frac{1.05^t}{0.97^t}$$

$$1.6 = 1.082^t$$

$\log_{1.05}$

$t = 5.93$ years

$t = 5.9$

$4000(0.97)^t = 2500(1.05)^t$

Score	Rationale
Bullet 1: 2 marks **Bullet 2:** 1.5 marks	In the first bullet, an accurate and detailed description of all of the numerical values in both equations is provided. The response clearly indicates that 0.97 represents a decrease of 3% each year and that 1.05 represents an increase of 5% each year. In the second bullet, the response contains some correct steps in an algebraic process. Even though the correct answer is given, algebraic evidence supporting the answer is incomplete (i.e., an appropriate application of a logarithm has been omitted from the solution).

Sample Response 5

Use the following information to answer written-reponse question 1.

Two rural Alberta towns experience changes in population due to the construction of a new highway. Equations modelling the population of each town, where P is population and t is the number of years after January 1, 2016, are shown below.

Town A $P_A = 4\,000(0.97)^t$

Town B $P_B = 2\,500(1.05)^t$

Written Response—5 marks

- **Describe** the significance of each numerical value in each equation listed above. **[2 marks]**

PA - Population of Town A → population ~~of~~ going down by 0.03 3% each year

PB - Population of Town B → pop. goes up 5% a year.

t Represents the time since January 1st 2016, meaning that the population of the two towns fluctuates each year.

- **Algebraically determine** the number of years, to the nearest tenth, that it will take for the population of the two towns to be the same. **[3 marks]**

$$4000(0.97)^t = 2500(1.05)^t$$

$$\frac{1.6(0.97)^t}{0.97^t} = \frac{(1.05)^t}{0.97^t}$$

$$1.6 = \frac{(1.05)^t}{(0.97)^t}$$

$$1.6 = 1.08^t$$

$$\log_{1.08} 1.6 = t$$

$$t = 6.1$$

Score	Rationale
Bullet 1: 1 mark **Bullet 2:** 2.5 marks	In the first bullet, an accurate and detailed description of the rates of change are included; however, the response does not include a description of the significance of the values of 4 000 or 2 500. In the second bullet, an equation representing the problem and several correct algebraic steps toward a solution are provided. A logarithm is applied correctly; however, the use of a rounded value in one of the calculations results in an incorrect final solution.

Written-response Question 2

Written Response—5 marks

- **Prove** that the equation $\dfrac{1}{\tan\theta} + \tan\theta = \dfrac{\sec^2\theta}{\tan\theta}$ is an identity using an algebraic approach. **[3 marks]**

A POSSIBLE SOLUTION

$\dfrac{1}{\tan\theta} + \tan\theta$	$\dfrac{\sec^2\theta}{\tan\theta}$
$\dfrac{1}{\frac{\sin\theta}{\cos\theta}} + \dfrac{\sin\theta}{\cos\theta}$	$\dfrac{1}{\cos^2\theta} \div \dfrac{\sin\theta}{\cos\theta}$
$\dfrac{\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{\cos\theta}$	$\dfrac{1}{\cos^2\theta} \times \dfrac{\cos\theta}{\sin\theta}$
$\dfrac{\cos\theta(\cos\theta)}{\sin\theta(\cos\theta)} + \dfrac{\sin\theta(\sin\theta)}{\cos\theta(\sin\theta)}$	$\dfrac{1}{\cos\theta\sin\theta}$
$\dfrac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta}$	
$\dfrac{1}{\cos\theta\sin\theta}$	

LS = RS

Because LS = RS, the identity is valid for all permissible values of θ.

Note: Many different proofs are possible but all proofs must end with a concluding equality statement.

- **Determine** all of the non-permissible values of the identity. **[2 marks]**

A POSSIBLE SOLUTION

To determine the non-permissible values, we must consider the denominators of any fractions on either side of the identity and any trigonometric ratios that have undefined values. Tanθ is in the denominator on both sides of the identity. Tanθ and sec θ also have undefined values for θ.

$$\tan\theta \neq 0 \text{ so } \theta \neq 0,\ \pi,\ 2\pi,\ \ldots$$

The general statement representing these values in the domain $\theta \in R$ is $\theta \neq n\pi,\ n \in I$.

$$\tan\theta \text{ cannot be undefined so } \theta \neq \frac{\pi}{2},\ \frac{3\pi}{2},\ \frac{5\pi}{2},\ \ldots$$

The general statement representing these values in the domain $\theta \in R$ is $\theta \neq \frac{\pi}{2} + n\pi,\ n \in I$.

$$\sec\theta \text{ cannot be undefined, which means that } \cos\theta \neq 0 \text{ and so } \theta \neq \frac{\pi}{2},\ \frac{3\pi}{2},\ \frac{5\pi}{2},\ \ldots$$

The general statement representing these values in the domain $\theta \in R$ is $\theta \neq \frac{\pi}{2} + n\pi,\ n \in I$.

Combining all of these components, all of the non-permissible values for the identity can be represented by the general statement

$$\theta \neq \frac{n\pi}{2},\ n \in I$$

Specific Scoring Guide for Written-response Question 2

Bullet 1:

Score	General Description	Specific Description
NR	No response is provided.	
0	In the response, the student does not address the question or provides a solution that is invalid.	The response does not contain the substitution of relevant trig identities or simplification steps that would lead to the proof of the trig identity.
0.5		*For example, the student could* • *complete one relevant substitution on one side of the identity.*
1	In the response, the student demonstrates minimal mathematical understanding of the problem by applying an appropriate strategy or some relevant mathematical knowledge to complete initial stages of a solution.	In the response, the student • completes one relevant substitution to both sides of the identity or one relevant substitution and one relevant simplification step to one side of the identity.
1.5		*For example, the student could* • *complete relevant substitution and simplification steps on both sides of the identity but equality between the sides is not established due to multiple errors or misunderstandings of the mathematics.*
2	In the response, the student demonstrates good mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a partial solution.	In the response, the student • applies relevant substitution and simplification steps to each side of the identity but equality between the sides is not established due to an error or a misunderstanding of the mathematics.
2.5		*For example, the student could* • *apply appropriate trig identities to illustrate how the LS of the identity is equivalent to the RS of the identity but equality between the two sides is not explicitly expressed.*
3	In the response, the student demonstrates complete mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a complete and correct solution.	In the response, the student • applies appropriate trig identities to illustrate how the LS of the identity is equivalent to the RS of the identity (equality between the two sides is explicitly expressed).

To go back after using an internal link, simultaneously press and hold Alt ← (in some browsers).

Bullet 2:

Score	General Description	Specific Description
NR	No response is provided.	
0	In the response, the student does not address the question or provides a solution that is invalid.	The response does not contain relevant evidence that would lead to determining the non-permissible values of the identity or the identification of any non-permissible values.
0.5		*For example, the student could* • *list some of the non-permissible values for a restricted domain without providing supporting evidence.*
1	In the response, the student demonstrates basic mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a partial solution.	In the response, the student • creates a correct statement that includes all of the non-permissible values for the identity but fails to support this statement with relevant evidence OR • creates a statement that includes some of the non-permissible values (or a list of non-permissible values for a restricted domain) and supports the statement with relevant evidence.
1.5		*For example, the student could* • *create a correct statement, supported by evidence, that includes all of the non-permissible values for the identity but an error is present.*
2	In the response, the student demonstrates complete mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a complete and correct solution.	In the response, the student • creates a correct statement that includes all of the non-permissible values for the identity and supports this statement with relevant evidence.

Sample Responses to Written-response Question 2

Sample Response 1

Written Response—5 marks

- **Prove** that the equation $\frac{1}{\tan\theta} + \tan\theta = \frac{\sec^2\theta}{\tan\theta}$ is an identity using an algebraic approach. **[3 marks]**

$$\cot\theta + \tan\theta$$

$$\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}$$

$$\frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta\cos\theta}$$

$$\frac{1}{\cos^2\theta} \div \frac{\sin\theta}{\cos\theta}$$

$$= \frac{1}{\cos^2\theta} \times \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\cos\theta}{\cos^2\theta\sin\theta}$$

$$= \frac{1}{\cos\theta\sin\theta}$$

$$= \frac{1}{\sin\theta\cos\theta}$$

$$LS = RS$$

- **Determine** all of the non-permissible values of the identity. **[2 marks]**

$\tan\theta \neq 0 \rightarrow \frac{\sin\theta}{\cos\theta} \rightarrow \cos\theta \neq 0 \rightarrow$  $\therefore \theta \neq \frac{\pi}{2} + \pi n$

$\sec^2\theta \neq 0 \rightarrow \frac{1}{\cos^2\theta} \rightarrow \cos^2\theta \neq 0 \rightarrow$  $\therefore \theta \neq \frac{\pi}{2} + \pi n$

Score	Rationale
Bullet 1: 3 marks **Bullet 2:** 1 mark	In the first bullet, appropriate trigonometric identites are substituted into each side of the given identity and are simplified correctly. Equality between the two sides is clearly illustrated and explicitly stated, resulting in full credit for this bullet. In the second bullet, only some of the non-permissible values are provided with relevant reasoning; however, the reasoning provided for the non-permissible values for tangent is incorrect.

Sample Response 2

Written Response—5 marks

- **Prove** that the equation $\frac{1}{\tan\theta} + \tan\theta = \frac{\sec^2\theta}{\tan\theta}$ is an identity using an algebraic approach. **[3 marks]**

LS $\left(\frac{1}{\tan\theta} + \tan\theta\right)$

$\frac{1}{\tan\theta} + \tan\theta$

$\frac{1}{\left(\frac{\sin\theta}{\cos\theta}\right)} + \frac{\sin\theta}{\cos\theta}$

$\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}$

$\frac{\cos^2\theta}{\sin\theta\cos\theta} + \frac{\sin^2\theta}{\cos\theta\sin\theta}$

$\frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta}$

$\frac{1}{\sin\theta\cos\theta}$

RS $\left(\frac{\sec^2\theta}{\tan\theta}\right)$

$\frac{\sec^2\theta}{\tan\theta}$

$\frac{\left(\frac{1}{\cos^2\theta}\right)}{\tan\theta}$

$\frac{1}{\tan\theta\cos^2\theta}$

$\frac{1}{\left(\frac{\sin\theta}{\cancel{\cos\theta}}\right)\cdot\cos^{\cancel{2}}\theta}$

$\frac{1}{\sin\theta\cos\theta}$

LS = RS

- **Determine** all of the non-permissible values of the identity. **[2 marks]**

$\sin\theta = 0 \;\}\; 0, \pi, 2\pi \;\}\; \theta = \pi n, n \in I$

$\cos\theta = 0 \;\}\; \frac{\pi}{2}, \frac{3\pi}{2} \;\}\; \theta = \frac{\pi}{2} + \pi n, n \in I$

Score	Rationale
Bullet 1: 3 marks **Bullet 2:** 2 marks	In the first bullet, equality between the two sides is clearly illustrated and explicitly stated through the substitution and simplification of trig identities on both sides of the identity. In the second bullet, fully supported statements that include all of the non-permissible values are provided.

To go back after using an internal link, simultaneously press and hold Alt ← (in some browsers).

Sample Response 3

Written Response—5 marks

- **Prove** that the equation $\frac{1}{\tan\theta} + \tan\theta = \frac{\sec^2\theta}{\tan\theta}$ is an identity using an algebraic approach. **[3 marks]**

$$\frac{1}{\frac{\sin\theta}{\cos\theta}} + \frac{\sin\theta}{\cos\theta}$$

$$\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}$$

$$\frac{\cos^2\theta + \sin^2\theta}{\cos\theta\sin\theta}$$

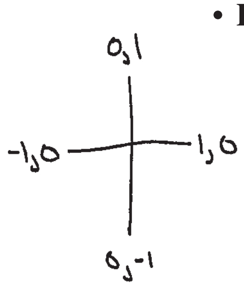
$$\frac{1}{\cos\theta\sin\theta}$$

$$\frac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}} = \frac{\frac{\cos\theta}{\cos^2\theta}}{\sin\theta} = \quad s$$

$$\frac{\frac{1}{\cos^2\theta}}{\frac{\sin\theta}{\cos\theta}}$$

$$\frac{1}{\cos^2\theta} \times \frac{\cos\theta}{\sin\theta} = \frac{\cos\theta}{\cos^2\theta\sin\theta}$$

- **Determine** all of the non-permissible values of the identity. **[2 marks]**



$\tan\theta \neq 0$

$\theta \neq 0 + 180°n,\ n \in I$

Score	Rationale
Bullet 1: 2 marks **Bullet 2:** 1 mark	In the first bullet, appropriate trigonometric identities are substituted into each side of the given identity and some simplification steps are completed correctly. Equality between the two sides is not reached due to an incorrect simplification step on the left side. In the second bullet, only some of the non-permissible values are given.

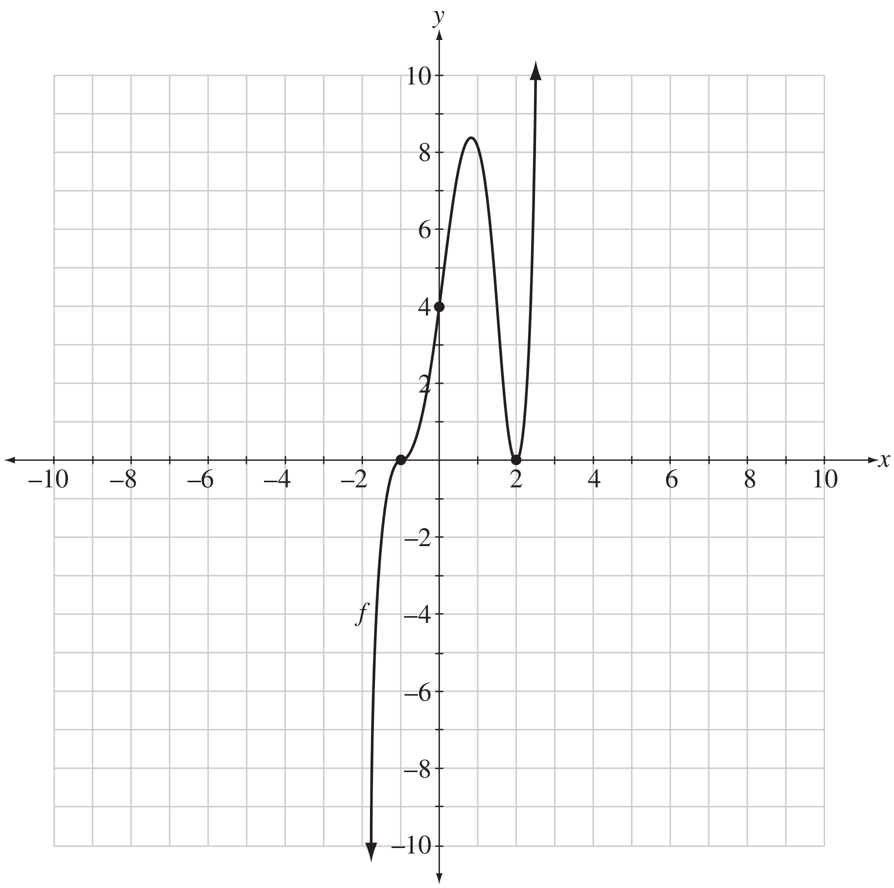
To go back after using an internal link, simultaneously press and hold Alt ← (in some browsers).

Written-response Question 3

Written Response—5 marks

- **Sketch** the graph and state the corresponding equation, in factored form, of a 5th-degree polynominal function with a minimum of two zeros. List the x- and y-intercepts below your graph. **[3 marks]**

A POSSIBLE SOLUTION



Equation: $y = (x - 2)^2(x + 1)^3$

x-intercepts: $x = 2$ and $x = -1$ or $(-1, 0)$ and $(2, 0)$

y-intercept: $y = 4$ or $(0, 4)$

Note: Many graphs are possible. To receive full marks, the equation, x-intercepts and y-intercept provided in the response must match the sketch that is drawn.

Use the following information to answer the next part of the written-response question.

The graph of the polynomial function $P(x) = a(x + b)^2 (x - c)^2$, where $b, c \in N$, is graphed on a Cartesian plane.

- **Explain** how changing the a-value from 3 to -6 would affect the graph of $P(x)$. Include specific details on how the change impacts the domain, range, y-intercept, and x-intercepts of the graph of $P(x)$ in your explanation. **[2 marks]**

A POSSIBLE SOLUTION

Changing the a-value from 3 to -6 will vertically stretch the graph about the x-axis by a factor of 2 and vertically reflect it about the x-axis.

These two transformations would not impact the domain of the function or the location of the x-intercepts. The domain would still be $\{x \mid x \in R\}$ and the x-intercepts would still be located at $x = -b$ and $x = c$.

However, the two transformations would change the range and the location of the y-intercept. The original graph was an upward-opening graph with a range of $\{y \mid y \geq 0\}$. The vertical reflection would change the direction of opening to downward, making the range $\{y \mid y \leq 0\}$.

The original graph also had a y-intercept located at $(0, ab^2c^2)$. The vertical stretch and reflection would require the y-coordinate of this point to be multiplied by -2, moving it from the positive arm of the y-axis to the negative arm of the y-axis and making it twice as far from the origin as the original.

Specific Scoring Guide for Written-response Question 3

Bullet 1:

Score	General Description	Specific Description
NR	No response is provided.	
0	In the response, the student does not address the question or provides a solution that is invalid.	The response does not contain a graph that represents the given characteristics, an appropriate equation or the identification of relevant points.
0.5		*For example, the student could* • *create a graph that represents a polynomial function with some of the required characteristics or provide a partially correct corresponding equation.*
1	In the response, the student demonstrates minimal mathematical understanding of the problem by applying an appropriate strategy or some relevant mathematical knowledge to complete initial stages of a solution.	In the response, the student • creates a graph that represents a polynomial function with some of the required characteristics; • provides a partially correct corresponding equation in factored form or the correct corresponding x- and y- intercepts; OR • creates a complete graph that represents a polynomial function with all of the required characteristics; but the correct corresponding equation, x-intercepts, and y-intercept are not provided.
1.5		*For example, the student could* • *create a graph that represents a polynomial function with all of the required characteristics and list the correct corresponding x- and y-intercepts; an equation in factored form has not been provided.*
2	In the response, the student demonstrates good mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a partial solution.	In the response, the student • creates a complete graph that represents a polynomial function with all of the required characteristics; • lists the correct corresponding x- and y-intercepts and provides a corresponding equation in factored form that contains errors or omissions (a correct element is present).
2.5		*For example, the student could* • *create a graph that represents a polynomial function with all of the required characteristics, list the correct corresponding x- and y-intercepts and provide a corresponding equation in factored form that contains one error or omission.*
3	In the response, the student demonstrates complete mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a complete and correct solution.	In the response, the student • creates a complete graph that represents a polynomial function with all of the required characteristics, the correct corresponding equation in factored form, and the correct corresponding x- and y- intercepts.

Bullet 2:

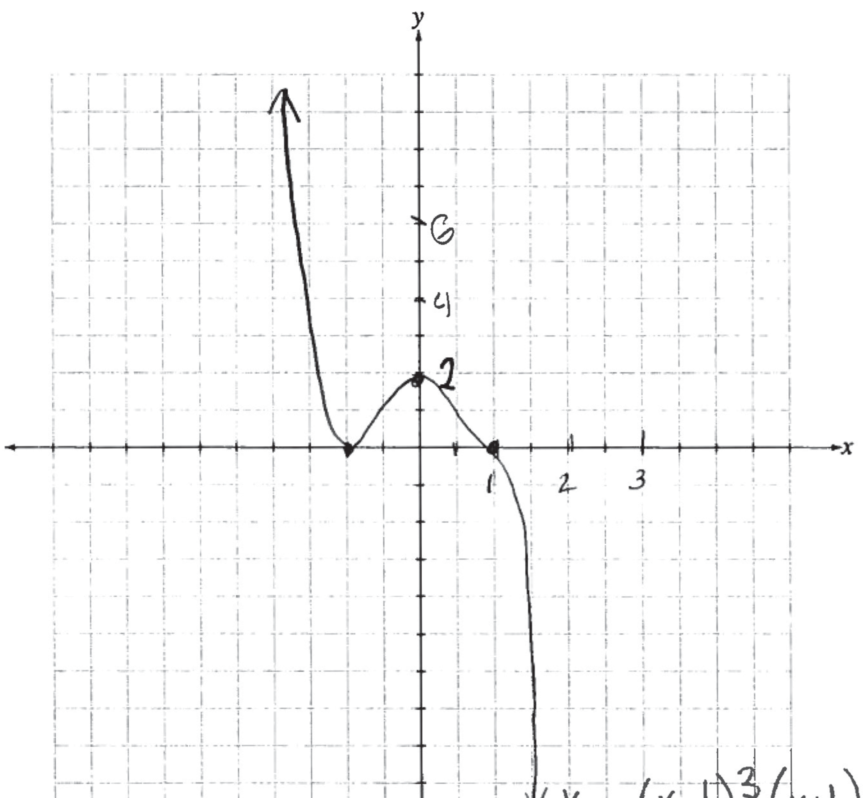
Score	General Description	Specific Description
NR	No response is provided.	
0	In the response, the student does not address the question or provides a solution that is invalid.	The response does not contain a relevant description of how the change in *a*-value impacts the characteristics of the corresponding graph.
0.5		*For example, the student could* • *partially describe how the graph is impacted by the change in a-value (i.e., the student identifies what transformations are occuring or lists the graph characteristics that are impacted).*
1	In the response, the student demonstrates basic mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a partial solution.	In the response, the student • partially describes how the graph is impacted by the change in *a*-value (i.e., the student identifies what transformations are occuring and lists the graph characteristics that are impacted).
1.5		*For example, the student could* • *partially describe how the graph is impacted by the change in a-value (i.e., the student identifies what transformations are occuring, lists the graph characteristics that are impacted and attempts to explain how the characteristics are impacted but the explanation is incomplete or partially incorrect).*
2	In the response, the student demonstrates complete mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a complete and correct solution.	In the response, the student • fully describes how the graph is impacted by the change in *a*-value (i.e., the student identifies what transformations are occuring, lists the graph characteristics that are impacted, and explains how the characteristics are impacted).

Sample Responses to Written-response Question 3

Sample Response 1

Written Response—5 marks

- **Sketch** the graph and state the corresponding equation, in factored form, of a 5th-degree polynominal function with a minimum of two zeros. List the *x*- and *y*-intercepts below your graph. **[3 marks]**



Equation: $y = -(x-1)^3(x+1)^2$

x-intercepts: $y = -(x-1)^3(x+1)^2$

y-intercept: $(-1,0)$ and $(1,0)$

$(0,2)$

Use the following information to answer the next part of the written-response question.

The graph of the polynomial function $P(x) = a(x + b)^2 (x - c)^2$, where $b, c \in N$, is graphed on a Cartesian plane.

- **Explain** how changing the a-value from 3 to -6 would affect the graph of $P(x)$. Include specific details on how the change impacts the domain, range, y-intercept, and x-intercepts of the graph of $P(x)$ in your explanation. **[2 marks]**

The domain will stay the same and the x-intercepts will stay the same because the graph is stretched vertically about the x-axis.

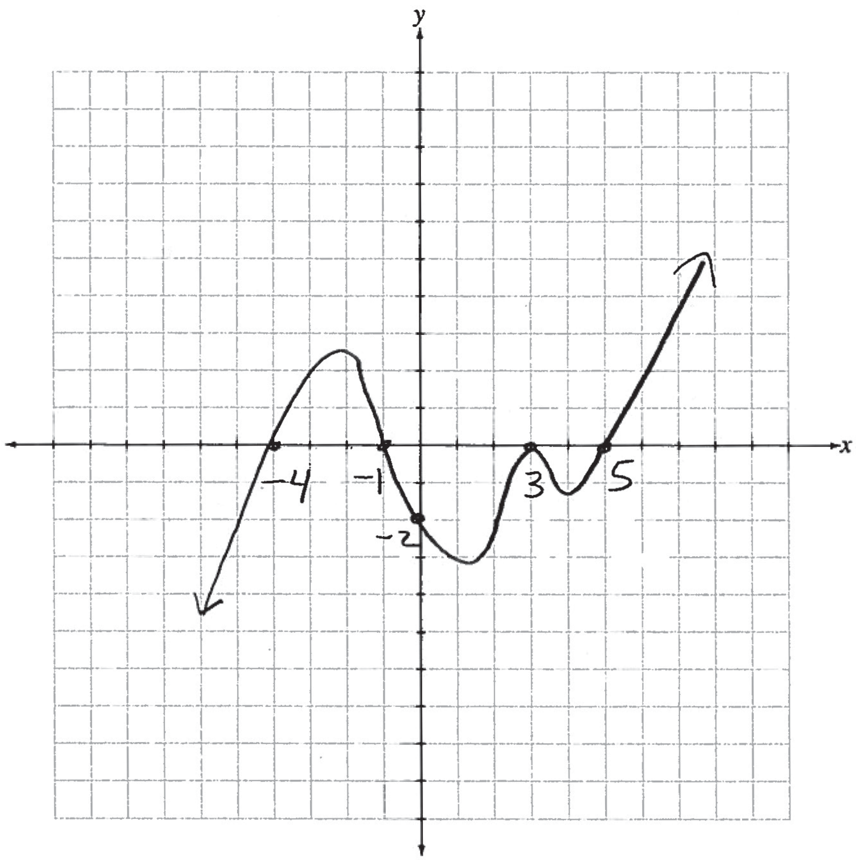
The range will be the opposite because the graph is flipped upside down. the flip will also change the y-intercept

Score	Rationale
Bullet 1: 2.5 marks **Bullet 2:** 1 mark	In the first bullet, a sketch that represents a 5th degree polynomial function with the required characteristics and a correct corresponding list of x- and y-intercepts are provided. However, the listed equation has an error that prevents it from matching the graph: the equation is missing an appropriate a-value. In the second bullet, the correct transformations and characteristics of the graph that are impacted have been listed but specific details about the characteristics have not been included in the response.

Sample Response 2

Written Response—5 marks

- **Sketch** the graph and state the corresponding equation, in factored form, of a 5th-degree polynominal function with a minimum of two zeros. List the *x*- and *y*-intercepts below your graph. **[3 marks]**



Equation: $y = (x-4)(x-1)(x+3)^2(x+5)$

x-intercepts: $-4, -1, 3, 5$

y-intercept: -2

Use the following information to answer the next part of the written-response question.

The graph of the polynomial function $P(x) = a(x + b)^2 (x - c)^2$, where $b, c \in N$, is graphed on a Cartesian plane.

- **Explain** how changing the a-value from 3 to -6 would affect the graph of $P(x)$. Include specific details on how the change impacts the domain, range, y-intercept, and x-intercepts of the graph of $P(x)$ in your explanation. **[2 marks]**

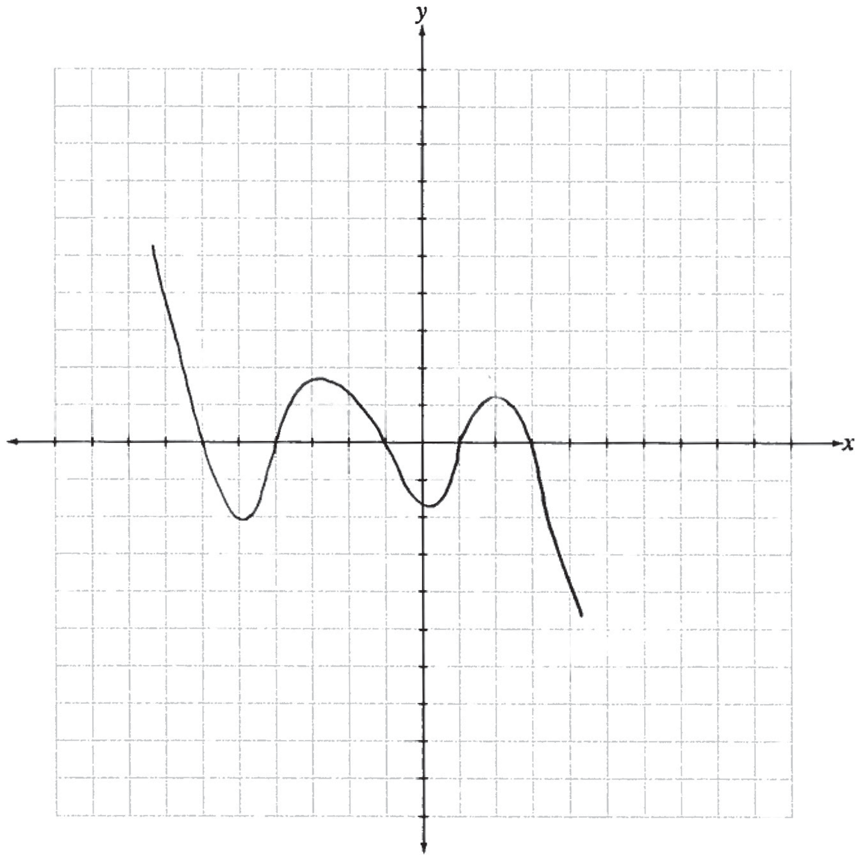
The new a value will make the graph be upside down and steeper. This will change the y-intercept to be twice as big and the graph crosses on the negative y-axis.

Score	Rationale
Bullet 1: 2 marks **Bullet 2:** 0.5 marks	In the first bullet, a sketch that represents a 5th degree polynomial function with the required characteristics and a correct corresponding list of x- and y-intercepts are provided. However, the listed equation has multiple errors that prevent it from matching the graph: the equation is missing an appropriate a-value and the signs in all of the binomial factors are incorrect. In the second bullet, the response addresses how the change to the equation impacts one of the characteristics of the graph but does not provide specific details about the remaining three characteristics.

Sample Response 3

Written Response—5 marks

- **Sketch** the graph and state the corresponding equation, in factored form, of a 5th-degree polynominal function with a minimum of two zeros. List the *x*- and *y*-intercepts below your graph. **[3 marks]**



Equation: $-(x+6)(x+4)(x+1)(x-1)(x-3)$

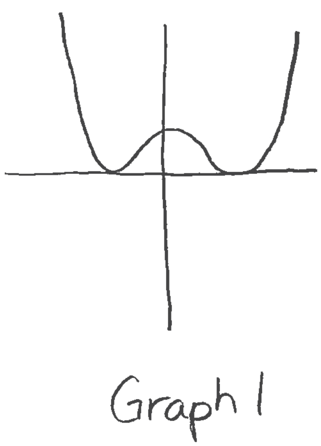
x-intercepts:

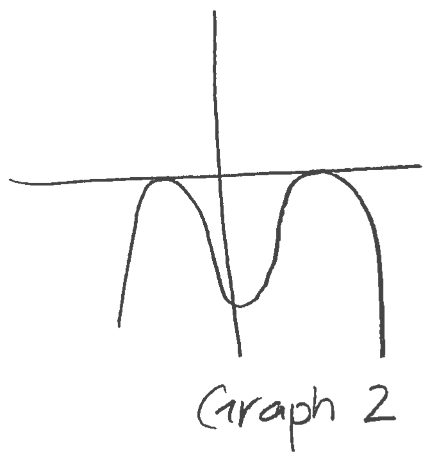
y-intercept:

Use the following information to answer the next part of the written-response question.

The graph of the polynomial function $P(x) = a(x + b)^2 (x - c)^2$, where $b, c \in N$, is graphed on a Cartesian plane.

- **Explain** how changing the a-value from 3 to -6 would affect the graph of $P(x)$. Include specific details on how the change impacts the domain, range, y-intercept, and x-intercepts of the graph of $P(x)$ in your explanation. **[2 marks]**



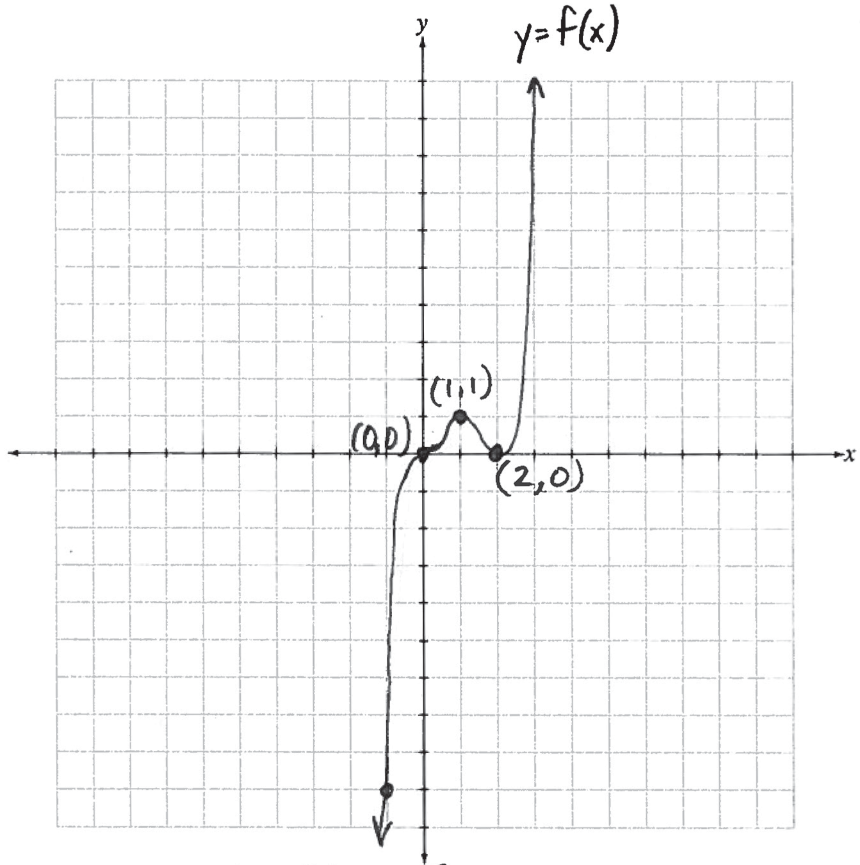


Score	Rationale
Bullet 1: 1 mark **Bullet 2:** 0.5 marks	In the first bullet, the sketch represents a 5th degree polynomial function and has an appropriate number of x-intercepts; however, an equation and the list of corresponding x- and y- intercepts have not been provided. Additionally, as a scale for the x- and y-axes has not been established, the assumed scale of increments of 1 matches the binomial factors in the provided expression but, when expanded, the constant of the expression does not correspond to the y-intercept. In the second bullet, the diagrams indicate an attempt to explain how the graph is impacted by the change to the equation but specific details about which characteristics are impacted or how they are impacted have not been included.

Sample Response 4

Written Response—5 marks

- **Sketch** the graph and state the corresponding equation, in factored form, of a 5th-degree polynominal function with a minimum of two zeros. List the *x*- and *y*-intercepts below your graph. **[3 marks]**



$y = x^3(x-2)^2$

Equation: $y = x^3(x^2-2x-4)$

$y = x^5 - 4x^4 - 4x^3$

x-intercepts:

$x = 0, 2$

y-intercept:

$y = 0$

To go back after using an internal link, simultaneously press and hold Alt ← (in some browsers).

Use the following information to answer the next part of the written-response question.

The graph of the polynomial function $P(x) = a(x + b)^2 (x - c)^2$, where $b, c \in N$, is graphed on a Cartesian plane.

- **Explain** how changing the a-value from 3 to -6 would affect the graph of $P(x)$. Include specific details on how the change impacts the domain, range, y-intercept, and x-intercepts of the graph of $P(x)$ in your explanation. **[2 marks]**

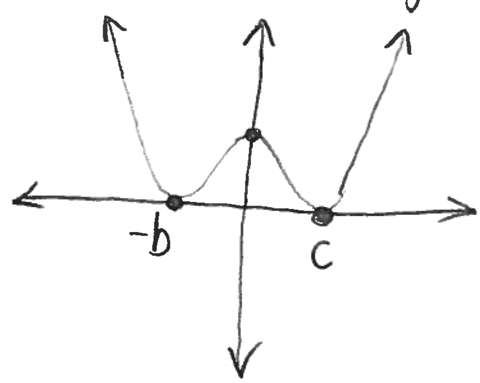
$P(x) = 3(x+b)^2(x-c)^2$

vertical stretch and a vertical reflection

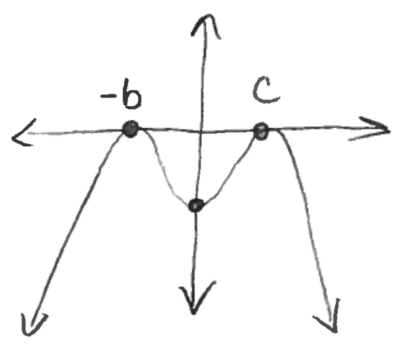
$P(x) = -6(x+b)^2(x-c)^2$

Vertical transformations do not affect the x-values so the domain and the x-intercepts do not change.

The old graph could look like:



The new graph looks like:



In the range, $y \geq 0$ becomes $y \leq 0$.

The y-intercept is now negative and is twice as far from the x-axis as it was before

Score	Rationale
Bullet 1: 3 marks **Bullet 2:** 2 marks	In the first bullet, a sketch that represents a 5th degree polynomial function with the required characteristics and a correct corresponding list of *x*- and *y*-intercepts are provided. Although an incorrect equation written in general form is included in the response, the correct equation written in factored form is provided. In the second bullet, the response lists which characteristics of the graph are impacted by the change to the equation and which ones are not. The response also includes an explanation of the transformations applied to the graph of the function and specific details on how the characteristics of the graph change.

Draft Mathematics Directing Words

In Provincial Assessment Sector use, mathematics directing words have the following definitions, which students are required to know. These words will be bolded in the written-response questions.

Algebraically	Using mathematical procedures that involve variables or symbols to represent values
Analyze	Make a mathematical examination of parts to determine the nature, proportion, function, interrelationships, and characteristics of the whole
Classify	Arrange items or concepts in categories according to shared qualities or characteristics
Compare	Examine the character or qualities of two things by providing characteristics of both that point out their mutual *similarities* **and** *differences*
Conclude	Make a logical statement based on reasoning and/or evidence
Describe	Give a written account of a concept
Design/Plan	Construct a detailed sequence of actions for a specific purpose
Determine	Find a solution, to a specified degree of accuracy, to a problem by showing appropriate formulas, procedures, and/or calculations
Evaluate	Find a numerical value or equivalent for an equation, formula or function
Explain	Make clear what is not immediately obvious or entirely known; give the cause of or reason for; make known in detail
Illustrate	Make clear by giving an example. The form of the example will be specified in the question: e.g., a word description, sketch, or diagram
Interpret	Provide a meaning of something; present information in a new form that adds meaning to the original data
Justify	Provide valid reasons, evidence and/or facts that support a position
Model	Represent a concept or situation in a concrete or symbolic way
Predict	State in advance on the basis of logic
Prove	Establish the truth or validity of a statement by giving factual evidence or logical argument
Sketch	Provide a drawing that represents the key features or characteristics of an object or graph
Solve	Give a solution to a problem
Verify	Establish, by substitution for a particular case or by geometric comparison, the truth of a statement

Explanation of Cognitive Levels

Procedural

The assessment of students' knowledge of mathematical procedures should involve recognition, execution, and verification of appropriate procedures and the steps contained within them. The use of technology can allow for conceptual understanding prior to specific skill development or vice versa. Students must appreciate that procedures are created or generated to meet specific needs in an efficient manner and thus can be modified or extended to fit new situations. Assessment of students' procedural knowledge will not be limited to an evaluation of their proficiency in performing procedures, but will be extended to reflect the skills presented above.

Conceptual

An understanding of mathematical concepts goes beyond a mere recall of definitions and recognition of common examples. Assessment of students' knowledge and understanding of mathematical concepts should provide evidence that they can compare, contrast, label, verbalize, and define concepts; identify and generate examples and counter-examples as well as properties of a given concept; recognize the various meanings and interpretations of concepts; and defend procedures and personal strategies. Students who have developed a conceptual understanding of mathematics can also use models, symbols, and diagrams to represent concepts. Appropriate assessment provides evidence of the extent to which students have integrated their knowledge of various concepts.

Problem Solving

Appropriate assessment of problem-solving skills is achieved by allowing students to adapt and extend the mathematics they know and by encouraging the use of strategies to solve unique and unfamiliar problems. Assessment of problem solving involves measuring the extent to which students use these strategies and knowledge and their ability to verify and interpret results. Students' ability to solve problems develops over time as a result of their experiences with relevant situations that present opportunities to solve various types of problems. Evidence of problem-solving skills is often linked to clarity of communication. Students demonstrating strong problem-solving skills should be able to clearly explain the process they have chosen, using appropriate language and correct mathematical notation and conventions.

To go back after using an internal link, simultaneously press and hold Alt ← (in some browsers).

Mathematics 30–1 Formula Sheet

For $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Relations and Functions

Graphing Calculator Window Format

x: $[x_{\text{min}},\ x_{\text{max}},\ x_{\text{scl}}]$

y: $[y_{\text{min}},\ y_{\text{max}},\ y_{\text{scl}}]$

Laws of Logarithms

$$\log_b(M \times N) = \log_b M + \log_b N$$

$$\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$$

$$\log_b(M^n) = n \log_b M$$

$$\log_b c = \frac{\log_a c}{\log_a b}$$

Growth/Decay Formula

$$y = ab^{\frac{t}{p}}$$

General Form of a Transformed Function

$$y = af\left[b(x - h)\right] + k$$

Permutations, Combinations, and the Binomial Theorem

$n! = n(n-1)(n-2)\ldots3 \times 2 \times 1$,
where $n \in N$ and $0! = 1$

$${}_nP_r = \frac{n!}{(n-r)!}$$

$${}_nC_r = \frac{n!}{(n-r)!r!} \qquad {}_nC_r = \binom{n}{r}$$

In the expansion of $(x + y)^n$, written in descending powers of x, the general term is $t_{k+1} = {}_nC_k x^{n-k} y^k$.

Trigonometry

$$\theta = \frac{a}{r}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

$$\sin(2\alpha) = 2\sin\alpha\cos\alpha$$

$$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha$$

$$\cos(2\alpha) = 2\cos^2\alpha - 1$$

$$\cos(2\alpha) = 1 - 2\sin^2\alpha$$

$$\tan(2\alpha) = \frac{2\tan\alpha}{1 - \tan^2\alpha}$$

$$y = a\sin\left[b(x - c)\right] + d$$

$$y = a\cos\left[b(x - c)\right] + d$$