Assessment Standards and Exemplars

Mathematics 30-1

2015–2016 Diploma Examinations Program

Alberta Government

This document was written primarily for:

Students	✓
Teachers	✓ of Mathematics 30–1
Administrators	✓
Parents	
General Audience	
Others	

Distribution: This document is posted on the Alberta Education website at education.alberta.ca.

Contents

Introduction

This resource is designed to support the implementation of the *Alberta Mathematics Grades 10–12 Program of Studies*, which can be found on the Alberta Education website at education.alberta.ca. Teachers are strongly encouraged to consult the program of studies for details about the philosophy of the program.

The examples shown in this document were chosen to illustrate the intent of particular outcomes of Mathematics 30–1 but will not necessarily be assessed on a diploma examination in the manner shown. All examples were developed and validated by classroom teachers of mathematics but have not been validated with students. For more examples, please go to the Quest A+ website at https://questaplus.alberta.ca.

To meet the outcomes of Mathematics 30–1, students will need access to an approved graphing calculator. In most classrooms, students will use a graphing calculator daily. Refer to the calculator policy in the *General Information Bulletin* or go to the Alberta Education website for a list of approved graphing calculators. Information about the diploma examinations each year can also be found in the *Mathematics 30–1 Information Bulletin*.

This document presents the current version of the curriculum and assessment standards. If you have comments or questions regarding this document, please contact Ross Marian by email at Ross.Marian@gov.ab.ca, by phone at (780) 427-0010, or by fax at (780) 422-4454.

The Provincial Assessment Sector would like to recognize and thank the many teachers throughout the province who helped to prepare this document. We would also like to thank the Programs of Study and Resources Sector and the French Language Education Services Branch for their input and assistance in reviewing these standards.

Standards for Mathematics 30−1

The word *and* used in the standards implies that both ideas should be addressed at the same time or in the same question.

The assessment standards for Mathematics 30–1 include an acceptable and an excellent level of performance.

Acceptable Standard

Students who attain the *acceptable standard* but not the *standard of excellence* will receive a final course mark between 50% and 79% inclusive. Typically, these students have a proficient level of number sense, algebra skills, mathematical literacy, reading, comprehension, and reasoning. They have gained new skills and a basic knowledge of the concepts and procedures relative to the general and specific outcomes defined for Mathematics 30–1 in the program of studies. They demonstrate mathematical skills as well as conceptual understanding, and can apply their knowledge to familiar problem contexts.

Standard of Excellence

Students who attain the *standard of excellence* will receive a final course mark of 80% or higher. Typically, these students have an advanced level of number sense, algebra skills, mathematical literacy, reading, comprehension, and reasoning. They have gained a breadth and depth of understanding regarding the concepts and procedures, as well as the ability to apply this knowledge and conceptual understanding to a broad range of familiar and unfamiliar problem contexts.

General Notes

- The seven mathematical processes [C, CN, ME, PS, R, T, V] should be used and integrated throughout as indicated in the specific outcomes.
- If technology [T] has not been specifically listed as a mathematical process for an outcome, teachers may, at their discretion, use it to assist students in exploring patterns and relationships when learning a concept. However, it is expected that technology will not be considered when assessing students' understandings of such outcomes.
- Most high school mathematics resources in North America use the letter *I* to represent the set of integers; however, students may encounter resources, especially at the post-secondary level, that use the letter *Z* to represent the set of integers. Both are correct.
- For students transitioning between French and English instruction in mathematics, teachers can reference the mathematical terminology of both languages using the English-French Mathematical Lexicons provided on the Alberta Education website under Support Materials at http://education.alberta.ca/teachers/program/math/educator/materials.aspx.

Performance Level Descriptors for Mathematics 30–1

These **Performance Level Descriptors** list attributes and abilities of students who attain each standard level.

Students who **just meet** *the Acceptable Standard are able to:*

- demonstrate knowledge of a particular concept through either visual or numerical representations
- demonstrate number sense, algebra skills, mathematical literacy, reading, comprehension, and reasoning
- solve knowledge-based questions by making basic connections (ask "how"); focus on algorithms/procedural knowledge and basic conceptual knowledge
- comprehend the characteristics of a relation or function when a graph is shown or the equation is given
- solve familiar problems
- use and state a strategy to solve problems
- use an algebraic process to solve problems, verify solutions or identify errors
- use technology to solve problems
- demonstrate mathematical processes that lead toward a complete solution when solving problems and/or equations

Students who **just meet** *the Standard of Excellence are able to:*

- demonstrate transferability of knowledge, making connections between topics, including abstract representations
- demonstrate an advanced level of number sense, algebra skills, mathematical literacy, reading, comprehension, and reasoning
- solve knowledge-based questions by making advanced connections (ask "why"); focus on conceptual/problem-solving knowledge
- apply the characteristics and/or changes in the context of a relation or function to create a graph or equation
- solve unique and unfamiliar problems
- use, interpret, and compare multiple strategies to solve problems
- use multiple algebraic processes to solve problems, verify solutions, and employ various methods of error identification
- solve problems and use technology to verify solutions
- demonstrate mathematical processes that result in a complete or correct solution when solving problems and/or equations

Relations and Functions

General Outcome

Develop algebraic and graphical reasoning through the study of relations.

General Notes:

- Transformations covered in Specific Outcomes 2 to 5 include the algebraic base functions: $y = \frac{1}{x}$, $y = x$, $y = x^2$, $y = x^3$, $y = \sqrt{x}$, $y = |x|$, $y = b^x$, and $y = \log_b x$, with appropriate restrictions, as well as other graphical representations of functions.
- Stretches and reflections are performed prior to translations unless otherwise stated.
- Analyzing a transformation for Specific Outcomes 2 to 5 includes, but is not limited to: determining and describing the effects of a transformation on the domain, range, intercepts, invariant points, and key points.
- Analyzing graphs for Specific Outcomes 9 and 12 to 14 includes, but is not limited to: determining and describing the domain, range, intercepts, invariant points, and key points.
- The term *key points* on a relation or function may include vertices, endpoints, maximum and minimum points, etc.
- Technology [T] is not one of the mathematical processes listed for specific outcomes 2 to 7, 10, and 11. While technology can be used to discover and investigate the concepts, students are expected to meet these outcomes without the use of technology.
- Students should understand mapping notation for transformations.
- Students should be able to express any domain or range in both interval notation and set builder notation. Be aware that both interval notation and set builder notation are different in French as detailed in the French version of this document.

Specific Outcomes

Specific Outcome 1

Demonstrate an understanding of operations on, and compositions of, functions. **[CN, R, T, V]** **[ICT: C6–4.1]**

Notes:

- The original functions used in operations and compositions should be limited to: linear, quadratic, cubic, radical (one linear radicand), rational (monomial, binomial), absolute value (first degree only), exponential, logarithmic, and piecewise functions.

- For composition of functions, students should be familiar with the following notation:

$$\left(f \circ g\right)(x) = f\left(g(x)\right)$$

- For operations on functions, students should be familiar with the following notation:

$$(f + g)(x) = f(x) + g(x)$$
$$(f - g)(x) = f(x) - g(x)$$
$$(f \bullet g)(x) = f(x)g(x)$$
$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

- Graphing technology may be used to analyze functions that result from operations or compositions that are beyond the scope of this course.
- The intent of this outcome is to focus on the conceptual understanding of operations and compositions rather than on lengthy algebraic processes.

(See examples 1–8)

Specific Outcome 2

Demonstrate an understanding of the effects of horizontal and vertical translations on the graphs of functions and their related equations. **[C, CN, R, V]**

(See examples 9–11)

Specific Outcome 3

Demonstrate an understanding of the effects of horizontal and vertical stretches on the graphs of functions and their related equations. **[C, CN, R, V]**

Notes:

- Stretches about a line parallel to the x- or y-axis are beyond the scope of this course.

(See examples 12, 13, 16, and 17)

Specific Outcome 4

Apply translations and stretches to the graphs and equations of functions. **[C, CN, R, V]**

(See examples 14, 15, and 19)

Specific Outcome 5

Demonstrate an understanding of the effects of reflections on the graphs of functions and their related equations, including reflections through the:

- x-axis
- y-axis
- line $y = x$ **[C, CN, R, V]**

Notes:

- Reflections about a line parallel to the x-axis or y-axis are beyond the scope of this course.
- Reflections in the line $y = x$ should not be combined with any other transformations.

(See examples 15–20 and 23)

Specific Outcome 6

Demonstrate an understanding of inverses of relations. **[C, CN, R, V]**

Notes:

- Students should be familiar with the notation $x = f(y)$.
- The notation $y = f^{-1}(x)$ should only be used if the inverse is also a function.
- When discussing the **equations** of inverse relations, the focus should primarily be on linear, quadratic, exponential, or logarithmic functions.
- When exploring inverse relations **graphically**, teachers may choose to explore various relations, such as polynomial, piecewise, radical, exponential, logarithmic, and absolute values.
- Students should be able to restrict the domain on the original function to obtain an inverse that is also a function.

(See examples 21–22)

Specific Outcome 7

Demonstrate an understanding of logarithms. **[CN, ME, R]**

(See examples 24–26)

Specific Outcome 8

Demonstrate an understanding of the product, quotient, and power laws of logarithms.
[C, CN, ME, R, T] [ICT: C6–4.1]

Notes:

- Change of base identity can be taught as a strategy for evaluating logarithms.

(See examples 27–30)

Specific Outcome 9

Graph and analyze exponential and logarithmic functions. **[C, CN, T, V]**
[ICT: C6–4.3, C6–4.4, F1–4.2]

Notes:

- When graphing $y = a(b)^{x-c} + d$, the value of b will be restricted to $b > 0, b \neq 1$.
- When graphing $y = a\log_b(x - c) + d$, the value of b will be restricted to $b > 1$.
- Natural logarithms and base e are beyond the scope of this course.

(See examples 31–34)

Specific Outcome 10

Solve problems that involve exponential and logarithmic equations. **[C, CN, PS, R]**

Notes:

- Logarithmic equations should be restricted to same bases.
- Formulas will be given for any problems involving logarithmic scales such as decibels, earthquake intensity, and pH.
- Formulas will be given unless the context fits the form $y = ab^{\frac{t}{p}}$, where y is the final amount, a is the initial amount, b is the growth/decay factor, t is the total time, and p is the period.
- Compound interest is an application of the formula $y = ab^{\frac{t}{p}}$. Students should be familiar with terms used for compound periods.

(See examples 35–40)

Specific Outcome 11

Demonstrate an understanding of factoring polynomials of degree greater than 2 (limited to polynomials of degree ≤ 5 with integral coefficients). **[C, CN, ME]**

Notes:

- Rational Zero Theorem is beyond the scope of this outcome; i.e., there will be at most two linear factors with a leading coefficient not equal to 1.

(See examples 41–44)

Specific Outcome 12

Graph and analyze polynomial functions (limited to polynomial functions of degree ≤ 5).
[C, CN, T, V] [ICT: C6–4.3, C6–4.4]

Notes:

- Students must understand the relationship between zeros of a function, roots of an equation, *x*-intercepts of a graph, and factors of a polynomial.
- Analyzing a polynomial function graphically includes: leading coefficient, maximum and minimum points, domain, range, *x*- and *y*-intercepts, zeros, multiplicity, odd and even degrees, and end behaviour.
- Students should be able to identify when no real roots exist, but the calculation of them is beyond the scope of this outcome.
- The terms *maximum point* and *minimum point* refer to the absolute maximum and absolute minimum points, respectively.

(See examples 45–51)

Specific Outcome 13

Graph and analyze radical functions (limited to functions involving one radical). **[CN, R, T, V] [ICT: C6–4.1, C6–4.3]**

Notes:

- Radical functions will be limited to square roots.
- This specific outcome includes sketching and analyzing the transformation of $y = f(x)$ to $y = \sqrt{f(x)}$. The function $y = f(x)$ may be a linear, quadratic, or piecewise function.

(See examples 52–54)

Specific Outcome 14

Graph and analyze rational functions (limited to numerators and denominators that are monomials, binomials, or trinomials). **[CN, R, T, V] [ICT: C6–4.1, C6–4.3, C6–4.4]**

Notes:

- Oblique or slant asymptotes are not a part of this outcome. All graphs are restricted to horizontal and vertical asymptotes.
- Numerators and denominators should be limited to degree two or less.
- This specific outcome does NOT include the transformation of $y = f(x)$ to $y = \frac{1}{f(x)}$.

(See examples 55–58)

Acceptable Standard

The student can:

- given their equations, sketch the graph of a function that is the sum, difference, product, quotient, or composition of two functions
- given their graphs, sketch the graph of a function that is the sum, difference, or product of two functions
- write a function, $h(x)$, as:
  - the sum or difference of two or more functions
  - the product or quotient of two functions
  - a single composition
    E.g., $h(x) = f\big(f(x)\big)$
    $h(x) = f\big(g(x)\big)$
- determine the domain and range of a function which results from the operation of two functions (i.e. sum, difference, product, or quotient)
- determine the value of operations or compositions of functions at a point
  E.g., $h(a) = (f \bullet g)(a)$
  $h(a) = f\big(f(a)\big)$
  $h(a) = (f \circ g)(a)$
  $h(a) = f\Big(g\big(h(a)\big)\Big)$
  $h(a) = g(a) + f\big(g(a)\big)$

Standard of Excellence

The student can also:

- given their graphs, sketch the graph of a function that is the quotient of two functions
- write a function, $h(x)$, as:
  - the product or quotient of three functions
  - the composition of functions involving two compositions
    E.g., $j(x) = (f \circ g \circ h)(x)$
    $h(x) = f\Big(g\big(f(x)\big)\Big)$
- write a function, $h(x)$, combining two or more functions through operations on, and/or compositions of, functions, limited to two operations
  E.g., $h(x) = g(x) + f\big(g(x)\big)$
  $h(x) = (f \bullet g)(x) - k(x)$
- determine the domain of a function that is the composition of two functions

- perform, analyze, and describe graphically or algebraically:
  - a combination of transformations involving stretches and/or translations
  - a combination of transformations involving reflections and/or translations
  - a combination of transformations involving reflections and/or stretches
  - a horizontal stretch and/or reflection in the y-axis and a translation where the parameter b is removed through factoring

  given the function in equation or graphical form or mapping notation
- perform, analyze, and describe a reflection in the line $y = x$, given the function or relation in graphical form
- determine the equation of the inverse of a linear, quadratic, exponential, or logarithmic function and analyze its graph
- determine an unknown parameter in a function, given information relating to one point on the graph of the function
- determine, without technology, the exact values of simple logarithmic expressions
- estimate the value of a logarithmic expression using benchmarks
- convert between $y = b^x$ and $\log_b y = x$
- simplify and/or expand logarithmic expressions using the laws of logarithms
- sketch and analyze (domain, range, intercepts, asymptote) the graphs of exponential or logarithmic functions and their transformations
- solve exponential equations that:
  - can be simplified to a common base
  - cannot be simplified to a common base and whose exponents are monomials

- perform, analyze, and describe graphically or algebraically:
  - a horizontal stretch and/or reflection in the y-axis and translation where the parameter b is not removed through factoring
  - a combination of transformations involving at least a reflection, a stretch, and a translation

  given the function in equation or graphical form or mapping notation
- determine restrictions on the domain of a function in order for its inverse to be a function, given the graph or equation
- convert between exponential and logarithmic forms involving more than two steps
- solve exponential equations that cannot be simplified to a common base, where the exponents are not monomials, or where there is a numerical coefficient

- solve logarithmic equations but cannot recognize when a solution is extraneous
- solve exponential and logarithmic real-world application problems
- solve for a value, such as an earthquake intensity, in comparison problems
- identify whether a binomial is a factor of a given polynomial
- completely factor a polynomial of degree 3, 4, or 5
- identify and explain whether a given function is a polynomial function
- find the zeros of a polynomial function and explain their relationship to the x-intercepts of the graph and the roots of an equation
- sketch and analyze polynomial functions (in terms of multiplicities, y-intercept, domain and range, etc.)
- provide a partial solution to solve a problem by modelling a given situation with a polynomial function
- determine the equation of a polynomial function in factored form, given its graph and/or key characteristics
- sketch and analyze (in terms of domain, range, invariant points, x- and y-intercepts) $y = \sqrt{f(x)}$ given the graph or equation of $y = f(x)$
- find the zeros of a radical function graphically and explain their relationship to the x-intercepts of the graph and the roots of an equation
- determine the equation of a radical function given its graph and/or key characteristics

- solve logarithmic equations and recognize when a solution is extraneous
- solve for an exponent in comparison problems
- provide a complete solution to a problem by modelling a given situation with a polynomial function
- determine the equation of a radical function involving all three types of transformations: reflection, stretch, and translation, given its graph and/or key characteristics

• sketch and analyze rational functions (in terms of vertical asymptotes, horizontal asymptote, x-coordinate of a point of discontinuity, domain, range, x- and y-intercepts)	• determine the y-coordinate of a point of discontinuity of a rational function
• find the zeros of a rational function graphically and explain their relationship to the x-intercepts of the graph and the roots of an equation	
• determine the equation of a rational function given its graph and/or key characteristics	• determine the equation of a rational function containing a point of discontinuity, given its graph and/or key characteristics
• participate in and contribute toward the problem-solving process for problems involving relations and functions studied in Mathematics 30-1	• complete the solution to problems involving relations and functions studied in Mathematics 30-1

Examples

Students who achieve the *acceptable standard* should be able to answer all of the following questions, except for any part of a question labelled ***SE***. Parts labelled ***SE*** are appropriate examples for students who achieve the *standard of excellence*.

Note: *In the multiple-choice questions that follow, * indicates the correct answer. Please be aware that the worked solutions show possible strategies; there may be other strategies that could be used.*

1. Given the functions $f(x) = 7 - x$ and $g(x) = 2x + 1$, sketch the graph of $h(x)$ for each question below **and** state the domain and range.

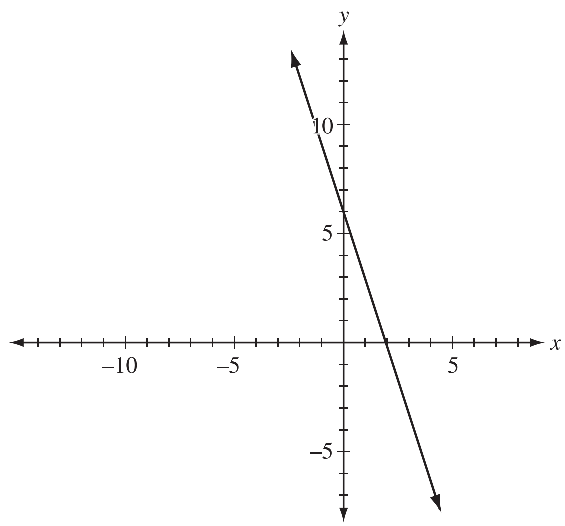
a) $h(x) = f(x) - g(x)$

Possible solution:

$h(x) = (7 - x) - (2x + 1)$

$h(x) = -3x + 6$

D:$\{x \mid x \in R\}$ and R:$\{y \mid y \in R\}$



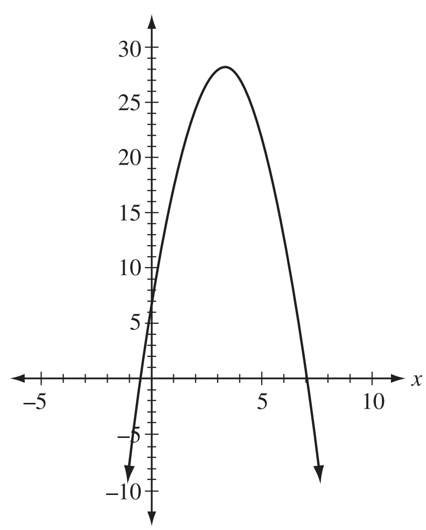
b) $h(x) = f(x)\,g(x)$

Possible solution:

$h(x) = (7 - x)(2x + 1)$

$h(x) = -2x^2 + 13x + 7$

D:$\{x \mid x \in R\}$ and R:$\{y \mid y \le 28.125\}$

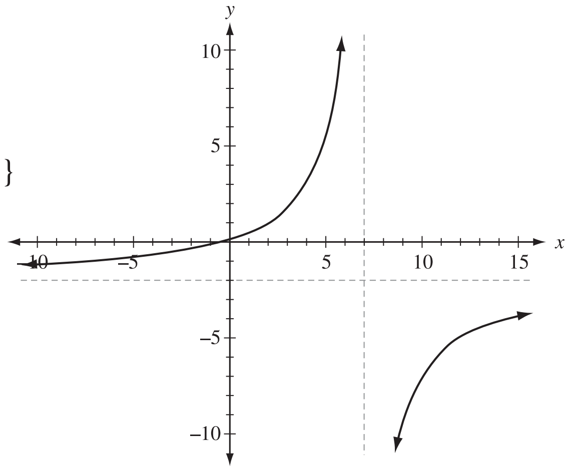


c) $h(x) = \left(\dfrac{g}{f}\right)(x)$

Possible solution:

$h(x) = \dfrac{2x+1}{7-x}$

D:$\{x \mid x \neq 7\}$ and R:$\{y \mid y \neq -2\}$



d) $h(x) = g(f(x))$

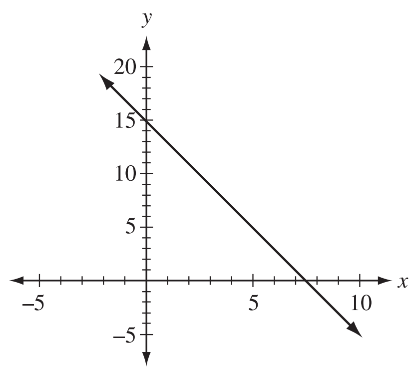
Possible solution:

$h(x) = g(7 - x)$

$h(x) = 2(7 - x) + 1$

$h(x) = 15 - 2x$

D:$\{x \mid x \in R\}$ and R:$\{y \mid y \in R\}$



SE **2.** Given $f(x) = \sqrt{x-1}$, $g(x) = x^2 + 3$, and $h(x) = 2x - 5$, if $k(x) = (h \circ g \circ f)(x)$, determine a simplified expression for $k(x)$ and state the domain of $k(x)$.

Possible solution: $k(x) = h\left(g\left(\sqrt{x-1}\right)\right)$

$k(x) = h\left(\left(\sqrt{x-1}\right)^2 + 3\right)$

$k(x) = h(x + 2)$

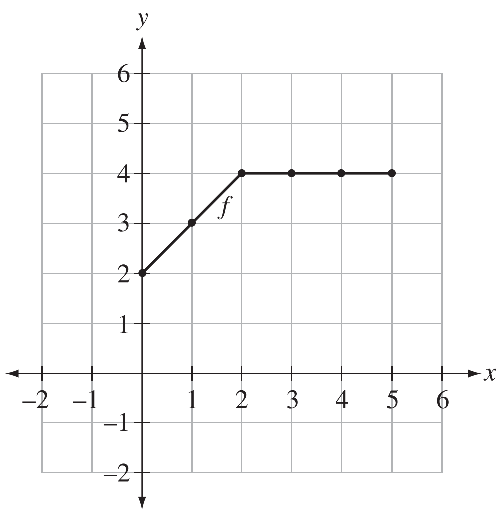
$k(x) = 2(x + 2) - 5$

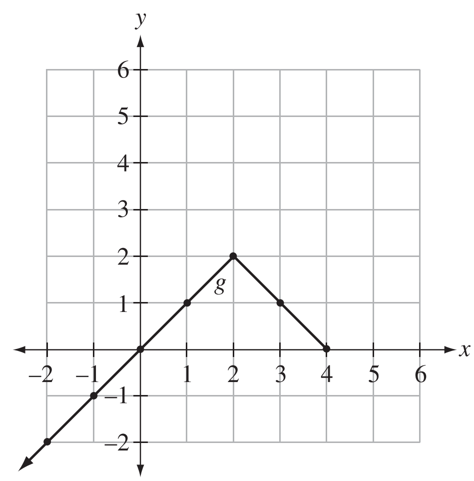
$k(x) = 2x - 1$, domain is $[1, \infty)$ since the domain of $f(x)$ is $[1, \infty)$

Note: This item is SE since it involves the combination of two compositions, as well as determining the restriction on the composite function.

Use the following information to answer the next question.

The graphs of the functions $y = f(x)$ and $y = g(x)$ are shown below.

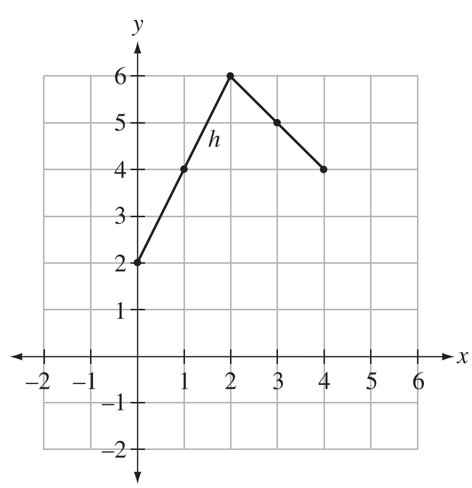




3. Sketch the graph of

a) $h(x) = f(x) + g(x)$

Possible solution:

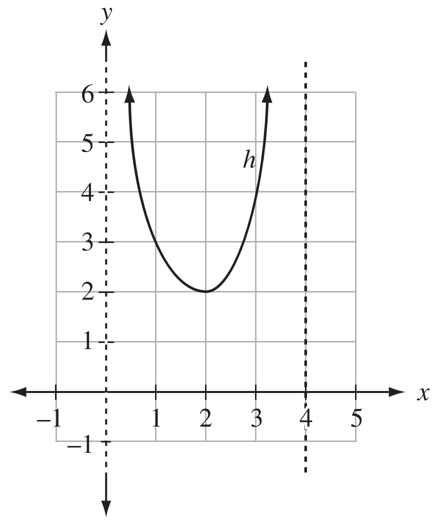


For each value of x that is common to both f and g, add the y-values from each function and plot the new points.

SE

b) $h(x) = \frac{f(x)}{g(x)}$

Possible solution:



Note: This item is SE since it involves sketching the graph of a function that is the quotient of two functions.

4. Given $f(x) = 7\log_2 x$ and $g(x) = |5 - 6x|$, determine the value of $f\big((f + g)(8)\big)$.

Possible solution: $f(8) + g(8) = 7\log_2(8) + |5 - 6(8)|$

$$f(8) + g(8) = 21 + 43$$

$$f(8) + g(8) = 64$$

$$f(64) = 7\log_2(64)$$

$$f(64) = 7\log_2(64)$$

$$f(64) = 42$$

$$\therefore f\big((f + g)(8)\big) = 42$$

Use the following information to answer the next question.

Alex is given the following list of functions, where $b > 1$. She is asked to determine a new function, $h(x)$, which is the quotient of two different functions below and where the domain of $h(x)$ is $\{x \in R\}$.

Function 1	$y = x - b$
Function 2	$y = x^2 + b$
Function 3	$y = x^3 + b$
Function 4	$y = \log_b x$
Function 5	$y = b^x$
Function 6	$y = \sqrt{x + b}$

5. If $h(x) = \dfrac{f(x)}{g(x)}$ and Alex selects Function 1 for $f(x)$, then the two functions that she could select for $g(x)$ are numbered _____ and _____.

Possible solution: 25 or 52

Function 1 is the numerator and cannot also be the denominator as $f(x)$ and $g(x)$ must be different functions.

In order for $h(x)$ to have the domain $x \in R$, $g(x)$ cannot equal zero with $b > 1$, and $g(x)$ must have a domain of $x \in R$. Only functions 2 and 5 fit this description.

Function 3 becomes zero when $x = \sqrt[3]{-b}$.

Function 4 has a limited domain of $x > 0$.

Function 6 becomes zero when $x = -b$ and has a limited domain of $x \geq -b$.

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SE **6.** Given $f(x) = x^2 - 7x$, $g(x) = x - 2$, and $h(x) = \dfrac{2x^2 + x}{x - 2}$, determine a simplified equation for $j(x)$, given that $j(x) = \dfrac{f(x)}{g(x)} + h(x)$. State the domain and range for $j(x)$.

Possible solution: $j(x) = \dfrac{x^2 - 7x}{x - 2} + \dfrac{2x^2 + x}{x - 2}$

$$j(x) = \frac{3x^2 - 6x}{x - 2}$$

$$j(x) = \frac{3x(x - 2)}{x - 2}$$

$$j(x) = 3x, \quad x \neq 2$$

$$\text{D:}\{x \mid x \neq 2, x \in R\} \text{ and } \text{R:}\{y \mid y \neq 6, x \in R\}$$

Note: This item is SE since it requires the y-coordinate of the point of discontinuity and because it involves two operations with functions.

SE **7.** Given that Point A (3, 4) lies on the graph of $g(x)$, and Point A' (3, 8) lies on the graph of $h(x)$, where $h(x) = (f \circ g)(x)$, the corresponding point that lies on the graph of $f(x)$ must be ______ .

Possible solution: The x-coordinate of $f(x)$ is the y-coordinate of $g(x)$, 4.

The y-coordinate of $f(x)$ is the y-coordinate of $h(x)$, 8.

$\therefore (4, 8)$ is the corresponding point on $f(x)$.

Possible solution:

$$\begin{tikzcd}
g(3) = 4 \arrow[rr] & & f(4) = 8 \\
& h(3) = 8 \arrow[ul] \arrow[ur] &
\end{tikzcd}$$

$\therefore (4, 8)$ is the corresponding point on $f(x)$.

Note: This item is SE since it requires the solution to problems involving relations and functions studied in Mathematics 30-1 as indicated in the last bullet on page 12.

Use the following information to answer the next question.

Assume that $f(x) = \sqrt{x-3}$ and $g(x) = \dfrac{x^2}{x^2 - 9}$.

Reference Number	**Domain**
1	$\{x \mid x \geq 3, x \in R\}$
2	$\{x \mid x > 3, x \in R\}$
3	$\{x \mid x \neq 12, x \in R\}$
4	$\{x \mid x \geq 3, x \neq 12, x \in R\}$
5	$\{x \mid x > 3, x \neq 12, x \in R\}$

SE **8.** The reference number for the domain of the graph of $h(x) = g\big(f(x)\big)$ is ______ .

Possible solution: 4

$$g\big(f(x)\big) = \frac{\left(\sqrt{x-3}\right)^2}{\left(\sqrt{x-3}\right)^2 - 9}$$

$$g\big(f(x)\big) = \frac{x-3}{x-12}$$

The domain of $f(x)$ is $x \geq 3$, and $h(x)$ is undefined at $x = 12$.

$\therefore \{x \mid x \geq 3, x \neq 12, x \in R\}$ and the correct reference number is 4.

Note: This item is SE since it requires the domain of a function that is the composition of two functions.

9. Given $y - k = a(x - h)^2$, $a = 1$, $h < 0$, $k > 0$, in which quadrant is the vertex?

- **A.** Quadrant I
- ***B.** Quadrant II
- **C.** Quadrant III
- **D.** Quadrant IV

10. Given the functions $f(x) = |x - 2| + 3$ and $g(x) = |x + 2| + 1$, the transformations that will transform $y = f(x)$ into $y = g(x)$ are a translation of

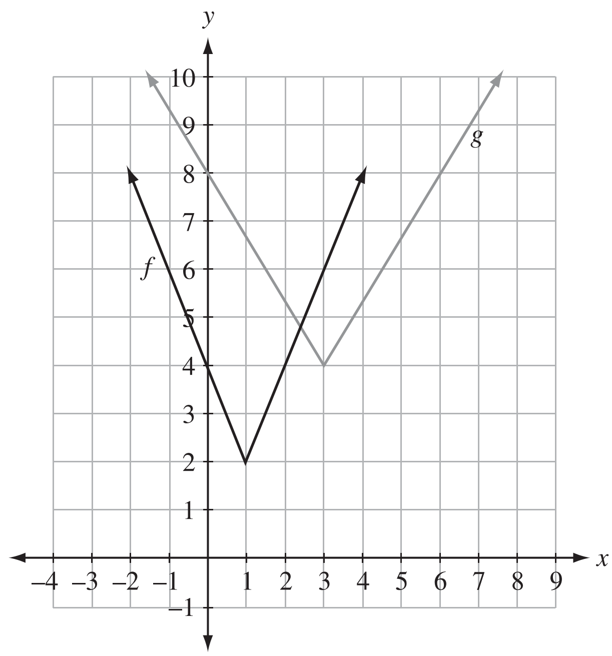
- ***A.** 4 units left and 2 units down
- **B.** 4 units right and 2 units up
- **C.** 1 unit left and 3 units up
- **D.** 2 units left and 4 units down

11. The transformation of the function $f(x) = x^3$ is described by the mapping notation $(x, y) \rightarrow (x - 4, y + 9)$. Describe the transformations on $y = f(x)$.

Possible solution: There will be a horizontal translation 4 units left and a vertical translation 9 units up.

Use the following information to answer the next question.

The graph of the function $y = f(x)$ is transformed to produce the graph of the function $y = g(x)$.



12. An equation for $g(x)$ in terms of $f(x)$ is

A. $g(x) = \frac{1}{2}f(3x)$

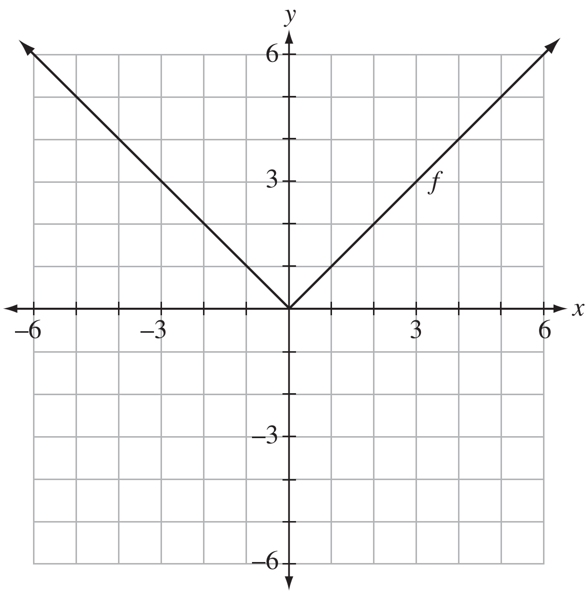
B. $g(x) = 2f(3x)$

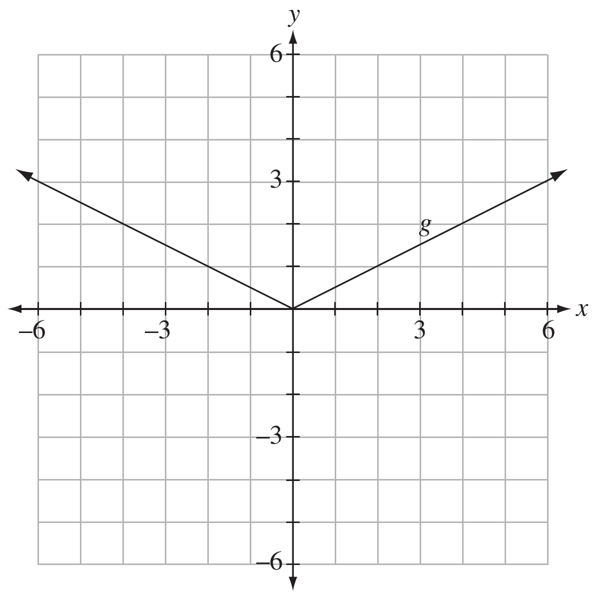
C. $g(x) = \frac{1}{2}f\left(\frac{1}{3}x\right)$

***D.** $g(x) = 2f\left(\frac{1}{3}x\right)$

Use the following information to answer the next question.

The graphs of $f(x) = |x|$ and $y = g(x)$ are shown below. The graph of $f(x)$ undergoes a single transformation to become the graph of $g(x)$.





13. Determine an equation for the function $g(x)$. (There is more than one correct answer.)

Possible solution: $g(x) = \frac{1}{2}|x|$ or $g(x) = \left|\frac{1}{2}x\right|$

The single transformation can be either a vertical stretch by a factor of $\frac{1}{2}$ about the x-axis or a horizontal stretch by a factor of 2 about the y-axis.

Use the following information to answer the next question.

The graph of $y = f(x)$ is transformed into the graph of $g(x) + 4 = 2f(x - 3)$. The domain and range of each function are shown below.

	Domain	**Range**
Graph of $f(x)$	[–1, 3]	[2, 6]
Graph of $g(x)$	[a, b]	[c, d]

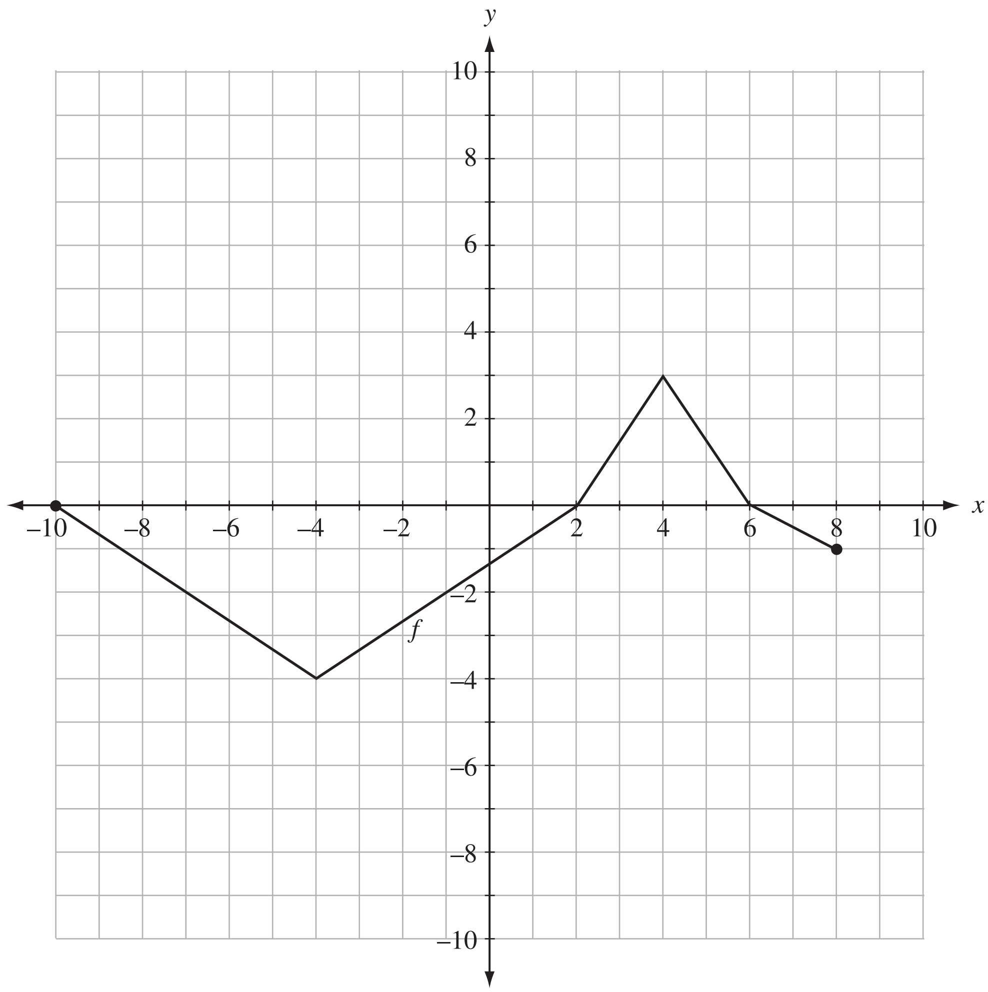
14. For the graph of $g(x)$, the values of ***a***, ***b***, ***c***, and ***d*** are, respectively, _____, _____, _____, and _____.

Possible solution: 2608

Since $(x, y) \rightarrow (x + 3, 2y - 4)$, D:[2, 6] and R:[0, 8] for the graph of $g(x)$.

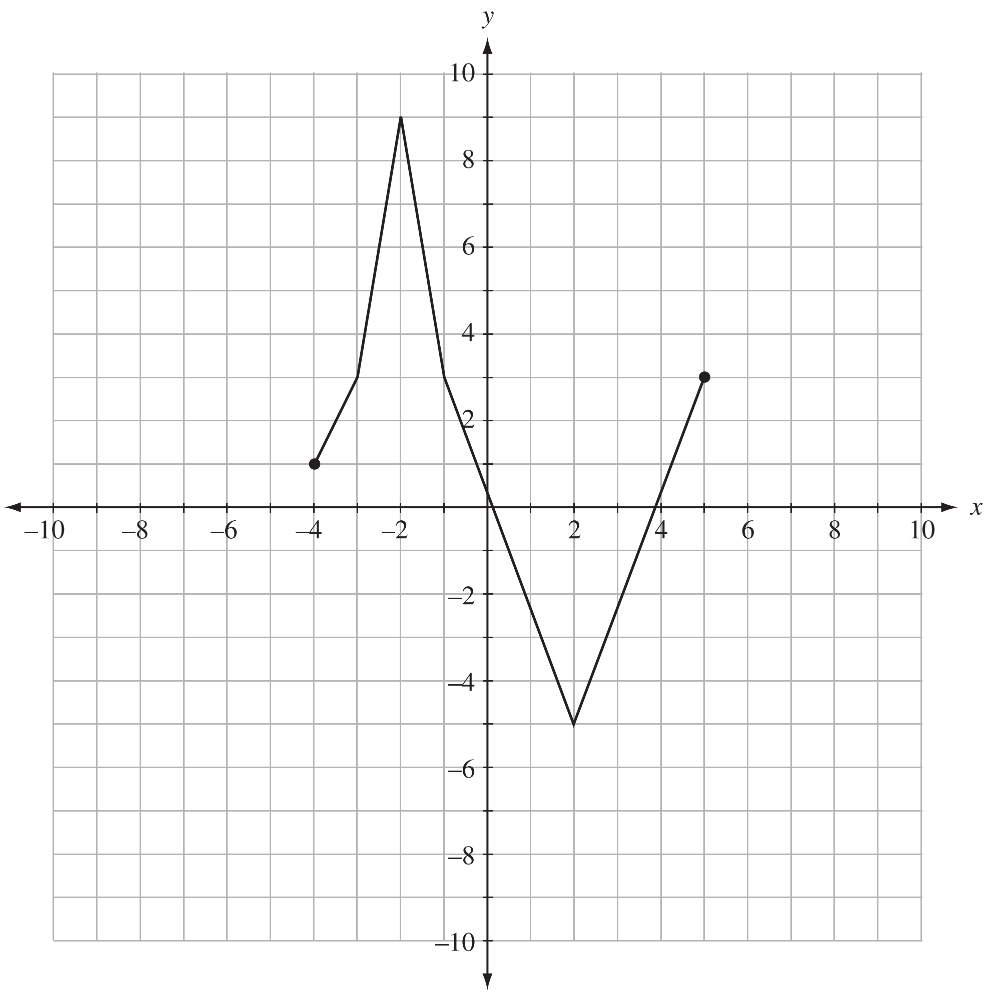
Use the following information to answer the next question.

The graph of $y = f(x)$ below is reflected in the y-axis, vertically stretched by a factor of 2 about the x-axis, horizontally stretched by a factor of $\frac{1}{2}$ about the line $x = 0$, and then translated 3 units up.



SE **15.** Sketch the graph of the new function.

Possible solution:



$$(x, y) \rightarrow \left(-\frac{1}{2}x,\ 2y + 3\right)$$

Note: This item is SE since it involves a reflection, a stretch, and a translation.

Use the following information to answer the next question.

The ordered pairs below represent possible transformations of Point $P(a, b)$ on the graph of the function $y = f(x)$.

Point 1: $(4a, b)$	**Point 3:** $(a, -b)$	**Point 5:** $\left(\frac{a}{4}, b\right)$
Point 2: $(-a, b)$	**Point 4:** $\left(a, \frac{b}{4}\right)$	**Point 6:** $(a, 4b)$

16. If $y = f(x)$ undergoes the following single transformations, identify the coordinates of the corresponding Point P on the new graph.

The corresponding point on the function $y = -f(x)$ is point number __________.

The corresponding point on the function $y = f\left(\frac{1}{4}x\right)$ is point number __________.

The corresponding point on the function $y = \frac{1}{4}f(x)$ is point number __________.

The corresponding point on the function $y = f(-x)$ is point number __________.

Solution: 3142

Use the following information to answer the next question.

The graph of $y = f(x)$ is reflected in the x-axis, stretched vertically about the x-axis by a factor of $\frac{1}{3}$, and stretched horizontally about the y-axis by a factor of 4 to create the graph of $y = g(x)$.

17. For Point $A(-3, 6)$ on the graph of $y = f(x)$, the corresponding image point, A', on the graph of $y = g(x)$ is

A. (9, 24)
B. (–12, –18)
C. (1, 24)
***D.** (–12, –2)

18. When the graph of $y = -x^2 + 4$ is reflected in the y-axis, the new equation will be

A. $y = x^2 + 4$
***B.** $y = -x^2 + 4$
C. $y = x^2 - 4$
D. $y = -x^2 - 4$

SE **19.** Describe a sequence of transformations required to transform the graph of $y = \sqrt{x}$ into the graph of $y = \sqrt{-\frac{1}{2}x - 4} + 10$.

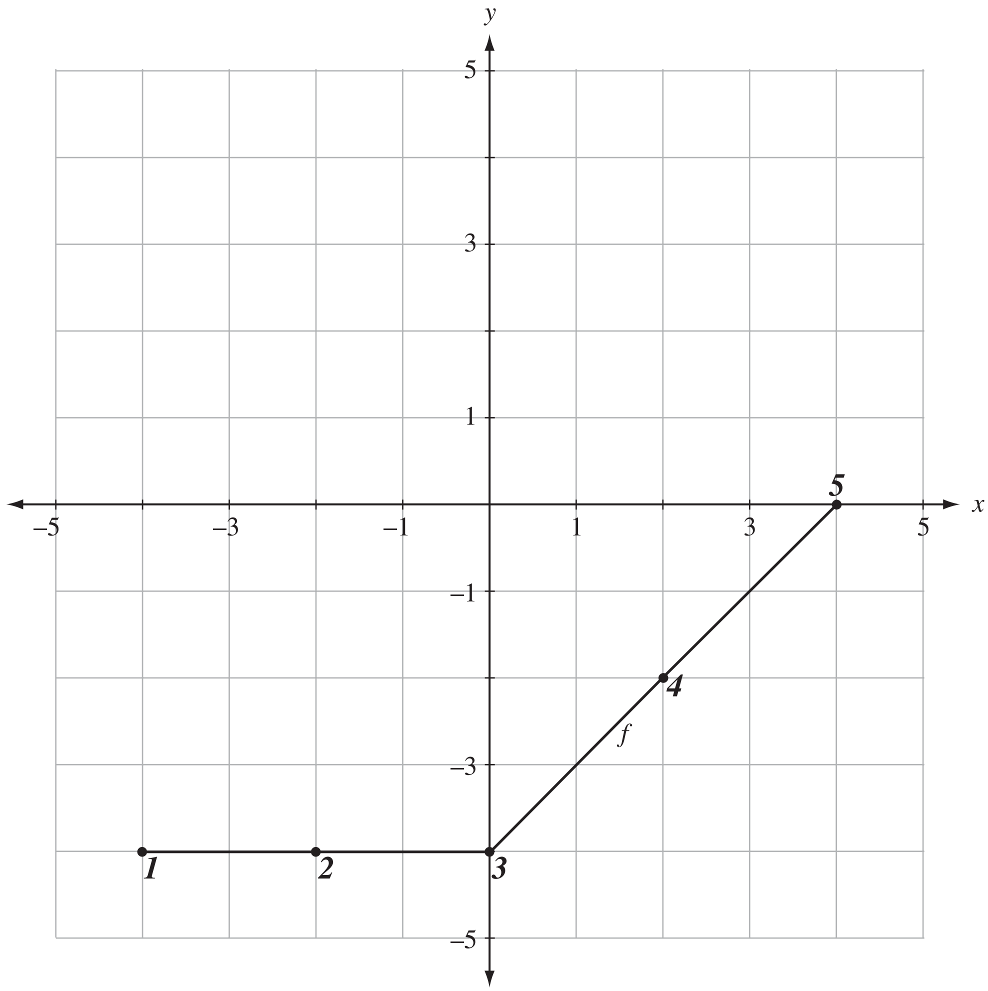
Possible solution: $y = \sqrt{-\frac{1}{2}(x + 8)} + 10$

The graph of $y = \sqrt{x}$ is reflected in the y-axis, horizontally stretched by a factor of 2 about the y-axis, and then translated 8 units left and 10 units up.

Note: This item is SE since the transformation involves factoring the b value and since it contains a reflection, a stretch, and translations.

Use the following information to answer the next question.

The graph of $y = f(x)$ is shown below.



20. For each transformation of $y = f(x)$ indicated below, the invariant point exists at point number:

________	________	________
$y = -f(x)$	$y = f(-x)$	$x = f(y)$

Solution: 531

21. Verify that $f(x) = 2x - 3$ is the inverse of $g(x) = \frac{1}{2}x + \frac{3}{2}$.

Possible solution: $x = 2y - 3$

$$x + 3 = 2y$$

$$\frac{x+3}{2} = y$$

$$\therefore f^{-1}(x) = \frac{1}{2}x + \frac{3}{2}$$

SE **22.** A restriction on the domain of $f(x) = x^2 + 4$, such that its inverse is also a function, could be:

A. $\{x \mid x \geq -4\}$
***B.** $\{x \mid x \geq 0\}$
C. $\{x \mid x \leq 2\}$
D. $\{x \mid x \leq 4\}$

Note: This item is SE since it involves a restriction on the domain in order for the inverse to be a function.

23. The graph of $y = 3^x$ is reflected in the line $y = x$. The equation of the new graph is

***A.** $y = \log_3 x$
B. $y = 3\log x$
C. $y = 3^{-x}$
D. $y = -3^x$

24. The value of $\log_5 625 + 3\log_7 49 + \log_2 \frac{1}{16} + \log_b b + \log_a 1$ is __________.

Possible solution: $4 + 3(2) + -4 + 1 + 0 = 7$

SE **25.** The equation $m\log_p n + 5 = q$ can be written in exponential form as

A. $p^{(q-5)} = mn$

***B.** $p^{(q-5)} = n^m$

C. $p^{(q-5)} = \frac{m}{n}$

D. $p^{(q-5)} = m^n$

Note: This item is SE since the conversion involves more than two steps.

26. Rank these logarithms in order from **least** to **greatest**: $\log_4 62$, $\log_6 36$, $\log_3 10$, $\log_5 20$.

Solution: $\log_5 20$, $\log_6 36$, $\log_3 10$, $\log_4 62$

Using benchmarks:

- Since $\log_5 25 = 2$, $\log_5 20 < 2$
- $\log_6 36 = 2$
- Also, $\log_3 9 = 2$, so $\log_3 10 > 2$
- Since $\log_4 16 = 2$ and $\log_4 64 = 3$, $2 < \log_4 62 < 3$ but its value is closer to 3.

27. The expression $\left(3^{\log x}\right)\left(3^{\log x}\right)$ is equivalent to

***A.** $3^{\log x^2}$

B. $9^{\log x^2}$

C. $3^{(\log x)^2}$

D. $9^{(\log x)^2}$

28. Written as a single logarithm, $2\log x - \dfrac{\log z}{2} + 3\log y$ is

***A.** $\log\left(\dfrac{x^2y^3}{\sqrt{z}}\right)$

B. $3\log\left(\dfrac{xy}{z}\right)$

C. $\log\left(\dfrac{x^2}{y^3\sqrt{z}}\right)$

D. $\log\left(x^2 - \sqrt{z} + y^3\right)$

29. Given that $\log_3 a = 6$ and $\log_3 b = 5$, determine the value of $\log_3\left(9ab^2\right)$.

Possible solution: $\log_3 9 + \log_3 a + 2\log_3 b = 2 + 6 + 2(5) = 18$

Use the following information to answer the next question.

A student's work to simplify a logarithmic expression is shown below, where $a > 1$.

Step 1 $2\log_a x^4 - 3\log_a x^2 + 4\log_a x^3$

Step 2 $\log_a x^8 - \log_a x^6 + \log_a x^{12}$

Step 3 $\log_a\left(\dfrac{x^8}{x^6 \times x^{12}}\right)$

Step 4 $\log_a\left(\dfrac{x^8}{x^{18}}\right)$

Step 5 $\log_a x^{10}$

30. The student's **first** recorded error is in Step

A. 2
***B.** 3
C. 4
D. 5

31. The equation of the asymptote for the graph of $y = \log_b(x-3) + 2$, where $b > 1$, is

A. $y = 2$
B. $y = -2$
***C.** $x = 3$
D. $x = -3$

Use the following information to answer the next question.

A student sketched the graphs of $f(x) = \log_a(x + 3) - 7$ and $g(x) = a^{(x-2)} + 5$, where $a > 1$, on a coordinate plane. She also drew the asymptotes of the two graphs using dotted lines.

32. The intersection point of the two dotted lines will be at

A. (3, 5)
***B.** (–3, 5)
C. (2, –7)
D. (–2, –7)

$\boxed{SE}$ **33.** For the graph of $y = \log_b(3x + 12)$, where $b > 1$, the domain is

***A.** $x > -4$
B. $x > 4$
C. $x > -12$
D. $x > 12$

Note: This item is SE since the *b* value is not factored out of the binomial.

34. The *y*-intercept on the graph of $f(x) = a^{(x+1)} + b$, where $a > 0$, $a \neq 1$, is

A. a
B. b
C. $1 + b$
***D.** $a + b$

35. Algebraically solve the equation $8^{(3x+4)} = 4^{(x-9)}$.

Possible solution: $2^{3(3x+4)} = 2^{2(x-9)}$

$$9x + 12 = 2x - 18$$

$$7x = -30$$

$$x = \frac{-30}{7}$$

SE **36.** Solve the equation $3^{(2x+1)} = \left(\frac{1}{5}\right)^{(x-3)}$ algebraically. Round to the nearest hundredth, if necessary.

Possible solution: $3^{(2x+1)} = 5^{(-x+3)}$

$$(2x + 1)\log 3 = (-x + 3)\log 5$$

$$2x \log 3 + \log 3 = -x \log 5 + 3 \log 5$$

$$2x \log 3 + x \log 5 = 3 \log 5 - \log 3$$

$$x(2 \log 3 + \log 5) = 3 \log 5 - \log 3$$

$$x = \frac{3 \log 5 - \log 3}{2 \log 3 + \log 5}$$

$$x = 0.97978\ldots$$

$$x \approx 0.98$$

Note: This item is SE since the powers have no common base, and because the exponents are not monomials.

A/SE

37. Solve algebraically $\log_7(x+1)+\log_7(x-5)=1.$

Possible solution: $\log_7(x+1)+\log_7(x-5)=1$

$\log_7\left(x^2-4x-5\right)=1$

$7=x^2-4x-5$

$0=x^2-4x-12$

$0=(x-6)(x+2)$

$x=6$ or $x=-2$ (Acceptable)

$x=-2$ is an extraneous root (Excellence)

$\therefore x=6$

Note: This item is SE if students are required to recognize that a solution is extraneous.

Use the following information to answer the next question.

Earthquake intensity is given by $I=I_0\times 10^M$, where I_0 is the reference intensity and M is the magnitude. An earthquake measuring 5.3 on the Richter scale is 125 times more intense than a second earthquake.

SE

38. Determine, to the nearest tenth, the Richter scale measure of the second earthquake.

Possible solution: $\dfrac{I_2}{I_1}=\dfrac{I_0\times 10^{5.3}}{I_0\times 10^x}$

$125=10^{5.3-x}$

$5.3-x=\log_{10}125$

$x=5.3-\log_{10}125$

$x\approx 3.2$

Note: This item is SE since it requires solving for an exponent in a comparison problem.

SE **39.** The population of a particular town on July 1, 2011, was 20 000. If the population decreases at an average annual rate of 1.4%, how long will it take for the population to reach 15 300?

Possible solution: $y = ab^{\frac{t}{p}}$

$$15\,300 = 20\,000(1 - 0.014)^{\frac{t}{1}}$$

$$0.765 = 0.986^{t}$$

$$t = \frac{\log 0.765}{\log 0.986}$$

$$t = 18.9999\ldots$$

$$t \approx 19 \text{ years}$$

Note: This item is SE since the exponential equation has a numerical coefficient.

40. Jordan needs \$6 000 to take his family on a trip. He is able to make an investment which offers an interest rate of 8%/a compounded semi-annually. How much should Jordan invest now, to the nearest dollar, so that he has enough money to go on a family trip in 3 years?

Possible solution: $y = ab^{\frac{t}{p}}$

$$6\,000 = a(1.04)^{\frac{3}{1/2}}$$

$$a = 4\,741.887\,15\ldots$$

Jordan should invest \$4 742 now.

41. Express the following polynomials in factored form.

a) $P(x) = x^3 - x^2 - 8x + 12$

Possible solution: Potential integral zeros: $\{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12\}$

$$P(2) = 2^3 - 2^2 - 8(2) + 12 = 0 \rightarrow (x - 2) \text{ is a factor}$$

$$\begin{array}{r|rrrr} -2 & 1 & -1 & -8 & 12 \\ & \downarrow & -2 & -2 & 12 \\ \hline & 1 & 1 & -6 & 0 \end{array}$$

$$x^2 + x - 6 = (x + 3)(x - 2)$$

$$\therefore\ P(x) = (x - 2)^2(x + 3)$$

b) $P(x) = 2x^4 + 3x^3 - 17x^2 - 27x - 9$

Possible solution: Potential integral zeros: $\{\pm 1, \pm 3, \pm 9\}$

$$P(-1) = 2(-1)^4 + 3(-1)^3 - 17(-1)^2 - 27(-1) - 9 = 0 \rightarrow (x + 1) \text{ is a factor}$$

$$\begin{array}{r|rrrrr} 1 & 2 & 3 & -17 & -27 & -9 \\ & \downarrow & 2 & 1 & -18 & -9 \\ \hline & 2 & 1 & -18 & -9 & 0 \end{array}$$

$$Q(x) = 2x^3 + x^2 - 18x - 9$$

$$Q(3) = 2(3)^3 + 3^2 - 18(3) - 9 = 0 \rightarrow (x - 3) \text{ is a factor}$$

$$\begin{array}{r|rrrr} -3 & 2 & 1 & -18 & -9 \\ & \downarrow & -6 & -21 & -9 \\ \hline & 2 & 7 & 3 & 0 \end{array}$$

$$2x^2 + 7x + 3 = (2x + 1)(x + 3)$$

$$\therefore\ P(x) = (x + 1)(x - 3)(2x + 1)(x + 3)$$

42. The polynomial function $P(x) = 4x^4 - x^3 - 8x^2 - 40$ has a linear factor of $x + 2$.

The remaining cubic factor is

A. $4x^3 + 7x^2 + 6x - 20$
B. $4x^3 + 7x^2 + 6x + 12$
***C.** $4x^3 - 9x^2 + 10x - 20$
D. $4x^3 - 9x^2 + 10x - 60$

43. Given that $x + 2$ is a factor of $f(x) = x^3 + 3x^2 - kx + 4$, determine the value of k.

Possible solution: $f(-2) = (-2)^3 + 3(-2)^2 - k(-2) + 4$

$0 = 2k + 8$

$k = -4$

44. *If* $P(x)$ *is a polynomial function where* $P\left(-\frac{2}{3}\right) = 0$ *and* $P(0) = 12$, *then* ___***i***___ *is a factor of* $P(x)$, *and* ___***ii***___ *is a constant term in the equation of* $P(x)$.

The statement above is completed by the information in row

Row	*i*	*ii*
A.	$(3x - 2)$	12
***B.**	$(3x + 2)$	12
C.	$(3x - 2)$	–12
D.	$(3x + 2)$	–12

Use the following information to answer the next question.

A list of five functions is given below.

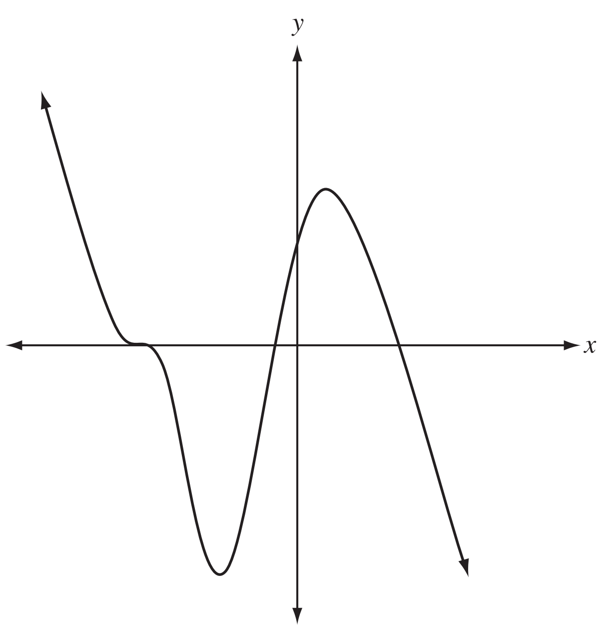
1	$y = x^4 + 10x^3 - 2x + 5$
2	$y = 3x^3 - 2x^2 + x^{-1} - 4$
3	$y = \sqrt{5}\,x + 3$
4	$y = 4x^3 + 2x^2 + \dfrac{1}{x}$
5	$y = -2x^5 + 7x^4 - 3x^3 + 2^x - 7$

45. Which of the functions above is a polynomial function? Explain why or why not.

Possible solution: Polynomials cannot have a variable in the denominator, a negative exponent on the variable, a variable as a radicand, a variable with a rational exponent, or a variable as an exponent.
Therefore, 1 and 3 are polynomial functions, and 2, 4, and 5 are not.

46. Sketch the graph of a fifth-degree polynomial function with one real zero of multiplicity 3 and with a negative leading coefficient.

Possible solution:



47. Given the function $y = \frac{1}{4}(x-2)(2x+5)(x+4)^2$,

a) Accurately sketch the graph, and label any key points.

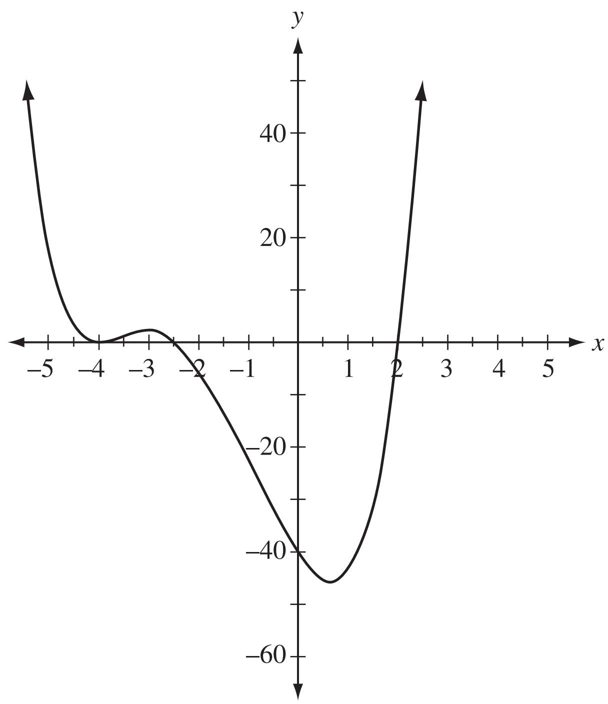
Possible solution:

Key points:

x-intercepts at $2, -\frac{5}{2}, -4$

y-intercept at -40

Using technology, the minimum value is at approximately -45.976 when $x = 0.659$.



b) State the domain and range.

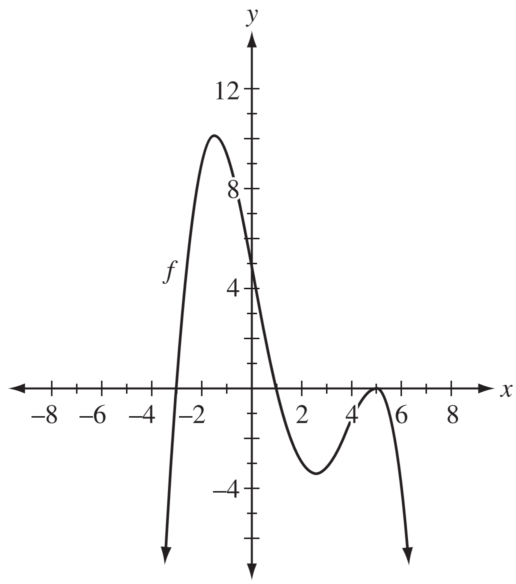
Possible solution: D:$\{x \mid x \in R\}$ and R:$\{y \mid y \geq -45.976\}$

c) Determine the zeros of the function.

Possible solution: zeros at $x = 2$, $x = -\frac{5}{2}$, $x = -4$ with multiplicity 2

Use the following information to answer the next question.

The graph of the polynomial function $y = f(x)$ is shown below.



48. What is the minimum possible degree for the polynomial function above? Determine an equation of the function in factored form.

Possible solution: Degree = 4

Since the zeros of the function are –3, 1, and 5 (multiplicity 2), the factors are $f(x) = a(x + 3)(x - 1)(x - 5)^2$. Use the y-intercept (0, 5) to find the leading coefficient.

$$5 = a(0 + 3)(0 - 1)(0 - 5)^2$$

$$5 = a(-75)$$

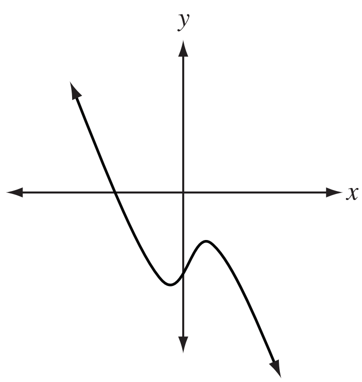
$$-\frac{1}{15} = a$$

$$\therefore f(x) = -\frac{1}{15}(x + 3)(x - 1)(x - 5)^2$$

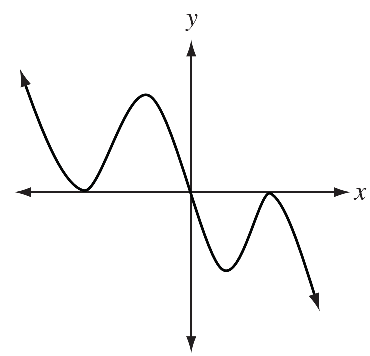
Use the following information to answer the next question.

The graphs of four polynomial functions are shown below.

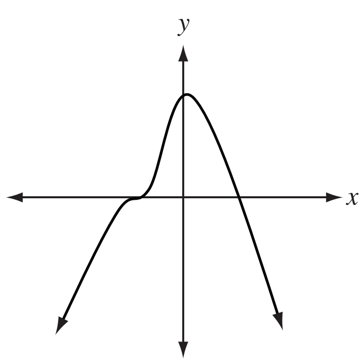
Graph 1



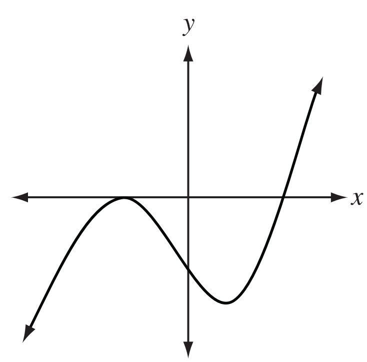
Graph 2



Graph 3



Graph 4



49. Match three of the graphs numbered above with a statement below that best describes the function.

The graph that has a positive leading coefficient is graph number __________.

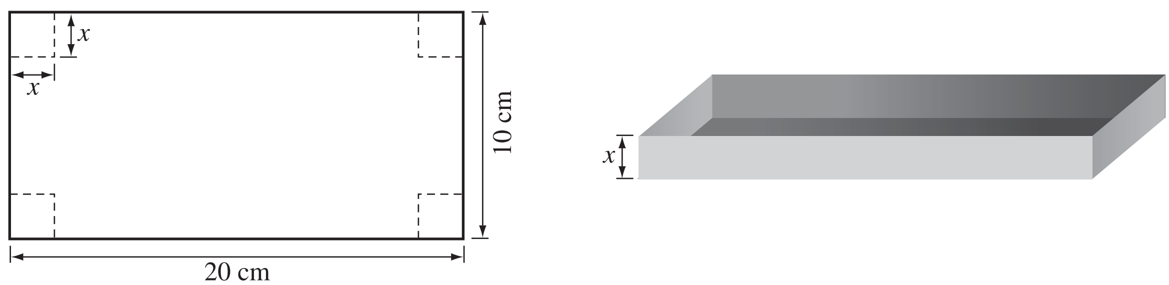
The graph of a function that has two different zeros, each with multiplicity 2, is graph number __________.

The graph that could be of a degree 4 function is graph number __________.

Solution: 423

Use the following information to answer the next question.

A box with no lid is made by cutting four squares of side length x from each corner of a 10 cm by 20 cm rectangular sheet of metal.



SE **50.** Using the information above, follow the directions below.

a) Find an expression that represents the volume of the box.

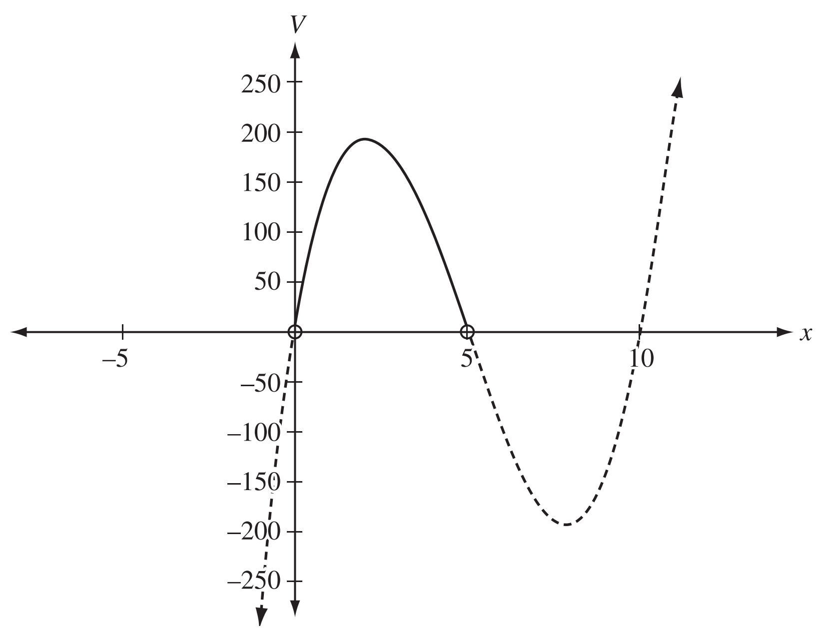
Possible solution: $V = x(10 - 2x)(20 - 2x)$

b) Sketch the graph of the function and state the restriction on the domain.

Possible solution:

Where $0 < x < 5$

Since the volume must be positive and one dimension of the sheet is 10 cm, the value of x cannot exceed 5 cm.



c) Find the value of x, to the nearest hundredth of a centimetre, that gives the maximum volume.

Possible solution: Using technology, the volume is a maximum when $x \approx 2.11$ cm.

d) What is the maximum volume of the box, to the nearest cubic centimetre?

Possible solution: Using technology, the maximum volume of the box is 192 cm^3.

Note: This item is SE since it requires a complete solution to a problem by modelling a given situation with a polynomial function.

51. Determine the roots of the equation $2x^3 - 3x^2 - 10x + 3 = 0$. Leave answers as exact values.

Possible solution: Potential integral zeros: $\{\pm 1, \pm 3\}$

$x = 3$

$2x^3 - 3x^2 - 10x + 3$

$= 2(3)^3 - 3(3)^2 - 10(3) + 3$

$= 0$

Therefore, $(x - 3)$ is a factor.

$$\begin{array}{r|rrrr} -3 & 2 & -3 & -10 & 3 \\ & \downarrow & -6 & -9 & 3 \\ \hline & 2 & 3 & -1 & 0 \end{array}$$

Since the remaining factor $(2x^2 + 3x - 1)$ cannot be factored, use the quadratic formula to determine the remaining roots.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

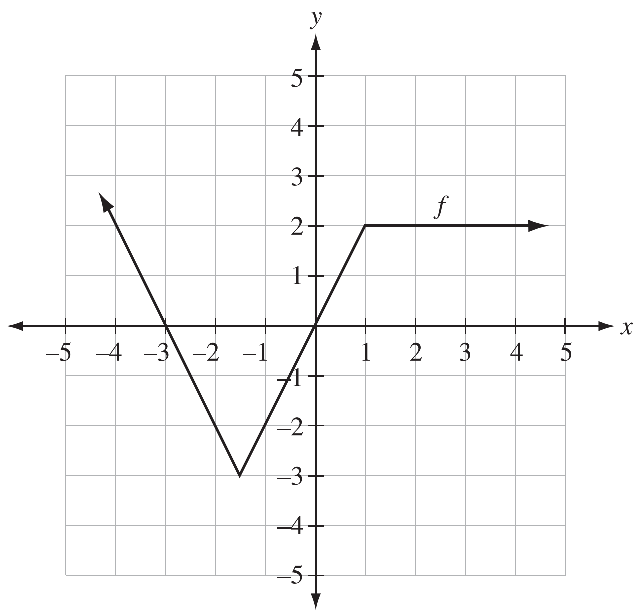
$$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-1)}}{2(2)}$$

$$x = \frac{-3 \pm \sqrt{17}}{4}$$

$$\therefore x = 3 \quad \text{and} \quad x = \frac{-3 \pm \sqrt{17}}{4}$$

Use the following information to answer the next question.

The graph of the function $y = f(x)$ is shown below.

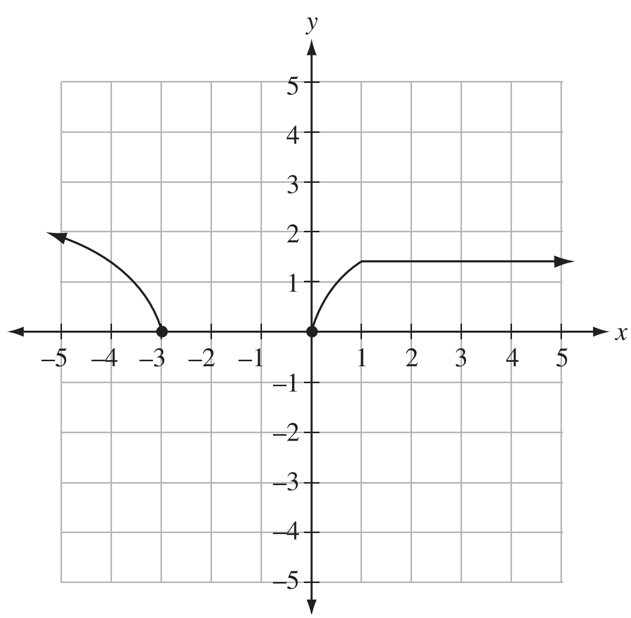


52. Sketch the graph of $y = \sqrt{f(x)}$ and state the domain and range.

Possible solution:

D: $(-\infty, -3]$ or $[0, \infty)$

R: $[0, \infty)$



53. State the coordinates of any invariant points when $f(x) = \frac{1}{2}x - 3$ is transformed into $y = \sqrt{f(x)}$.

Possible solution: When $y = f(x)$ is transformed into $y = \sqrt{f(x)}$, the invariant points exist where $f(x) = 0$ and $f(x) = 1$.

$\therefore$ The coordinates of the invariant points are $(6, 0)$ and $(8, 1)$.

54. Determine the x-intercept of $y = -2\sqrt{x+4} + 3$, to the nearest hundredth, and explain its relationship to the zero of the function.

Possible solution: x-intercept is –1.75, which is the same as the zero of the function.

55. Sketch the graph of the following functions and determine the following characteristics for each function below: domain, x- and y-intercepts, and equations of vertical asymptotes.

a) $y = \dfrac{3x}{x^2 + 2x - 8}$

Possible solution:

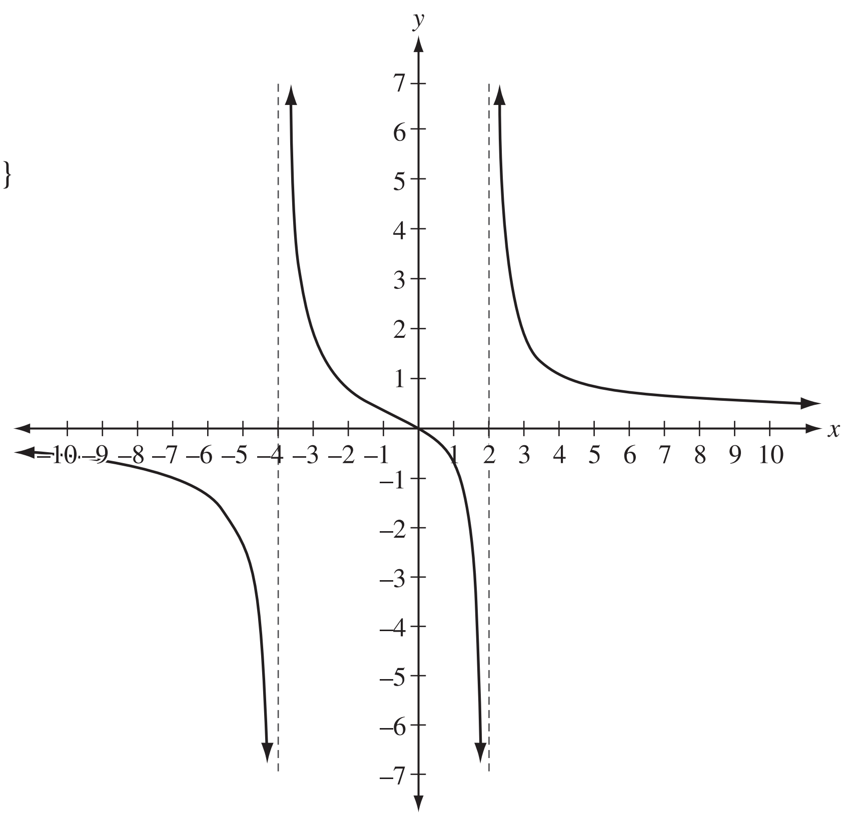
$y = \dfrac{3x}{(x+4)(x-2)}$

D:$\{x \mid x \neq -4, 2, x \in R\}$

x-intercept: 0

y-intercept: 0

Equation of vertical asymptotes: $x = -4$ and $x = 2$



b) $y = \dfrac{x+3}{x^2 - 9}$

Possible solution:

$y = \dfrac{x + 3}{(x + 3)(x - 3)}$

$y = \dfrac{1}{x - 3}$

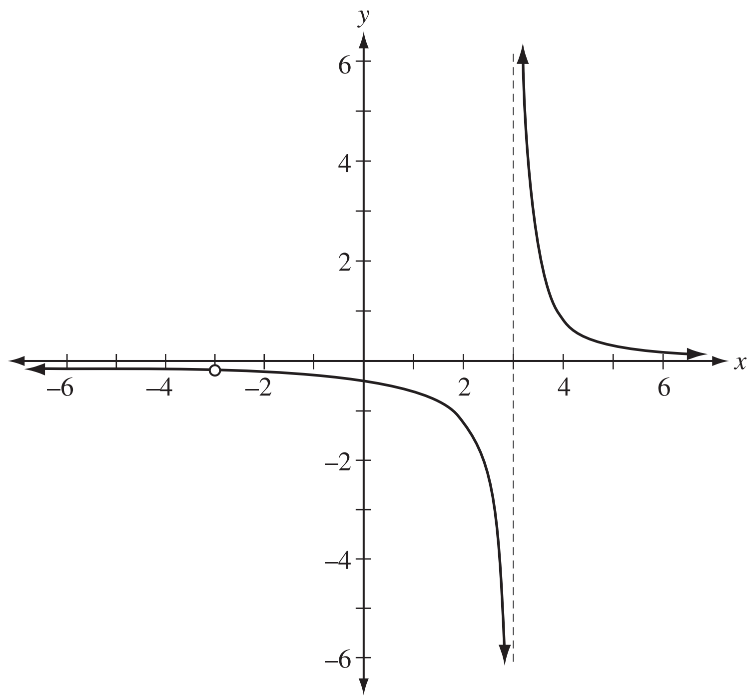
D:$\{x \mid x \neq \pm 3, x \in R\}$

No x-intercept

y-intercept: $-\dfrac{1}{3}$

Equation of vertical asymptote: $x = 3$

There is a point of discontinuity at $x = –3$.



56. For the graph of $y = \dfrac{3x + 7}{2x + 5}$, determine the equation of the horizontal asymptote and the range.

Possible solution: $y(2x + 5) = 3x + 7$

$$2xy + 5y = 3x + 7$$

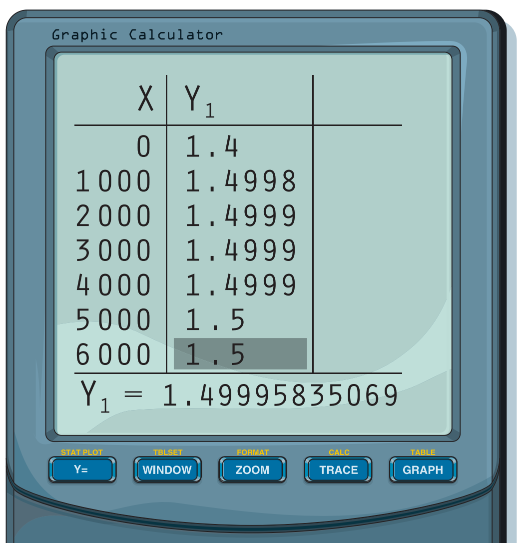
$$2xy - 3x = 7 - 5y$$

$$x(2y - 3) = 7 - 5y$$

$$x = \frac{7 - 5y}{2y - 3}$$

Since $y \neq \dfrac{3}{2}$, the equation of the horizontal asymptote is $y = \dfrac{3}{2}$ and the range is $\left\{y \mid y \neq \dfrac{3}{2},\ y \in R\right\}$.

Possible solution: Using the function $y = \dfrac{3x + 7}{2x + 5}$, examine the y-values corresponding to large values of x, either using a table of values, or by analyzing the graph.



As x gets larger, the y-value approaches, but is never equal to, 1.5.

(**Note:** while the calculator may appear to round the y-value to 1.5, the y-coordinate is a value very close to 1.5; in this case at $x = 6\ 000$, a cursor placed on the y-coordinate displays a value of 1.49995835069...)

Therefore, the horizontal asymptote is $y = 1.5$, and the range is $\left\{y \mid y \neq 1.5,\ y \in R\right\}$.

SE **57.** Determine the coordinates of the point of discontinuity on the graph of $f(x) = \frac{2x^2 - 15x + 7}{x - 7}$.

Possible solution: $f(x) = \frac{(2x-1)(x-7)}{x-7}$

There is a point of discontinuity when $x = 7$.

$\therefore f(x) = 2x - 1$, for $x \neq 7$

$y = 2(7) - 1$

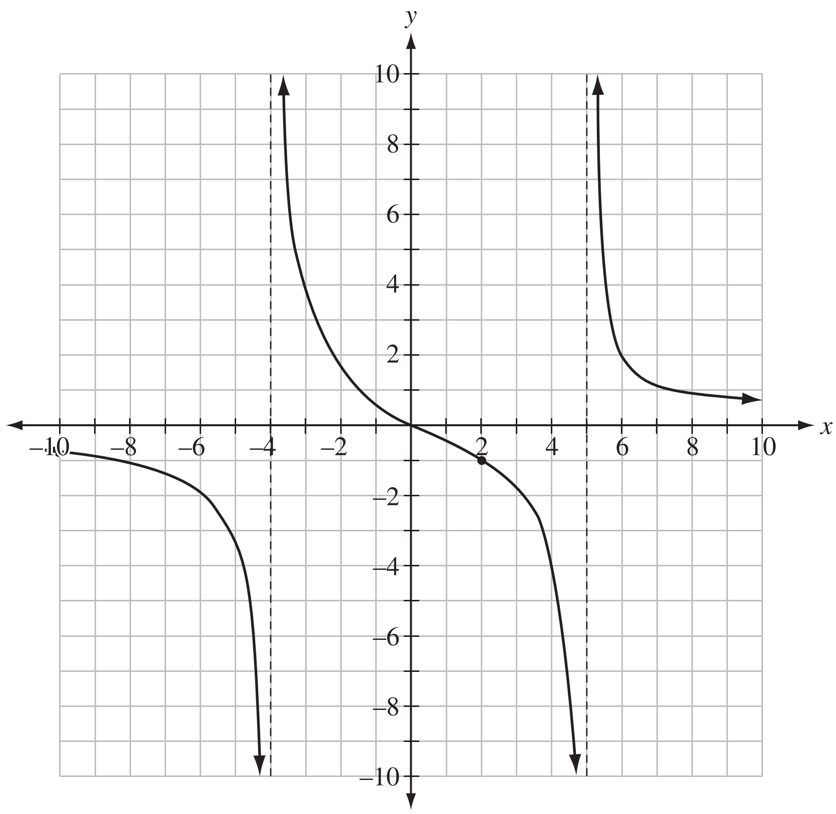
$y = 13$

So the point of discontinuity is (7, 13).

Note: This item is SE since the y-coordinate of the point of discontinuity is required.

Use the following information to answer the next question.

The graph of the function below can be expressed in the form $f(x) = \dfrac{ax}{x^2 + bx + c}$, where $f(2) = -1$.



58. Determine the values of ***a***, ***b***, and ***c***.

Possible solution: $a = 9$, $b = -1$, and $c = -20$

Since the vertical asymptotes exist at $x = -4$ and $x = 5$,

$y = \dfrac{ax}{(x+4)(x-5)}$.

Using the point $(2, -1)$, $-1 = \dfrac{a(2)}{(2+4)(2-5)}$

$$18 = 2a$$

$$9 = a$$

$$\therefore y = \frac{9x}{x^2 - x - 20}$$

Trigonometry

General Outcome

Develop trigonometric reasoning.

General Notes:

- Students need to differentiate between "exact value" and "approximate value" in problems.
- Students should be able to express exact value answers with either a rationalized or unrationalized denominator.
- Students should be able to perform, analyze, and describe transformations on sinusoidal functions.
- When appropriate, technology could include graphing calculators, computer graphing applications, and applets.
- The restricted domain may be something other than $0° \leq \theta < 360°$ or $0 \leq \theta < 2\pi$; for example, $-\pi \leq \theta \leq \frac{3\pi}{2}$.

Specific Outcomes

Specific Outcome 1

Demonstrate an understanding of angles in standard position, expressed in degrees and radians. **[CN, ME, R, V]**

Notes:

- The understanding of radian measure $\theta = \frac{a}{r}$ is intended to be a part of this outcome.

(See examples 1–5, 11, and 13)

Specific Outcome 2

Develop and apply the equation of the unit circle. **[CN, R, V]**

Notes:

- The intent of this outcome is that students have a deeper understanding of how points on the unit circle relate to the equation of the unit circle and the trigonometric ratios.

(See examples 6 and 7)

Specific Outcome 3

Solve problems, using the six trigonometric ratios for angles expressed in radians and degrees.
[ME, PS, R, T, V] [ICT : C6–4.1]

(See examples 7–13)

Specific Outcome 4

Graph and analyze the trigonometric functions sine, cosine, and tangent to solve problems.
[CN, PS, T, V] [ICT : C6–4.1, C6–4.3]

Notes:

- Students should write the sinusoidal function in the form $y = a\sin\left[b(x - c)\right] + d$ or $y = a\cos\left[b(x - c)\right] + d$.
- Transformations of tangent graphs are beyond the scope of this outcome.
- Graphing reciprocal trigonometric functions is beyond the scope of this outcome.
- Analyzing the characteristics of a sinusoidal function includes but is not limited to: determining and describing amplitude, period, horizontal phase shift, vertical displacement, intercepts, domain, and range.
- Analyzing the characteristics of a tangent function includes but is not limited to: determining and describing period, asymptotes, intercepts, domain, and range.

(See examples 14–19)

Specific Outcome 5

Solve, algebraically and graphically, first- and second-degree trigonometric equations with the domain expressed in degrees and radians. **[CN, PS, R, T, V]   [ICT : C6–4.1, C6–4.4]**

Notes:

- Solving equations using single trigonometric identity substitutions should be limited to reciprocal, quotient, Pythagorean, double-angle identities, and sum or difference identities.
- Solving a double-angle equation algebraically will only be approached by identity substitution, resulting in the removal of the double-angle identity. All other multiple-angle equations are beyond the scope of this outcome.

(See examples 20–24)

Specific Outcome 6

Prove trigonometric identities, using:

- reciprocal identities
- quotient identities
- Pythagorean identities
- sum or difference identities (restricted to sine, cosine, and tangent)
- double-angle identities (restricted to sine, cosine, and tangent) **[R, T, V] [ICT: C6–4.1, C6–4.4]**

Notes:

- Students should understand that proving an identity is different from solving an equation.

(See examples 25–30)

Acceptable Standard	Standard of Excellence
The student can:	The student can also:
• demonstrate an understanding of the radian measure of an angle as a ratio of the subtended arc to the radius of a circle	
• convert from radians to degrees and vice versa	
• solve problems involving arc length, radius, and angle measure in either radians or degrees	• solve multi-step problems based on the relationship $\theta = \frac{a}{r}$ (angle conversion is not considered a step)
• determine the measures, in degrees or radians, of all angles that are co-terminal with a given angle in standard position, within a specified domain	
• determine the missing coordinate of a point $P(x, y)$ that lies on the unit circle	
• find the approximate values of trigonometric ratios of angles, θ, where $\theta \in R$	
• find the exact values of trigonometric ratios of special angles, θ, where $\theta \in R$	
• determine the exact values of all the trigonometric ratios, given the value of one trigonometric ratio in a restricted domain or the coordinates of a point on the terminal arm of an angle in standard position	
• determine the measures of the angles, θ, in degrees or radians, given the value of a trigonometric ratio, where $0 \le \theta < 2\pi$ or $0° \le \theta < 360°$, or given a point on the terminal arm of an angle in standard position	
• sketch the graphs of $y = \sin x$, $y = \cos x$, and $y = \tan x$, and analyze the characteristics of the graph	
• describe the characteristics of sinusoidal functions of the form $y = a\sin[b(x - c)] + d$ and $y = a\cos[b(x - c)] + d$, and sketch the graph	• describe the characteristics of sinusoidal functions where the parameter b must be factored, and sketch the graph

• give partial explanations of the relationships between equation parameters and transformations of sinusoidal functions	• give full explanations of the relationships between equation parameters and transformations of sinusoidal functions
• determine a partial equation for a sinusoidal curve given the graph, the characteristics, or a real-world situation	• determine the complete equation, finding all 4 parameters, for a sinusoidal curve given the graph, the characteristics, or a real-world situation
• provide a partial explanation of how the characteristics of the graph of a trigonometric function relate to the conditions in a contextual situation	• provide a complete explanation of how the characteristics of the graph of a trigonometric function relate to the conditions in a contextual situation
• given a trigonometric equation, identify restrictions on the variable in the domain $0 \le \theta < 2\pi$ or $0° \le \theta < 360°$	• given a trigonometric equation, identify restrictions on the variable in the domain $\theta \in R$
• determine, in a restricted domain, the graphical solution for any trigonometric equation	
• algebraically determine the solution set of first-degree trigonometric equations within a restricted domain or the general solution	
• in a restricted domain within $0 \le \theta < 2\pi$ or $0° \le \theta < 360°$, algebraically determine the solution set of second-degree trigonometric equations	• in a restricted domain outside $0 \le \theta < 2\pi$ or $0° \le \theta < 360°$, algebraically determine the solution set of second-degree trigonometric equations • algebraically determine, in a restricted domain, the solution set of trigonometric equations involving sum and difference, double-angle, or Pythagorean trigonometric identity substitutions • determine the general solution of – second-degree trigonometric equations – trigonometric equations involving sum and difference, double-angle, or Pythagorean trigonometric identity substitutions
• explain the difference between a trigonometric identity and a trigonometric equation	

• explain the difference between verifying for a given value and proving an identity for all permissible values	
• verify a trigonometric identity graphically or numerically for a given value	• determine the non-permissible values of a trigonometric identity
• algebraically simplify and prove simple identities, and recognize that there may be non-permissible values	• algebraically simplify and prove more-difficult identities which include sum and difference identities, double-angle identities, conjugates, or the extensive use of rational operations
• determine the exact value of a trigonometric ratio using the sum, difference, and double-angle identities of sine and cosine	• determine the exact value of a trigonometric ratio using the sum, difference, and double-angle identities of a tangent
• participate in and contribute to the problem-solving process for problems that require analysis of trigonometry studied in Mathematics 30–1	• complete the solutions to problems that require the analysis of trigonometry studied in Mathematics 30–1

Examples

Students who achieve the *acceptable standard* should be able to answer all of the following questions, except for any part of a question labelled ***SE***. Parts labelled ***SE*** are appropriate examples for students who achieve the *standard of excellence.*

Note: *In the multiple-choice questions that follow, * indicates the correct answer. Please be aware that the worked solutions show possible strategies; there may be other strategies that could be used.*

1. The angle $\frac{15\pi}{4}$, converted to degrees, is __________°.

Possible solution: $\frac{15\pi}{4} \times \frac{180}{\pi} = 675°$

2. An angle, in radians, that is co-terminal with 30° is

A. $-\frac{5\pi}{6}$

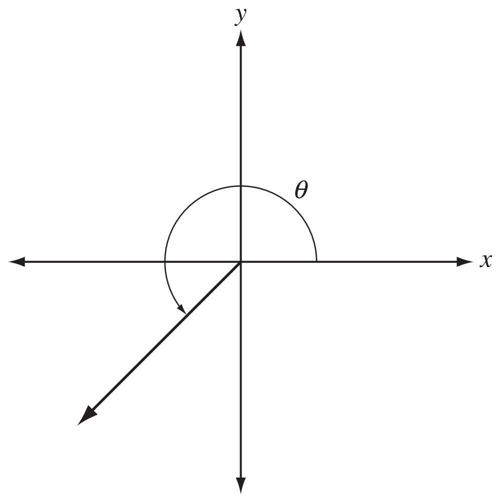
B. $-\frac{13\pi}{6}$

C. $\frac{7\pi}{6}$

***D.** $\frac{25\pi}{6}$

Use the following information to answer the next question.

An angle, θ, in standard position, is shown below.

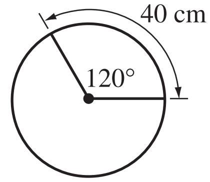


3. The best estimate of the rotation angle θ is

A. 1.25 radians
B. 3.12 radians
***C.** 4.01 radians
D. 5.38 radians

Use the following information to answer the next question.

Mary is given the diagram below, showing an angle rotation of 120°. The arc length of the sector is 40 cm.



Statement 1 The radius of the circle, to the nearest centimetre, is 19 cm.

Statement 2 An equivalent angle rotation is $\frac{4\pi}{3}$.

Statement 3 If the arc length on this circle increases to 80 cm, then the central angle must be 240°.

Statement 4 Mary can determine the radius of the circle by dividing the given angle by the arc length.

4. The two statements above that are correct are numbered _____ and _____.

Possible solution: 13 or 31

Statement 1: *True*

$$\theta = \frac{a}{r}$$

$$r = \frac{40}{\left(120^\circ \times \frac{\pi}{180^\circ}\right)}$$

$$r \approx 19 \text{ cm}$$

Statement 2: *False*

$$120^\circ \times \frac{\pi}{180^\circ} = \frac{2\pi}{3}$$

Statement 3: *True*

$$\theta = \frac{a}{r}$$

$$\theta = \frac{80}{19} \times \frac{180^\circ}{\pi}$$

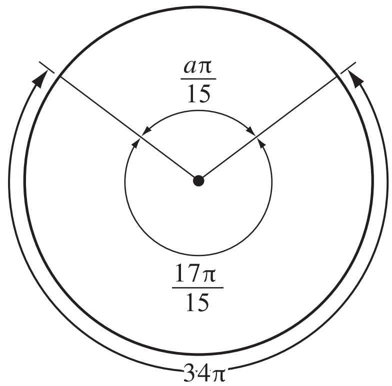
$$\theta = 240^\circ$$

Statement 4: *False*

The radius of a circle is determined by dividing the arc length by the given angle.

Use the following information to answer the next question.

A circle with a radius r, an indicated arc length of 34π, and two central angles of $\frac{a\pi}{15}$ and $\frac{17\pi}{15}$ is shown below.



The value of a in the angle $\frac{a\pi}{15}$ is ***bc***.

The length of the radius, r, of the circle, to the nearest whole number, is ***de***.

SE **5.** The values of ***b***, ***c***, ***d***, and ***e*** are, respectively, _____, _____, _____, and _____.

Possible solution: 1330

Angles

$$\frac{a\pi}{15} + \frac{17\pi}{15} = 2\pi$$

$$\frac{a\pi}{15} = 2\pi - \frac{17\pi}{15}$$

$$\frac{a\pi}{15} = \frac{13\pi}{15}$$

Therefore, $a = 13$.

Radius

$$\theta = \frac{a}{r}$$

$$r = \frac{34\pi}{\frac{17\pi}{15}}$$

Therefore, $r = 30$.

Note: This item is SE since it involves multiple steps.

Use the following information to answer the next question.

If the point $P(0.2, k)$ lies on a circle with a centre at the origin and a radius of 1, then the exact value of k can be expressed as $\pm\sqrt{b}$.

6. The value of b, to the nearest hundredth, is __________.

Possible solution: $x^2 + y^2 = r^2$

$$(0.2)^2 + k^2 = (1)^2$$

$$k = \pm\sqrt{0.96}$$

$$b = 0.96$$

7. On a unit circle, Point $P\left(-\frac{5}{13}, \frac{12}{13}\right)$ lies on the terminal arm of angle θ in standard position. What are the exact values of the 6 trigonometric ratios for angle θ?

Possible solution: Since Point P is on the unit circle, the x-coordinate is the ratio for cosine, the y-coordinate is the ratio for sine, and $\frac{y}{x}$ is the ratio for tangent.

$$\therefore \sin\theta = \frac{12}{13}; \cos\theta = -\frac{5}{13}; \tan\theta = -\frac{12}{5}; \csc\theta = \frac{13}{12}; \sec\theta = -\frac{13}{5}; \cot\theta = -\frac{5}{12}$$

8. Given that $\csc\theta = \frac{8}{5}$, where $\frac{\pi}{2} < \theta < \pi$, determine the **exact** value of $\tan\theta$.

Possible solution: $x^2 + (5)^2 = 8^2$

$$x^2 = 39$$

$$x = \pm\sqrt{39}$$

Since θ is a second quadrant angle, $x = -\sqrt{39}$.

Therefore $\tan\theta = -\frac{5}{\sqrt{39}}$ or $\tan\theta = -\frac{5\sqrt{39}}{39}$.

9. Determine the **exact** value of $\sin\left(-\frac{\pi}{6}\right)+\cos\left(\frac{7\pi}{4}\right)$.

Possible solution: $\sin\left(-\frac{\pi}{6}\right)+\cos\left(\frac{7\pi}{4}\right)$

$$= -\frac{1}{2}+\frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{2}-1}{2}$$

10. If $\tan\theta=\frac{5}{2}$, where $0\le\theta<2\pi$, then the largest positive value of θ, to the nearest tenth, is __________ rad.

Possible solution: $\theta=\tan^{-1}\left(\frac{5}{2}\right)$

$\theta\approx 1.19$

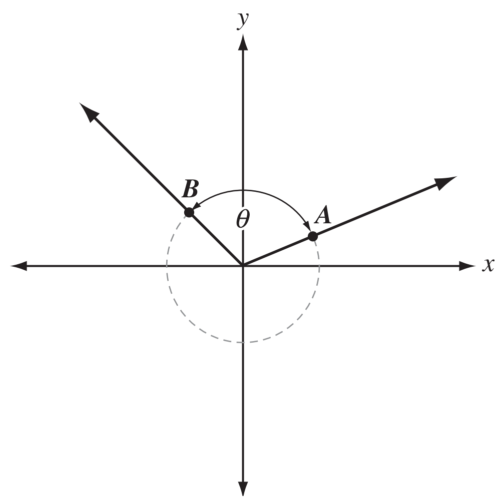
Tangent is positive in quadrants 1 and 3; therefore, the largest positive value for the angle in the given domain is:

$\theta=\pi+1.19$

$\theta\approx 4.3$

Use the following information to answer the next question.

Point $A\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ and Point $B\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ lie on the terminal arm of two different angles in standard position. The angle, θ, where $0 < \theta < \pi$, can be expressed in the form $\frac{a\pi}{b}$.



11. The values of a and b are, respectively, _____ and _____.

Possible solution: In standard position, $\angle A = \frac{\pi}{6}$ and $\angle B = \frac{3\pi}{4}$; therefore, $\theta = \angle B - \angle A$.

$$\theta = \frac{3\pi}{4} - \frac{\pi}{6}$$

$$\theta = \frac{7\pi}{12}$$

$a = 7$ and $b = 12$

Use the following information to answer the next question.

Each of the trigonometric ratios listed below results in a value of zero, or is undefined.

$$\tan\left(\frac{\pi}{2}\right)$$

$$\cot\left(\frac{3\pi}{2}\right)$$

$$\sin\pi$$

$$\csc(2\pi)$$

12. Use the following code to indicate that the value of the ratio is zero, or that the ratio is undefined.

1 = The value of the ratio is zero.
2 = The ratio is undefined.

	____	____	____	____
Ratio:	$\tan\left(\frac{\pi}{2}\right)$	$\cot\left(\frac{3\pi}{2}\right)$	$\sin\pi$	$\csc(2\pi)$

Solution: 2112

Use the following information to answer the next question.

For the angles $\frac{\pi}{6}$, $\frac{5\pi}{6}$, $\frac{7\pi}{6}$, and $\frac{11\pi}{6}$, the following statements are given.

Statement 1 They all have the same reference angle.

Statement 2 These angles in degrees are, respectively, 30°, 150°, 210°, and 300°.

Statement 3 They are all part of the solution set $\theta = \frac{\pi}{6} + 2n\pi, n \in I$.

Statement 4 The values of $\sin\left(\frac{7\pi}{6}\right)$ and $\cos\left(\frac{5\pi}{6}\right)$ are both negative.

13. The two statements that are true from the list above are numbered _____ and _____.

Possible solution: 14 or 41

Statement 1: *True*

They all have a reference angle of $\frac{\pi}{6}$.

Statement 2: *False*

$$\frac{11\pi}{6} \times \frac{180°}{\pi} = 330°$$

Statement 3: *False*

The solution set is

$$\theta = \frac{\pi}{6} + n\pi \text{ and } \theta = \frac{5\pi}{6} + n\pi, n \in I.$$

Statement 4: *True*

Sine is negative in Quadrant 3
and cosine is negative in Quadrant 2.

14. For the function $y = a\cos\theta + d$, the range is $[-4, 10]$. What are the values of a and d?

Possible solution: $a = 7$ and $d = 3$

Since the maximum value is 10 and the minimum value is -4,
the amplitude is $\frac{10 - (-4)}{2}$.
The vertical displacement is maximum – amplitude = 10 – 7.

SE **15.** For the function $y = \sin(3x + \pi) + 7$, what are the phase shift and period of the corresponding graph?

Possible solution: Since $y = \sin\left[3\left(x + \frac{\pi}{3}\right)\right] + 7$, there is a phase shift $\frac{\pi}{3}$ left.

The period is $\frac{2\pi}{b} = \frac{2\pi}{3}$.

Note: This item is SE since the b value must be factored out in order to identify the phase shift.

16. Given that $f(\theta) = \cos(n\theta)$ has the same period as the graph of $g(\theta) = \tan\theta$, the value of n is __________.

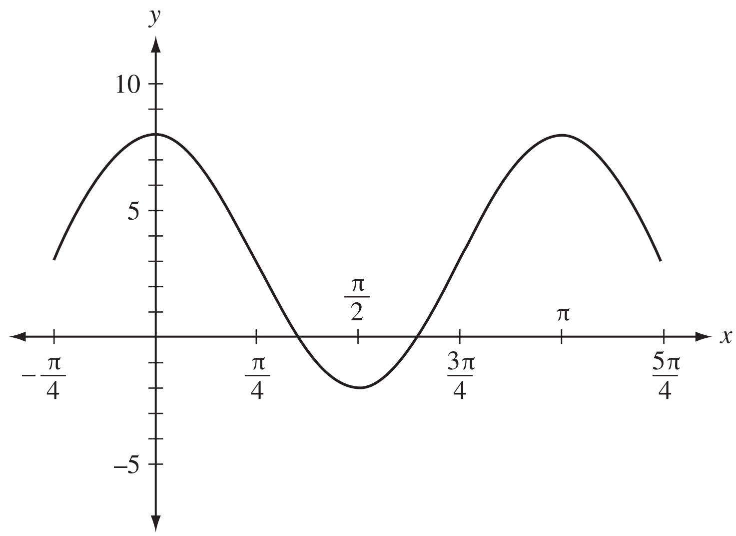
Possible solution: Period of $g(\theta) = \tan\theta$ is π.

Period of $f(\theta) = \cos(n\theta)$ is $\frac{2\pi}{n}$.

In order for the two functions to have the same period of π, $n = 2$.

Use the following information to answer the next question.

The partial graph of the cosine function below has a minimum point at $\left(\frac{\pi}{2}, -2\right)$ and a maximum point at $(\pi, 8)$. The equation of the function can be expressed in the form $y = a\cos\big(b(x - c)\big) + d,\ a, b, c, d \in W$.



SE **17.** The values of ***a***, ***b***, ***c***, and ***d*** are, respectively, ______, ______, ______, and ______.

Possible solution: $a = \frac{8 - (-2)}{2} = 5$

$$b = \frac{2\pi}{\text{period}} = \frac{2\pi}{\pi} = 2$$

$$c = 0$$

$$d = 8 - 5 = 3$$

$\therefore$ solution is 5203

Note: This item is SE since all 4 parameters must be determined.

Use the following information to answer the next question.

For the graph of the function $f(x) = -3\sin\left[2(x-5)\right] + d$ the following statements were made.

Statement 1	The amplitude is 3.
Statement 2	The maximum value is $(d - 3)$.
Statement 3	The period is 2π.
Statement 4	When compared to the graph of $g(x) = -3\sin(2x) + d$, the graph of $y = f(x)$ has been horizontally translated 5 units to the right.
Statement 5	If $d > 3$, then the graph of $y = f(x)$ will have no x-intercepts.

18. The true statements are

A. 1, 5
B. 2, 3
***C.** 1, 4, 5
D. 2, 4, 5

Possible solution:

Statement 1: *True*

The value of $|a|$ identifies the amplitude of the function.

Statement 2: *False*

The value of d identifies the vertical displacement. The maximum value is $(d + 3)$.

Statement 3: *False*

The period is $\frac{2\pi}{2} = \pi$.

Statement 4: *True*

The value of c identifies the phase shift.

Statement 5: *True*

If the vertical displacement is greater than the amplitude, then the graph will not touch or cross the x-axis.

Use the following information to answer the next question.

> The height of a point on a Ferris wheel, h, in metres above the ground, as a function of time, t, in seconds can be represented by a sinusoidal function. The maximum height of the Ferris wheel above the ground is 17 m and the minimum height is 1 m. It takes the Ferris wheel 60 seconds to complete two full rotations.

SE

19. Assuming that the particular point starts at the minimum height above the ground, write an equation for the height of this point on the Ferris wheel, h, as a function of time, t, in the form $h = a\cos\left[b(t-c)\right] + d$.

Possible solution: Amplitude is 8 m. Period is 30 s.

$$a = \frac{17-1}{2} = 8$$

$$b = \frac{2\pi}{30} = \frac{\pi}{15}$$

$$c = 15 \text{ s to the right or to the left} = \pm 15$$

$$d = \frac{17+1}{2} = 9$$

$$\therefore\ h = 8\cos\left[\frac{\pi}{15}(t-15)\right] + 9 \quad \text{or} \quad h = 8\cos\left[\frac{\pi}{15}(t+15)\right] + 9$$

Note: This item is SE since all 4 parameters of the function must be determined.

20. Solve for θ, where $180° \le \theta < 360°$, in the equation $2\cos^2\theta + \cos\theta = 0$.

Possible solution: $2\cos^2\theta + \cos\theta = 0$

$$\cos\theta(2\cos\theta + 1) = 0$$

$$\cos\theta = 0 \quad \text{or} \quad \cos\theta = -\frac{1}{2}$$

$$\theta = 90°, 270° \qquad \theta = 120°, 240°$$

$$\therefore\ \theta = 240° \text{ and } 270°$$

SE **21.** Determine a general solution of $\tan^2\theta - 1 = 0$, expressed in radians.

Possible solution: $\tan\theta = \pm 1$

$$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \text{etc.}$$

Therefore, the general solution is $\theta = \frac{\pi}{4} + \frac{n\pi}{2}, n \in I$.

Note: This item is SE since a general solution for a second-degree equation is required.

SE **22.** Determine the solution set for the equation $2\cos^2 x + \sin x - 1 = 0$, where $-\pi \le x \le \pi$.

Possible solution: $2\cos^2 x + \sin x - 1 = 0$

$$2(1 - \sin^2 x) + \sin x - 1 = 0$$

$$2 - 2\sin^2 x + \sin x - 1 = 0$$

$$2\sin^2 x - \sin x - 1 = 0$$

$$(2\sin x + 1)(\sin x - 1) = 0$$

$$\sin x = -\frac{1}{2} \text{ or } \sin x = 1$$

$$\left\{-\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{\pi}{2}\right\}$$

Note: This item is SE since it involves a Pythagorean trigonometric identity substitution and since the restricted domain is outside $[0, 2\pi]$.

23. Graphically solve for θ, where $-180° \le \theta \le 0°$, given $\left(2 - \sqrt{3}\sec\theta\right)\left(\sec\theta + 3\right) = 0$. State answers to the nearest degree.

Possible solution: $\theta = -109°$ and $-30°$

Enter the following function into the calculator.

$$y_1 = \left(2 - \frac{\sqrt{3}}{\cos x}\right)\left(\frac{1}{\cos x} + 3\right)$$

A window that could be used is x:[–180, 0, 30], y:[–5, 5, 1].

The x-intercepts are the solutions to the original equation.

Use the following information to answer the next question.

A Mathematics 30–1 class was asked to determine a general solution to the equation $\sin(2\theta) - \cos\theta = 0$, in degrees. The answers provided by four different students are shown below.

Student 1 $\theta = 60° + n(120°),\ n \in I$

Student 2 $\theta = 90° + n(360°),\ n \in I$, and $\theta = 30° + n(120°),\ n \in I$

Student 3 $\theta = n(180°)$, $\theta = 60° + n(360°)$, and $\theta = 300° + n(360°),\ n \in I$

Student 4 $\theta = 90° + n(180°)$, $\theta = 30° + n(360°)$, and $\theta = 150° + n(360°),\ n \in I$

SE **24.** The two students who provided a correct general solution are numbered

A. 1 and 3
B. 1 and 4
C. 2 and 3
***D.** 2 and 4

Note: This item is SE since it involves a double-angle trigonometric identity substitution and determining a general solution for a second-degree equation.

25. Three students were given the identity $\dfrac{\sin^2\theta - 1}{\cos\theta} = -\cos\theta$, where $\cos\theta \neq 0$.

a) Student A substituted $\theta = \dfrac{\pi}{3}$ into both sides of the equation and got LS = RS. Student B entered LS into y_1 and RS into y_2 and concluded that the graphs are exactly the same. Explain why these methods are not considered a proof of this identity.

Possible solution: Student A is verifying the identity using a single value of θ, whereas a proof is valid for all permissible values of θ. Student B is verifying that the graphs of each side of the identity are the same for all values of θ, but there are points of discontinuity on the graph of y_1, whereas there are no points of discontinuity on the graph of y_2.

b) Student C correctly completed an algebraic process to show LS = RS. Show a process Student C might have used.

Possible solution:

$$\text{LS} = \frac{\sin^2\theta - 1}{\cos\theta} \qquad\qquad \text{RS} = -\cos\theta$$

$$= \frac{-\cos^2\theta}{\cos\theta}$$

$$= -\cos\theta$$

$$\therefore \text{LS} = \text{RS}$$

SE

c) Which non-permissible values of θ should be stated for this identity?

Possible solution: $\cos\theta \neq 0$

$$\theta \neq \frac{\pi}{2}, \frac{3\pi}{2}, \text{etc.}$$

$$\theta \neq \frac{\pi}{2} + n\pi,\ n \in I$$

Note: This item is SE since since non-permissible values are required.

SE **26.** The expression $\dfrac{\cot x + \csc x}{\sec x + 1}$, where $\sec x \neq -1$, is equivalent to

A. $\sin x$

B. $\tan x$

C. $\csc x$

***D.** $\cot x$

Possible solution:
$$\frac{\dfrac{\cos x}{\sin x} + \dfrac{1}{\sin x}}{\dfrac{1}{\cos x} + 1} = \frac{\dfrac{\cos x + 1}{\sin x}}{\dfrac{1 + \cos x}{\cos x}}$$
$$= \frac{\cos x + 1}{\sin x} \times \frac{\cos x}{1 + \cos x}$$
$$= \frac{\cos x}{\sin x}$$
$$= \cot x$$

Note: This item is SE since it entails simplifying an identity involving rational operations.

Use the following information to answer the next question.

Each trigonometric expression below can be simplified to a single numerical value.

1 $\cot^2 x - \csc^2 x$

2 $\sec^2 x - \tan^2 x$

3 $\sin x - \dfrac{\tan x}{\sec x}$

4 $\dfrac{1}{7}\cos^2 x + \dfrac{1}{7}\sin^2 x$

27. When the numerical values of the simplified expressions are arranged in ascending order, the expression numbers are _____, _____, _____, and _____.

Possible solution: 1342

1 $\cot^2 x - \csc^2 x = -1$

2 $\sec^2 x - \tan^2 x = 1$

3 $\sin x - \dfrac{\tan x}{\sec x} = \sin x - \dfrac{\sin x}{\cos x} \times \dfrac{\cos x}{1} = \sin x - \sin x = 0$

4 $\dfrac{1}{7}\cos^2 x + \dfrac{1}{7}\sin^2 x = \dfrac{1}{7}\left(\cos^2 x + \sin^2 x\right) = \dfrac{1}{7}$

SE **28.** What is the **exact** value of $\tan 75°$?

Possible solution: $\sqrt{3} + 2$ or $\dfrac{\sqrt{3}+1}{\sqrt{3}-1}$

$$\begin{aligned}
\tan 75° &= \tan(45° + 30°) \\
&= \frac{\tan 45° + \tan 30°}{1 - \tan 45° \tan 30°} \\
&= \frac{1 + \frac{1}{\sqrt{3}}}{1 - 1\left(\frac{1}{\sqrt{3}}\right)} \\
&= \frac{\frac{\sqrt{3}+1}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}} \\
&= \frac{\sqrt{3}+1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}-1} \\
&= \frac{\sqrt{3}+1}{\sqrt{3}-1} \text{ or } \sqrt{3} + 2 \text{ (rationalize the denominator)}
\end{aligned}$$

Note: This item is SE since it involves the sum identity of a tangent.

SE **29.** Prove algebraically that $\dfrac{2\tan x}{1-\tan^2 x} = \dfrac{\sin(2x)}{\cos^2 x - \sin^2 x}$, where $x \neq \dfrac{\pi}{4} + \dfrac{n\pi}{2}$, $n \in I$.

Possible solution:

Left Side	**Right Side**
$\dfrac{2\tan x}{1-\tan^2 x}$	$\dfrac{\sin(2x)}{\cos^2 x - \sin^2 x}$
$\tan(2x)$	$\dfrac{\sin(2x)}{\cos(2x)}$
	$\tan(2x)$

LS = RS

Note: This item is SE since it involves a double-angle tangent identity.

30. Given that $\sin\theta = -\frac{2}{7}$ and $\cot\theta < 0$, determine the **exact** value of $\cos\left(\theta - \frac{2\pi}{3}\right)$.

Possible solution: $x^2 + (-2)^2 = 7^2$

$$x^2 = \sqrt{45}$$

$$x = \pm 3\sqrt{5}$$

Since θ is in Quadrant IV, $x = 3\sqrt{5}$ and $\cos\theta = \frac{3\sqrt{5}}{7}$.

$$\cos\left(\theta - \frac{2\pi}{3}\right) = \cos\theta\cos\frac{2\pi}{3} + \sin\theta\sin\frac{2\pi}{3}$$

$$= \left(\frac{3\sqrt{5}}{7}\right)\left(-\frac{1}{2}\right) + \left(-\frac{2}{7}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{-3\sqrt{5}}{14} - \frac{2\sqrt{3}}{14}$$

$$= \frac{-3\sqrt{5} - 2\sqrt{3}}{14}$$

Permutations, Combinations, and Binomial Theorem

General Outcome

Develop algebraic and numeric reasoning that involves combinatorics.

General Notes:

- Students are expected to simplify expressions involving ${}_nP_r$, ${}_nC_r$, and $n!$ algebraically.
- Probability is beyond the scope of this course.

Specific Outcomes

Specific Outcome 1

Apply the fundamental counting principle to solve problems. **[C, PS, R, V] [ICT: C6–2.3]**

Notes:

- Alternative methods of problem solving involving pictorial or visual explanations (tree diagrams, lists) are appropriate.

(See examples 1–3)

Specific Outcome 2

Determine the number of permutations of *n* elements taken *r* at a time to solve problems.
[C, PS, R, V]

Notes:

- Permutation problems could involve repetition of like elements and constraints.
- Circular and ring permutations are beyond the scope of this outcome.
- Single 2-dimensional pathways may be used as an application of repetition of like elements.

(See examples 4–6)

Specific Outcome 3

Determine the number of combinations of n different elements taken r at a time to solve problems. **[C, PS, R, V]**

Notes:

- Students are expected to know both ${}_{n}C_{r}$ and $\binom{n}{r}$ notation.

(See examples 7–9)

Specific Outcome 4

Expand powers of a binomial in a variety of ways, including using the binomial theorem (restricted to exponents that are natural numbers). **[CN, R, V]**

Notes:

- Teachers may choose to show the relationship between the rows of Pascal's triangle and the numerical coefficients of the terms in the expansion of a binomial $(x + y)^n$, $n \in N$.
- Students are expected to recognize various patterns in the binomial expansion.

(See examples 10–14)

Acceptable Standard	**Standard of Excellence**
The student can:	The student can also:
• apply the fundamental counting principle to various problems involving at most two cases or constraints	• apply the fundamental counting principle to various problems involving three or more cases or constraints
• understand and use factorial notation	
• recognize and address problems involving the terms *and* or *or*	• recognize and address problems involving the terms *at least* or *at most*
	• recognize and address scenarios involving cases where items *cannot be together*
• solve problems involving permutations or combinations	• solve problems involving both permutations and combinations
• solve problems involving permutations when two or more elements are identical (repetitions), with at most one constraint	• solve problems involving permutations when two or more elements are identical (repetitions), with more than one constraint
• solve for n in equations involving one occurrence of ${}_nP_r$ or ${}_nC_r$ given r, where $r \le 3$, and identify extraneous solutions	
• obtain solutions to problems involving at most two cases or constraints	• obtain solutions to problems involving three or more cases or constraints
• demonstrate an understanding of patterns that exist in the binomial expansion	
• expand $(x + y)^n$ or determine a specified term in the expansion of a binomial with linear terms	• expand $(x + y)^n$ or determine a specified term in the expansion of a binomial with non-linear terms
	• determine an unknown value in $(x + y)^n$ given a specified term in its expansion
• provide a partial solution and explain simple mathematical strategies in a problem-solving context involving combinatorics studied in Mathematics 30-1	• provide a complete solution and explain complex mathematical strategies in a problem-solving context involving combinatorics studied in Mathematics 30-1

Examples

Students who achieve the acceptable standard should be able to answer all of the following questions, except for any part of a question labelled ***SE***. Parts labelled ***SE*** are appropriate examples for students who achieve the standard of excellence.

Note: *Please be aware that the worked solutions show possible strategies; there may be other strategies that could be used.*

1. How many arrangements of all of the letters of the word REASON are there if the arrangement must start with an S?

Possible solution: $1 \times 5! = 120$

2. If all of the letters in the word DIPLOMA are used, then how many different arrangements are possible that begin and end with an I, O, or A?

Possible solution: $3 \times 5 \times 4 \times 3 \times 2 \times 1 \times 2 = 720$

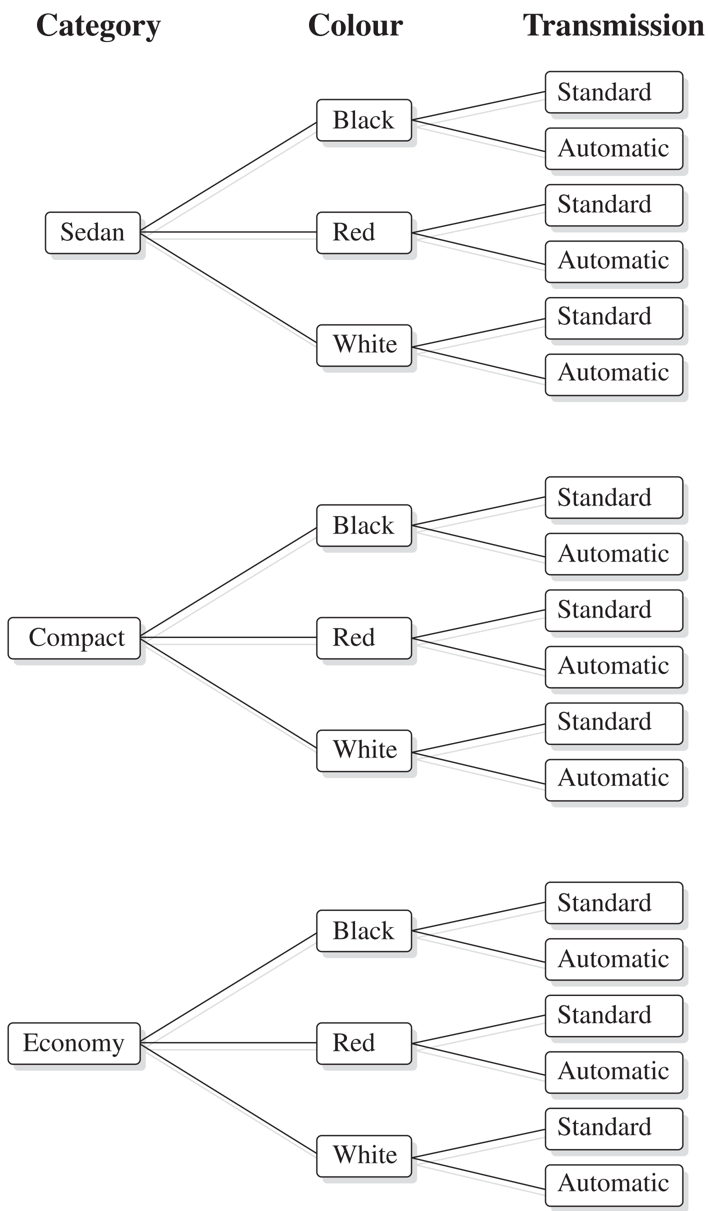
Possible solution: ${}_3P_2 \times {}_5P_5 = 720$

Use the following information to answer the next question.

Josh wants to rent a car. He has narrowed his choices to a sedan, a compact, or an economy car. The colours available are black, red, or white. He may also choose between a standard and an automatic transmission.

3. Determine the total number of options Josh has.

Possible solution: 18



Use the following information to answer the next question.

A volleyball team made up of 6 players stands in a line facing the camera for a picture.

4. If Joan and Emily must be together, then how many different arrangements are possible for the picture?

Possible solution: $2! \times 5! = 240$

SE

5. Determine the number of different arrangements using all the letters of the word ACCESSES that

a) begin with at least two S's.
b) begin with exactly two S's.
c) Explain why the answers in questions a) and b) are different.

Possible solution: a) $\dfrac{3 \times 2 \times 5 \times 5!}{3!2!2!} + \dfrac{3 \times 2 \times 1 \times 5!}{3!2!2!} = 150 + 30 = 180$

b) $\dfrac{3 \times 2 \times 5 \times 5!}{3!2!2!} = 150$

c) The answers are different because the 3rd letter cannot be an S in part b).

Possible solution: a) Must begin with two S's
Therefore we only need to arrange the remaining letters: AEECCS

$\dfrac{6!}{2!2!} = 180$

b) Arrange the letters AEECCS as in part a) above
Then subtract the arrangements that begin with a third S

$\dfrac{6!}{2!2!} - \dfrac{5!}{2!2!} = 150$

c) The answers are different because the 3rd letter cannot be an S in part b).

Note: This item is SE since it involves problems using the term "at least".

Use the following information to answer the next question.

At a car dealership, the manager wants to line up 10 cars of the same model in the parking lot. There are 3 red cars, 2 blue cars, and 5 green cars.

SE

6. If all 10 cars are lined up in a row facing forward, determine the number of possible car arrangements if the blue cars **cannot** be together.

Possible solution: $\underbrace{\dfrac{10!}{3!2!5!}}_{\text{Total possible arrangements}} - \underbrace{\dfrac{9!}{3!5!}}_{\text{Arrangements with blue together}} = 2\ 016$

Possible solution: $\dfrac{8! \times {}_9P_2}{3!2!5!} = 2\ 016$

Note: This item is SE since it involves a scenario in which items cannot be together.

Use the following information to answer the next question.

If 14 different types of fruit are available, how many different fruit salads could be made using exactly 5 types of fruit?

Student 1 Kevin used $\dfrac{14!}{5!}$ to solve the problem.

Student 2 Ron suggested using ${}_{14}P_5$.

Student 3 Michelle solved the problem using ${}_{14}C_9$.

Student 4 Jackie thought that 5! would give the correct answer.

Student 5 Stan decided to use $\binom{14}{5}$.

7. The correct solution would be obtained by student number _____ and student number _____.

Possible solution: 35 or 53

The order of fruits being added to the salad does not make a difference; therefore, this must be a combination. Both Michelle and Stan are correct since ${}_{14}C_5 = {}_{14}C_9$. Choosing 5 fruits from the 14 available is the same as not choosing 9 fruits from the 14 available.

Use the following information to answer the next question.

At a meeting, every person shakes hands with every other person exactly once.

8. If there are 36 handshakes in total, how many people were at the meeting?

Possible solution:

$$_{n}C_{2} = 36$$

$$\frac{n!}{(n-2)!2!} = 36$$

$$n(n-1) = 72$$

$$n^2 - n - 72 = 0$$

$$(n-9)(n+8) = 0$$

$$n = 9, \cancel{-8}$$

There were 9 people at the meeting.

SE

9. How many different 4-letter arrangements are possible using any 2 letters from the word SMILE and any 2 letters from the word FROG?

Possible solution: $\left({}_{5}C_{2} \times {}_{4}C_{2}\right) \times 4! = 1\,440$

Choose 2 letters from each word first, and then arrange the 4 letters.

Note: This item is SE since it involves both a combination and a permutation.

10. Find the value of *a* if the expansion of $(2x + 3)^{(2a-5)}$ has 18 terms.

Possible solution:

$$18 = (2a - 5) + 1$$

$$17 = 2a - 5$$

$$11 = a$$

Use the following information to answer the next question.

A student made the following statements regarding the expansion of $(a + b)^4$, written in descending powers of a.

Statement 1 The total number of terms is 5.

Statement 2 The middle term is $6a^2b^2$.

Statement 3 The sum of the leading coefficients of all the terms is 14.

Statement 4 For the term $4a^3b^m$, the value of m is 1.

Statement 5 The leading coefficient of the first term is ${}_4C_1$.

11. The three statements above that are true are numbered _____, _____, and _____.

Possible solution: 124 (list in any order)

According to the Binomial Theorem, the terms of the binomial expansion will be:

$$(a + b)^4 = {}_4C_0(a^4) + {}_4C_1(a^3b^1) + {}_4C_2(a^2b^2) + {}_4C_3(a^1b^3) + {}_4C_4(b^4)$$

$$= a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

Statement 1: *True*

The number of terms in the expansion of $(a + b)^n$ is $n + 1$; therefore, there are 5 terms.

Statement 2: *True*

The middle term is $6a^2b^2$.

Statement 3: *False*

The sum of all the leading coefficients in the expansion of $(a + b)^n$ is 2^n; therefore, the sum is $2^4 = 16$.

Statement 4: *True*

The sum of the exponents in each term must equal n; therefore, $m = 1$.

Statement 5: *False*

The leading coefficient of the first term is ${}_4C_0$.

SE **12.** In the expansion of $\left(3a-b^2\right)^{10}$, what is the coefficient of the term containing a^4b^{12}?

Possible solution: Since the term must include a^4b^{12}, using the general term, $k = 6$,

$$t_{6+1} = {}_{10}C_6(3a)^{10-6}(-b^2)^6$$
$$t_7 = 210(81a^4)(b^{12})$$
$$t_7 = 17\ 010a^4b^{12}$$

Therefore, the coefficient of the term is 17 010.

Note: This item is SE since the binomial contains non-linear terms.

SE **13.** In the expansion of the binomial $\left(2a+\frac{1}{a}\right)^8$, the constant term is ______.

Possible solution: The exponent of the variable for a constant term must be zero, i.e. a^0.

$${}_8C_4(2a)^4\left(\frac{1}{a}\right)^4$$

$$70\left(16a^4\right)\left(\frac{1}{a^4}\right)$$

1 120

Therefore the constant term is 1 120.

Note: This item is SE since the binomial contains non-linear terms.

SE **14.** Given that a term in the expansion of $(ax-y)^6$ is $-252xy^5$, determine the numerical value of a.

Possible solution: Using the general term, $k = 5$,

$$t_{5+1} = {}_6C_5(ax)^{6-5}(-y)^5$$
$$-252xy^5 = 6(ax)\left(-y^5\right)$$
$$-252 = -6a$$
$$42 = a$$

Note: This item is SE since it requires a parameter in the binomial to be found, given a specified term.

Website Links

Publication/Resource Mathematics 30–1 Program of Studies	Website
Mathematics Grades 10–12 Program of Studies	*education.alberta.ca*, via this pathway: *Teachers > (Programs of Study) > Mathematics > Educators > Programs of Study*
Mathematics 30–1 Bulletin	*education.alberta.ca*, via this pathway: *Teachers > (Additional Programs and Services) Diploma Exams > Information Bulletins*
Using Calculators and Computers	*education.alberta.ca*, via this pathway: *Teachers > (Additional Programs and Services) Diploma Exams > Diploma General Information Bulletin > Using Calculators and Computers*
General Information Bulletin	*education.alberta.ca*, via this pathway: *Teachers > (Additional Programs and Services) Diploma Exams > Diploma General Information Bulletin*
Quest A+	*https://questaplus.alberta.ca*
Mathematics and Science Directing Words	*education.alberta.ca*, via this pathway: *Teachers > (Additional Programs and Services) Diploma Exams > Information Bulletins > Mathematics and Science Directing Words*
FAQs for Educators	*education.alberta.ca*, via this pathway: *Teachers > (Programs of Study) > Mathematics > Educators > FAQs for Educators*
The High School Information Package	*education.alberta.ca*, via this pathway: *Teachers > (Programs of Study) > Mathematics > Educators > Fact Sheets and Useful Links*

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