Outcomes with Assessment Standards for Mathematics 20-1

2013

This resource is intended to assist teachers with the provincial implementation of Mathematics 20-1.

Alberta Government

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The primary audience for this resource is:

Teachers	✓
Administrators	
Students	
Parents	

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INTRODUCTION

Mathematics 20-1 was provincially implemented in September 2011. Teachers participating in focus groups during the development of the program of studies expressed a need for a common understanding of the curriculum and assessment standards. In response to this need, and in keeping with Alberta Education's goal of establishing and effectively communicating clear outcomes and high standards, this standards resource was developed.

This resource is designed to support the implementation of the Alberta Mathematics Grades 10–12 Program of Studies, which can be found at http://education.alberta.ca/media/655889/math10to12.pdf. Teachers are strongly encouraged to consult the program of studies for details about the philosophy of the program.

PURPOSE

Outcomes with Assessment Standards for Mathematics 20-1 links the achievement indicators for the specific outcomes from the program of studies with information and commentaries about standards. Its purpose is to provide teachers of Mathematics 20-1 with clearly stated standards to use as guidelines in their classroom instruction and assessment practices.

DEFINITIONS AND TERMINOLOGY

Standards

A *standard* is a reference point used in planning and evaluation. In evaluating educational performance, the following standards apply:

- *Curriculum* and *assessment standards* apply to the assessment of individual students.
- *Achievement standards* apply to the assessment of student populations.

In this resource, only curriculum and assessment standards are discussed.

Curriculum Standards

Curriculum standards are outcomes for a course within a program. The curriculum standards for Mathematics 20-1 are defined by the general and specific outcomes outlined in the program of studies. They are further clarified by the achievement indicators, which reflect the scope of each specific outcome.

Outcomes

General outcomes are concise statements identifying what it is that students are expected to know and be able to do upon completion of a course within a program.

Specific outcomes are statements identifying the component knowledge, skills and attitudes of a general outcome. Specific outcomes identify a range of contexts in which the general outcomes apply.

In the specific outcomes, the word *including* indicates that any ensuing items must be addressed to fully meet the learning outcome. The phrase *such as* indicates that the ensuing items are provided for clarification and are not requirements that must be addressed to fully meet the learning outcome.

The word *and* used in an outcome indicates that both ideas must be addressed to fully meet the learning outcome, although not necessarily at the same time or in the same question.

Achievement Indicators

Achievement indicators are samples of how students may demonstrate their achievement of the goals of a specific outcome. The range of samples provided is meant to reflect the scope of the specific outcome.

The word *and* used in an achievement indicator implies that both ideas should be addressed at the same time or in the same question.

Assessment Standards

Assessment standards are the criteria used for judging individual student achievement relative to the curriculum standards.

STANDARDS FOR MATHEMATICS 20-1

Mathematics 20-1 is designed to follow directly from Mathematics 10C, so students taking Mathematics 20-1 are presumed to have reached the acceptable standard or better in the outcomes of Mathematics 10C.

The assessment standards for Mathematics 20-1 include an acceptable and an excellent level of performance. Student performance should be measured on a range of tasks, some of which are routine and obvious tasks in familiar contexts, and others which are nonroutine tasks in unfamiliar contexts. In many cases, a correlated example from the authorized resources is referenced to assist in assessing student performance. The authorized resources for Mathematics 20-1, published by McGraw-Hill Ryerson, are:

- Pre-Calculus 11: Student Resource
- Pre-Calculus 11: Teacher's Resource.

Acceptable Standard

The *acceptable standard* of achievement in Mathematics 20-1 is met by students who receive a course mark between and including 50 percent and 79 percent. Typically, these students have gained new skills and a basic knowledge of the concepts and procedures relative to the general and specific outcomes defined for Mathematics 20-1 in the program of studies. These students can apply this knowledge to a limited range of familiar problem contexts.

Standard of Excellence

The *standard of excellence* for achievement in Mathematics 20-1 is met by students who receive a course mark at or above 80 percent. Typically, these students have gained a breadth and depth of understanding regarding the concepts and procedures, as well as the ability to apply this knowledge to a broad range of familiar and unfamiliar problem contexts.

Description of Standards

The following statements describe what is expected of Mathematics 20-1 students who meet the *acceptable standard* or the *standard of excellence* on independent work. The statements represent the standards against which student achievement is measured.

Acceptable Standard	***Standard of Excellence***
Students who meet the *acceptable standard* in Mathematics 20-1 consistently perform acceptable work on routine and obvious tasks in familiar contexts.	Students who meet the *standard of excellence* in Mathematics 20-1 consistently perform excellent work on routine and obvious tasks in familiar contexts, and acceptable work on nonroutine tasks in unfamiliar contexts.
These students have a basic understanding of the concepts and procedures outlined in the program of studies. They demonstrate their understanding in concrete, pictorial and symbolic modes, and can translate from one mode to another. They perform the mathematical operations and procedures that are fundamental to Mathematics 20-1 and apply what they know in daily living contexts.	These students have a comprehensive understanding of the concepts and procedures outlined in the program of studies. They demonstrate their understanding in concrete, pictorial and symbolic modes, and can translate from one mode to another. They perform the mathematical operations and procedures that are fundamental to Mathematics 20-1, apply what they know in daily living contexts and provide alternative solution procedures to verify results.
To meet the *acceptable standard*, students communicate about mathematical situations in an understandable way, using appropriate everyday and mathematical terms. They understand mathematical questions containing objects, diagrams or numbers in familiar contexts, and they construct mathematical models.	To meet the *standard of excellence*, students communicate about mathematical situations in a clear way, using numbers, diagrams and appropriate mathematical terms. They understand mathematical questions containing objects, diagrams or numbers in familiar and unfamiliar contexts, and they construct mathematical models using multiple representations.
Students meeting the *acceptable standard* apply what they know in solving straightforward problems in familiar settings and in analyzing simple mathematical models. They describe the steps they used to solve a particular problem, and verify and defend their solution to the problem.	Students meeting the *standard of excellence* apply what they know in solving routine and nonroutine problems in a broad range of settings. They describe the steps they used to solve a particular problem, defend their solution to the problem, and, where appropriate, provide alternative solution procedures to verify results.
Students meeting the *acceptable standard* have a positive attitude toward mathematics and a sense of personal competence in using mathematics. They demonstrate confidence when using common mathematical procedures and when applying problem-solving strategies in familiar settings.	Students meeting the *standard of excellence* have a positive attitude toward mathematics and show confidence in using mathematics meaningfully. They are self-motivated risk takers who persevere when solving novel problems. They take initiative in trying new methods and are creative in their approach to problem solving.

GENERAL NOTES

- All mathematical processes should be used and integrated throughout the outcomes.
- Technology [T], including calculators and computers, has been listed as one of the mathematical processes to be emphasized for some outcomes, with the expectation that students will have access to technology when completing the outcomes. If technology has not been specifically listed for a particular outcome, teachers may, at their discretion, use it to assist students in exploring patterns and relationships when learning a concept. It is expected, however, that technology will not be considered when assessing students' understandings of such outcomes.
- Each specific outcome is accompanied by notes that address some of the questions that may arise when teaching the concept. The assessment standards for each outcome are described in a chart that indicates, for each achievement indicator, whether the acceptable standard, the standard of excellence or, in some cases, both standards may be applicable (✓). Some check marks are accompanied by qualifying statements. Shaded regions indicate that the standard does not apply for the given achievement indicator. In many cases, a correlated example from the authorized resources is referenced in the chart to illustrate the standards.
- A *partial solution* to a problem is a solution in which a student demonstrates a basic understanding of the problem and the mathematical concepts required in solving the problem. However, the student is unable to complete the solution correctly for a variety of reasons, such as not being able to correctly connect the concepts involved or not being able to avoid procedural errors. For example, in solving a problem using the cosine law, given the measure of the three sides of a triangle, a student may be able to draw a diagram to correctly represent the situation and identify the appropriate equation needed to solve the problem but then makes procedural errors in solving for the measure of an angle. Note that assessment of student learning is the responsibility of the teacher, and what is considered a partial solution may vary according to the question or task presented.

Topic: Algebra and Number

General Outcome: Develop algebraic reasoning and number sense.

Specific Outcome

It is expected that students will:

1. Demonstrate an understanding of the absolute value of real numbers.
   [R, V]

Notes

- This is the first time that the concept of the absolute value of a number is formally addressed. Students will benefit from having a thorough understanding of this outcome before addressing absolute value functions that are covered in Relations and Functions, Mathematics 20-1, Specific Outcome (SO) 2.

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

Achievement Indicators	Acceptable Standard	Standard of Excellence
1.1 Determine the distance of two real numbers of the form $\pm a$, $a \in R$, from 0 on a number line, and relate this to the absolute value of a ($\lvert a \rvert$).	✓ STUDENT RESOURCE p. 359, Investigate Absolute Value	
1.2 Determine the absolute value of a positive or negative real number.	✓ STUDENT RESOURCE p. 360, Example 1	
1.3 Explain, using examples, how distance between two points on a number line can be expressed in terms of absolute value.	✓ STUDENT RESOURCE p. 363, #5	
1.4 Determine the absolute value of a numerical expression.	✓ STUDENT RESOURCE p. 363, #6	
1.5 Compare and order the absolute values of real numbers in a given set.	✓ STUDENT RESOURCE p. 361, Example 2	

Algebra and Number (continued)

Specific Outcome

It is expected that students will:

2. Solve problems that involve operations on radicals and radical expressions with numerical and variable radicands.
   [CN, ME, PS, R]

Notes

- Prior knowledge from previous grade levels/courses includes:
  - perfect squares and square roots (Grade 8)
  - introduction to radicals (Mathematics 10C, Algebra and Number, SO2)
  - factoring a difference of squares (Mathematics 10C, Algebra and Number, SO5)
  - solving linear inequalities in one variable (Grade 9).
- The focus of this specific outcome is square and cube roots; however, indices greater than 3 can be explored in order to provide students with an opportunity to generalize patterns.
- Technology [T] has not been identified as one of the mathematical processes to be emphasized in completing this outcome. Technology may be used, where appropriate, in calculating decimal approximations of irrational numbers.
- Rationalizing denominators should be limited to radicals with an index of 2. Monomial cube roots can be explored to develop a conceptual understanding.
- The topic of radical equations is introduced in SO3; radical functions are introduced in Mathematics 30-1, Relations and Functions, SO13.
- In Achievement Indicator 2.8, radicands should be limited to single variable monomial or binomial expressions.

Achievement Indicators

The following set of indicators ***may*** *be used to determine whether students have met the corresponding specific outcome.*

Achievement Indicators	Acceptable Standard	Standard of Excellence
2.1 Compare and order radical expressions with numerical radicands in a given set.	✓ STUDENT RESOURCE p. 276, Example 3	
2.2 Express an entire radical with a numerical radicand as a mixed radical.	✓ Write mixed radicals up to an index of 3. STUDENT RESOURCE p. 278, #2a, c	✓ Write mixed radicals with an index greater than 3.
2.3 Express a mixed radical with a numerical radicand as an entire radical.	✓ Write entire radicals up to an index of 3. STUDENT RESOURCE p. 274, Example 1a	✓ Write entire radicals with an index greater than 3. STUDENT RESOURCE p. 304, #2b
2.4 Perform one or more operations to simplify radical expressions with numerical or variable radicands.	✓ Simplify radical expressions that involve only square roots. STUDENT RESOURCE p. 279, #9a, b, c p. 284, Example 1a, b, c	✓ Simplify radical expressions that involve radicals with indices greater than two. STUDENT RESOURCE p. 279, #9d p. 284, Example 1d

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Achievement Indicators	Acceptable Standard	Standard of Excellence
2.5 Rationalize the denominator of a rational expression with monomial or binomial denominators.	✓ Rationalize expressions with a square root monomial numerator and a monomial denominator. STUDENT RESOURCE p. 290, #8a, b, c ✓ Rationalize expressions with a square root monomial numerator and binomial denominator. STUDENT RESOURCE p. 290, #10a, b, c	✓ Rationalize expressions with a square root binomial numerator and denominator. STUDENT RESOURCE p. 290, #10d ✓ Rationalize expressions with a denominator involving a cube root monomial. STUDENT RESOURCE p. 290, #8d
2.6 Describe the relationship between rationalizing a binomial denominator of a rational expression and the product of the factors of a difference of squares expression.	✓ Provide a partial description of the relationship. STUDENT RESOURCE p. 292, #29	✓ Provide a complete description of the relationship. STUDENT RESOURCE p. 292, #29
2.7 Explain, using examples, that $(-x)^2 = x^2$, $\sqrt{x^2} = \lvert x \rvert$ and $\sqrt{x^2} \neq \pm x$; e.g., $\sqrt{9} \neq \pm 3$.	✓ STUDENT RESOURCE p. 281, #25	
2.8 Identify the values of the variable for which a given radical expression is defined.	✓ Determine the domain for radicals that involve monomial or binomial linear radicands. STUDENT RESOURCE p. 291, #19a	✓ Determine the domain for radicals that involve binomial quadratic radicands.
2.9 Solve a problem that involves radical expressions.	✓ Provide a partial solution to the problem. STUDENT RESOURCE p. 285, Example 2	✓ Provide a complete solution to the problem. STUDENT RESOURCE p. 285, Example 2

Algebra and Number (continued)

Specific Outcome

It is expected that students will:

3. Solve problems that involve radical equations (limited to square roots).
   [C, PS, R]

Notes

- Prior knowledge from previous grade levels/courses includes:
  - perfect squares and square roots (Grade 8)
  - introduction to radicals (Mathematics 10C, Algebra and Number, SO2).
- Students should have an understanding of solving quadratic equations (Relations and Functions, SO5) prior to studying this topic.
- Graphing and analyzing radical functions is introduced in Mathematics 30-1, Relations and Functions, SO13.
- It is intended that the equations will have no more than two radicals.

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome. (It is intended that the equations will have no more than two radicals.)*

Achievement Indicators	Acceptable Standard	Standard of Excellence
3.1 Determine any restrictions on values for the variable in a radical equation.	✓ Determine restrictions for radicals that involve monomial or binomial linear radicands. STUDENT RESOURCE p. 296, Example 1a	✓ Determine restrictions for radicals that involve binomial quadratic radicands. STUDENT RESOURCE p. 306, #9
3.2 Determine the roots of a radical equation algebraically, and explain the process used to solve the equation.	✓ Determine the roots of a radical equation algebraically that involves squaring the equation once, and explain the process. STUDENT RESOURCE p. 296, Example 1b	✓ Determine, algebraically, the roots of a radical equation that involves squaring the equation twice, and explain the process. STUDENT RESOURCE p. 298, Example 3
3.3 Verify, by substitution, that the values determined in solving a radical equation algebraically are roots of the equation.	✓ STUDENT RESOURCE p. 300, #4	
3.4 Explain why some roots determined in solving a radical equation algebraically are extraneous.	✓ Provide an explanation that is limited to verification of extraneous roots through substitution. STUDENT RESOURCE p. 303, #25	✓ Provide an explanation that includes restrictions on the variable in the radicand. STUDENT RESOURCE p. 303, #25
3.5 Solve problems by modelling a situation using a radical equation.	✓ Solve a problem that involves squaring the equation once. STUDENT RESOURCE p. 302, #16	✓ Solve a problem that involves squaring the equation twice.

Algebra and Number (continued)

Specific Outcome

It is expected that students will:

4. Determine equivalent forms of rational expressions (limited to numerators and denominators that are monomials, binomials or trinomials).
   [C, ME, R]

Notes

- Prior knowledge from previous grade levels/courses includes:
  - operations on rational numbers (Grades 7, 8, 9)
  - operations on polynomials (Grade 9; Mathematics 10C, Algebra and Number, SO4)
  - factoring polynomials (Mathematics 10C, Algebra and Number, SO5).

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

Achievement Indicators	Acceptable Standard	Standard of Excellence
4.1 Compare the strategies for writing equivalent forms of rational expressions to the strategies for writing equivalent forms of rational numbers.	✓ STUDENT RESOURCE p. 317, #1	
4.2 Explain why a given value is non-permissible for a given rational expression.	✓ STUDENT RESOURCE p. 317, #4	
4.3 Determine the non-permissible values for a rational expression.	✓ Determine non-permissible values for monomial denominators. ✓ Determine non-permissible values for binomial and trinomial denominators containing one variable. STUDENT RESOURCE p. 312, Example 1	✓ Determine non-permissible values for binomial and trinomial denominators containing two variables. STUDENT RESOURCE p. 315, Example 3a
4.4 Determine a rational expression that is equivalent to a given rational expression by multiplying the numerator and denominator by the same factor (limited to a monomial or a binomial), and state the non-permissible values of the equivalent rational expression.	✓ STUDENT RESOURCE p. 317, #2b, d	
4.5 Simplify a rational expression.	✓ STUDENT RESOURCE p. 314, Example 2	
4.6 Explain why the non-permissible values of a given rational expression and its simplified form are the same.	✓ STUDENT RESOURCE p. 318, #9	
4.7 Identify and correct errors in a simplification of a rational expression, and explain the reasoning.	✓ Identify and correct errors and provide a partial explanation. STUDENT RESOURCE p. 318, #13	✓ Identify and correct errors and provide a complete explanation. STUDENT RESOURCE p. 318, #13

Algebra and Number (continued)

Specific Outcome

It is expected that students will:

5. Perform operations on rational expressions (limited to numerators and denominators that are monomials, binomials or trinomials).
   [CN, ME, R]

Notes

- Prior knowledge from previous grade levels/courses includes:
  - operations on rational numbers (Grades 7, 8, 9)
  - operations on polynomials (Grade 9; Mathematics 10C, Algebra and Number, SO4)
  - factoring polynomials (Mathematics 10C, Algebra and Number, SO5).
- Students may have several different strategies for operations with rational numbers. It would be beneficial to determine students' personal strategies prior to covering this outcome and to discuss with students the merits of each strategy in the context of rational expressions.
- Complex rational expressions $\left(\text{in the form } \frac{\frac{a}{b}}{\frac{c}{d}}\right)$ are beyond the scope of this outcome. These expressions should be shown as a division in the form $\frac{a}{b} \div \frac{c}{d}$.

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

Achievement Indicators	Acceptable Standard	Standard of Excellence
5.1 Compare the strategies for performing a given operation on rational expressions to the strategies for performing the same operation on rational numbers.	✓ STUDENT RESOURCE p. 330, #21 p. 339, #24	
5.2 Determine the non-permissible values when performing operations on rational expressions.	✓ STUDENT RESOURCE p. 336, #3 p. 355, #6	
5.3 Determine, in simplified form, the sum or difference of rational expressions with the same denominator.	✓ STUDENT RESOURCE p. 333, Example 1a, b	
5.4 Determine, in simplified form, the sum or difference of rational expressions in which the denominators are not the same and which may or may not contain common factors.	✓ Simplify expressions where the lowest common denominator is limited to at most: • three monomial factors • two monomial factors and one binomial factor • two binomial factors. STUDENT RESOURCE p. 334, Example 2a, b	✓ Simplify expressions where the lowest common denominator is composed of at most three linear binomial factors. STUDENT RESOURCE p. 336, #7b
5.5 Determine, in simplified form, the product or quotient of rational expressions.	✓ STUDENT RESOURCE pp. 324–325, Examples 1 and 2	
5.6 Simplify an expression that involves two or more operations on rational expressions.	✓ Simplify an expression limited to three rational expressions, and operations that do not require the use of order of operations. STUDENT RESOURCE p. 353, #11, #15e	✓ Simplify an expression given that the operations require the use of order of operations. STUDENT RESOURCE p. 338, #15

Algebra and Number (continued)

Specific Outcome

It is expected that students will:

6. Solve problems that involve rational equations (limited to numerators and denominators that are monomials, binomials or trinomials).
   [C, PS, R]

Notes

- Prior knowledge from previous grade levels includes:
  - solving simple rational equations of the type $\frac{a}{x} = b, x \neq 0$ (Grade 9).
- Students should have an understanding of solving quadratic equations (Relations and Functions, SO5) prior to studying this topic.
- Graphing and analyzing rational functions is introduced in Mathematics 30-1, Relations and Functions, SO14.
- It is intended that the rational equations be those that can be simplified to linear and quadratic equations.

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
(It is intended that the rational equations be those that can be simplified to linear and quadratic equations.)

Achievement Indicators	Acceptable Standard	Standard of Excellence
6.1 Determine the non-permissible values for the variable in a rational equation.	✓ STUDENT RESOURCE p. 342, Example 1	
6.2 Determine the solution to a rational equation algebraically, and explain the process used to solve the equation.	✓ Determine the solution, and provide a partial explanation. STUDENT RESOURCE p. 351, #24b	✓ Determine the solution, and provide a complete explanation. STUDENT RESOURCE p. 351, #24b
6.3 Explain why a value obtained in solving a rational equation may not be a solution of the equation.	✓ STUDENT RESOURCE p. 348, #4	
6.4 Solve problems by modelling a situation using a rational equation.	✓ Provide a complete solution to a problem, given an equation, or a partial solution to a problem when an equation is not given. STUDENT RESOURCE p. 348, #7 p. 346, Example 4	✓ Provide a complete solution to a problem when an equation is not given. STUDENT RESOURCE p. 346, Example 4

Topic: Trigonometry

General Outcome: Develop trigonometric reasoning.

Specific Outcome

It is expected that students will:

1. Demonstrate an understanding of angles in standard position (0° to 360°).
   [R, V]

Notes

- Prior knowledge from previous grade levels includes:
  – points on the coordinate plane (Grades 7, 8, 9)
  – degree measurement (Grade 9).
- Visualization [V] of the concepts in this outcome can be supported concretely using tools such as compasses, rulers and protractors, or pictorially using dynamic geometry software such as GeoGebra or Geometer's Sketchpad.
- This outcome leads to the introduction of circular functions in Mathematics 30-1.
- Radian measure is **not** part of this outcome.

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

Achievement Indicators	Acceptable Standard	Standard of Excellence
1.1 Sketch an angle in standard position, given the measure of the angle.	✓ STUDENT RESOURCE p. 83, #4	
1.2 Determine the reference angle for an angle in standard position.	✓ STUDENT RESOURCE p. 83, #5	
1.3 Explain, using examples, how to determine the angles from 0° to 360° that have the same reference angle as a given angle.	✓ STUDENT RESOURCE p. 130, #13	
1.4 Illustrate, using examples, that any angle from 90° to 360° is the reflection in the x-axis and/or the y-axis of its reference angle.	✓ STUDENT RESOURCE p. 81, Example 3	
1.5 Determine the quadrant in which a given angle in standard position terminates.	✓ STUDENT RESOURCE p. 83, #3	
1.6 Draw an angle in standard position given any point $P(x, y)$ on the terminal arm of the angle.	✓ STUDENT RESOURCE p. 96, #1	
1.7 Illustrate, using examples, that the points $P(x, y)$, $P(-x, y)$, $P(-x, -y)$ and $P(x, -y)$ are points on the terminal sides of angles in standard position that have the same reference angle.	✓ STUDENT RESOURCE p. 126, #7	

Trigonometry (continued)

Specific Outcome

It is expected that students will:

2. Solve problems, using the three primary trigonometric ratios for angles from 0° to 360° in standard position.
   [C, ME, PS, R, T, V]
   [ICT: C6–4.1]

Notes

- Prior knowledge from previous grade levels/courses includes:
  - the Pythagorean theorem (Grade 8)
  - introduction to trigonometric ratios (Mathematics 10C, Measurement, SO4).
- Radian measure is **not** part of this outcome.
- Special triangles **are** a part of this outcome.
- The processes of Technology [T] and Visualization [V] can be supported concretely using tools such as compasses, rulers and protractors, or pictorially using dynamic geometry software such as GeoGebra or Geometer's Sketchpad.
- Although rationalizing the denominator is an outcome of Mathematics 20-1, rationalizing denominators of trigonometric ratios such as $\frac{1}{\sqrt{2}}$ is **not** a necessary part of this outcome.
- For Achievement Indicator 2.1, note that students have not yet been formally introduced to the distance formula.
- The general solution of first degree trigonometric equations is **not** part of this outcome.
- Coordinates containing variables are beyond the scope of this outcome.

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

Achievement Indicators	Acceptable Standard	Standard of Excellence
2.1 Determine, using the Pythagorean theorem or the distance formula, the distance from the origin to a point P (x, y) on the terminal arm of an angle.	✓ STUDENT RESOURCE p. 126, #6b	
2.2 Determine the value of sin θ, cos θ or tan θ, given any point P (x, y) on the terminal arm of angle θ.	✓ STUDENT RESOURCE p. 91, Example 1	
2.3 Determine, without the use of technology, the value of sin θ, cos θ or tan θ, given any point P (x, y) on the terminal arm of angle θ, where $\theta = 0^\circ, 90^\circ, 180^\circ, 270^\circ$ or 360°.	✓ STUDENT RESOURCE p. 97, #10	
2.4 Determine the sign of a given trigonometric ratio for a given angle, without the use of technology, and explain.	✓ STUDENT RESOURCE p. 96, #6	
2.5 Solve, for all values of θ, an equation of the form sin $\theta = a$ or cos $\theta = a$, where $-1 \leq a \leq 1$, and an equation of the form tan $\theta = a$, where a is a real number.	✓ STUDENT RESOURCE p. 127, #10	
2.6 Determine the exact value of the sine, cosine or tangent of a given angle with a reference angle of 30º, 45º or 60º.	✓ STUDENT RESOURCE p. 98, #19	
2.7 Describe patterns in and among the values of the sine, cosine and tangent ratios for angles from 0° to 360°.	✓ STUDENT RESOURCE p. 98, #21	
2.8 Sketch a diagram to represent a problem.	✓ STUDENT RESOURCE p. 82, Example 4	
2.9 Solve a contextual problem, using trigonometric ratios.	✓ Provide a partial solution to the problem. STUDENT RESOURCE p. 84, #13	✓ Provide a complete solution to the problem. STUDENT RESOURCE p. 84, #13

Trigonometry (continued)

Specific Outcome

It is expected that students will:

3. Solve problems, using the cosine law and sine law, including the ambiguous case.
   [C, CN, PS, R, T]
   [ICT: C6–4.1]

Notes

- Prior knowledge from previous grade levels/courses includes:
  - the Pythagorean theorem (Grade 8)
  - introduction to trigonometric ratios (Mathematics 10C, Measurement, SO4)
  - solve systems of linear equations (Mathematics 10C, Relations and Functions, SO 9).

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

Achievement Indicators	Acceptable Standard	Standard of Excellence
3.1 Sketch a diagram to represent a problem that involves a triangle without a right angle.	✓ STUDENT RESOURCE p. 109, #10	
3.2 Solve, using primary trigonometric ratios, a triangle that is not a right triangle.	✓ Solve a triangle that does not require a system of equations. STUDENT RESOURCE pp. 102–103, Example 1, Method 1	✓ Solve a triangle requiring the use of a system of equations. STUDENT RESOURCE p. 109, #13
3.3 Explain the steps in a given proof of the sine law or cosine law.	✓ Explain the steps in a given proof of the sine law. STUDENT RESOURCE p. 111, #19	✓ Explain the steps in a given proof of the cosine law. STUDENT RESOURCE p. 123, #22
3.4 Sketch a diagram and solve a problem, using the cosine law.	✓ STUDENT RESOURCE p. 120, #9	

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Achievement Indicators	Acceptable Standard	Standard of Excellence
3.5 Sketch a diagram and solve a problem, using the sine law.	✓ Solve problems, excluding the ambiguous case. STUDENT RESOURCE p. 109, #10	✓ Solve problems, including the ambiguous case. STUDENT RESOURCE p. 110, #17
3.6 Describe and explain situations in which a problem may have no solution, one solution or two solutions.	✓ Describe the number of solutions in a specific situation. STUDENT RESOURCE p. 108, #6	✓ Generalize the conditions in which the ambiguous case occurs. STUDENT RESOURCE p. 127, #16

Topic: Relations and Functions

General Outcome: Develop algebraic and graphical reasoning through the study of relations.

Specific Outcome

It is expected that students will:

1. Factor polynomial expressions of the form:
   - $ax^2 + bx + c, a \neq 0$
   - $a^2x^2 - b^2y^2, a \neq 0, b \neq 0$
   - $a(f(x))^2 + b(f(x)) + c, a \neq 0$
   - $a^2(f(x))^2 - b^2(g(y))^2, a \neq 0, b \neq 0$

   where a, b and c are rational numbers.
   [CN, ME, R]

Notes

- Prior knowledge from previous courses includes:
  – introduction to polynomial factoring (Mathematics 10C, Algebra and Number, SO5).
- Polynomial division, synthetic division, the factor theorem and the remainder theorem are **not** part of this outcome and will be introduced in Mathematics 30-1.

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

Achievement Indicators	Acceptable Standard	Standard of Excellence
1.1 Factor a given polynomial expression that requires the identification of common factors.	✓ STUDENT RESOURCE p. 220, Example 1a, b	
1.2 Determine whether a given binomial is a factor for a given polynomial expression, and explain why or why not.	✓ STUDENT RESOURCE p. 232, #18	
1.3 Factor a given polynomial expression of the form: • $ax^2 + bx + c, a \neq 0$ • $a^2x^2 - b^2y^2, a \neq 0, b \neq 0.$	✓ STUDENT RESOURCE p. 229, #2 p. 230, #4	
1.4 Factor a given polynomial expression that has a quadratic pattern, including: • $a(f(x))^2 + b(f(x)) + c, a \neq 0$ • $a^2(f(x))^2 - b^2(g(y))^2, a \neq 0, b \neq 0.$	✓ Factor when $f(x)$ or $g(y)$ is a monomial. STUDENT RESOURCE p. 230, #6c	✓ Factor when $f(x)$ or $g(y)$ is a binomial. STUDENT RESOURCE p. 222, Example 2

Relations and Functions (continued)

Specific Outcome

It is expected that students will:

2. Graph and analyze absolute value functions (limited to linear and quadratic functions) to solve problems.
   [C, PS, R, T, V]
   [ICT: C6–4.1, C6–4.3]

Notes

- Prior knowledge from previous courses includes:
  - introduction to intercepts, domain and range (Mathematics 10C, Relations and Functions, SO5).
- Students should be familiar with the concepts in Algebra and Number SO1 and Relations and Functions SO4 prior to the introduction of this outcome.
- Piecewise notation is an important concept that will be used in Mathematics 31.

Achievement Indicators

The following set of indicators ***may*** *be used to determine whether students have met the corresponding specific outcome.*

Achievement Indicators	Acceptable Standard	Standard of Excellence
2.1 Create a table of values for $y = \vert f(x)\vert$, given a table of values for $y = f(x)$.	✓ STUDENT RESOURCE p. 375, #1	
2.2 Generalize a rule for writing absolute value functions in piecewise notation.	✓ STUDENT RESOURCE p. 379, #27	
2.3 Sketch the graph of $y = \vert f(x)\vert$; state the intercepts, domain and range; and explain the strategy used.	✓ Provide an accurate sketch, state the characteristics and provide a partial explanation of the strategy used to graph. STUDENT RESOURCE p. 370, Example 1	✓ Provide an accurate sketch, state the characteristics and provide a complete explanation of the strategy used to graph. STUDENT RESOURCE p. 370, Example 1
2.4 Solve an absolute value equation graphically, with or without technology.	✓ STUDENT RESOURCE p. 382, Example 1	

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Achievement Indicators	Acceptable Standard	Standard of Excellence
2.5 Solve, algebraically, an equation with a single absolute value, and verify the solution.	✓ Solve absolute value equations that involve linear expressions, including verification of solutions and identification of extraneous roots, if any. ✓ Provide a partial solution to an equation that involves a quadratic expression. STUDENT RESOURCE p. 389, #4 p. 389, #6	✓ Solve absolute value equations that involve quadratic expressions, including verification of solutions and identification of extraneous roots, if any. STUDENT RESOURCE p. 389, #6
2.6 Explain why the absolute value equation $\lvert f(x)\rvert < 0$ has no solution.	✓ STUDENT RESOURCE p. 381, Investigate Absolute Value Equations #6	
2.7 Determine and correct errors in a solution to an absolute value equation.	✓ Identify and correct the error to the solution to an equation involving a linear expression. STUDENT RESOURCE p. 390, #15	✓ Identify and correct the error to the solution to an equation involving a quadratic expression. STUDENT RESOURCE p. 413, #9
2.8 Solve a problem that involves an absolute value function.	✓ Provide solutions to routine problems. STUDENT RESOURCE p. 378, #17	✓ Provide solutions to nonroutine problems. STUDENT RESOURCE p. 414, #11

Specific Outcome

It is expected that students will:

3. Analyze quadratic functions of the form $y = a(x - p)^2 + q$ and determine the:
   - vertex
   - domain and range
   - direction of opening
   - axis of symmetry
   - x- and y-intercepts.

   [CN, R, T, V]
   [ICT: C6–4.3, C7–4.2]

Notes

- Prior knowledge from previous courses includes:
  - introduction to intercepts, domain and range (Mathematics 10C, Relations and Functions, SO5).
- For Achievement Indicators 3.2–3.5, students are expected to use an inductive reasoning process to determine the rules and generalizations for quadratic functions. A complete study of deductive and inductive reasoning is **not** required for this outcome.
- Quadratic functions of the form $y = a(x - p)^2 + q$ are sometimes referred to as functions in *vertex form*.

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

Achievement Indicators	Acceptable Standard	Standard of Excellence
3.1 Explain why a function given in the form $y = a(x - p)^2 + q$ is a quadratic function.	✓ STUDENT RESOURCE p. 199, #10a	
3.2 Compare the graphs of a set of functions of the form $y = ax^2$ to the graph of $y = x^2$, and generalize, using inductive reasoning, a rule about the effect of a.	✓ STUDENT RESOURCE p. 143 Investigate Graphs: Part A	
3.3 Compare the graphs of a set of functions of the form $y = x^2 + q$ to the graph of $y = x^2$, and generalize, using inductive reasoning, a rule about the effect of q.	✓ STUDENT RESOURCE p. 143 Investigate Graphs: Part B	
3.4 Compare the graphs of a set of functions of the form $y = (x - p)^2$ to the graph of $y = x^2$, and generalize, using inductive reasoning, a rule about the effect of p.	✓ STUDENT RESOURCE p. 144 Investigate Graphs: Part C	
3.5 Determine the coordinates of the vertex for a quadratic function of the form $y = a(x - p)^2 + q$, and verify with or without technology.	✓ STUDENT RESOURCE p. 157, #6a	
3.6 Generalize, using inductive reasoning, a rule for determining the coordinates of the vertex for quadratic functions of the form $y = a(x - p)^2 + q$.	✓	
3.7 Sketch the graph of $y = a(x - p)^2 + q$, using transformations, and identify the vertex, domain and range, direction of opening, axis of symmetry and x- and y-intercepts.	✓ STUDENT RESOURCE p. 157, #4	
3.8 Explain, using examples, how the values of a and q may be used to determine whether a quadratic function has zero, one or two x-intercepts.	✓ STUDENT RESOURCE p. 162, #25	
3.9 Write a quadratic function in the form $y = a(x - p)^2 + q$ for a given graph or a set of characteristics of a graph.	✓ Write a quadratic function when the vertex is given. STUDENT RESOURCE p. 158, #8, #9	✓ Write a quadratic function when the vertex is not given. STUDENT RESOURCE p. 161, #21b

Relations and Functions (continued)

Specific Outcome

It is expected that students will:

4. Analyze quadratic functions of the form $y = ax^2 + bx + c$ to identify characteristics of the corresponding graph, including:
   - vertex
   - domain and range
   - direction of opening
   - axis of symmetry
   - x- and y-intercepts

   and to solve problems.
   [CN, PS, R, T, V]
   [ICT: C6–4.1, C6–4.3]

Notes

- Prior knowledge from previous courses includes:
  - introduction to intercepts, domain and range (Mathematics 10C, Relations and Functions, SO5).
- Quadratic functions of the form $y = ax^2 + bx + c$ are often referred to as functions in *standard form*.
- For Achievement Indicators 4.7 and 4.8, minimum and maximum problems are examples of problems that can be modelled by quadratic functions.

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

Achievement Indicators	Acceptable Standard	Standard of Excellence
4.1 Explain the reasoning for the process of completing the square as shown in a given example.	✓ STUDENT RESOURCE pp. 185–186, Example 1, Method 2	
4.2 Write a quadratic function given in the form $y = ax^2 + bx + c$ as a quadratic function in the form $y = a(x - p)^2 + q$ by completing the square.	✓ Complete the square for quadratic functions where b/a is an integer. STUDENT RESOURCE p. 193, #4	✓ Complete the square for quadratic functions where b/a is not an integer. STUDENT RESOURCE p. 193, #8
4.3 Identify, explain and correct errors in an example of completing the square.	✓ Identify and correct errors, and provide a partial explanation. STUDENT RESOURCE p. 194, #12	✓ Identify and correct errors, and provide a complete explanation. STUDENT RESOURCE p. 194, #12
4.4 Determine the characteristics of a quadratic function given in the form $y = ax^2 + bx + c$, and explain the strategy used.	✓ STUDENT RESOURCE p. 193, #11	
4.5 Sketch the graph of a quadratic function given in the form $y = ax^2 + bx + c$.	✓ STUDENT RESOURCE p. 199, #11a	
4.6 Verify, with or without technology, that a quadratic function in the form $y = ax^2 + bx + c$ represents the same function as a given quadratic function in the form $y = a(x - p)^2 + q$.	✓ STUDENT RESOURCE p. 193, #5	

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Achievement Indicators	Acceptable Standard	Standard of Excellence
4.7 Write a quadratic function that models a given situation, and explain any assumptions made.	✓ Write a quadratic function and provide a partial explanation of the assumptions. STUDENT RESOURCE p. 197, #31a, c	✓ Write a quadratic function and provide a full explanation of the assumptions. STUDENT RESOURCE p. 197, #31a, c
4.8 Solve a problem, with or without technology, by analyzing a quadratic function.	✓ Provide a complete solution to a problem given an equation, or a partial solution to a problem when an equation is not given. STUDENT RESOURCE p. 203, #13 p. 200, #17	✓ Provide a complete solution to a problem when an equation is not given. STUDENT RESOURCE p. 200, #17

Relations and Functions (continued)

Specific Outcome

It is expected that students will:

5. Solve problems that involve quadratic equations.
   [C, CN, PS, R, T, V]
   [ICT: C6–4.1]

Notes

- Prior knowledge from previous grade levels/courses includes:
  - square roots (Grade 8)
  - factoring quadratic expressions (Mathematics 10C, Algebra and Number, SO5)
  - simplifying radicals (Mathematics 10C, Algebra and Numbers, SO2).
- Students should be introduced to Algebra and Number SO3 and SO4 prior to studying this outcome.

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

Achievement Indicators	Acceptable Standard	Standard of Excellence
5.1 Explain, using examples, the relationship among the roots of a quadratic equation, the zeros of the corresponding quadratic function and the *x*-intercepts of the graph of the quadratic function.	✓ STUDENT RESOURCE p. 264, #7	
5.2 Derive the quadratic formula, using deductive reasoning.	✓ Derive the formula partially. STUDENT RESOURCE p. 244, Investigate the Quadratic Formula #2	✓ Derive the formula completely. STUDENT RESOURCE p. 244, Investigate the Quadratic Formula #2
5.3 Solve a quadratic equation of the form $ax^2 + bx + c = 0$ by using strategies such as: • determining square roots • factoring • completing the square • applying the quadratic formula • graphing its corresponding function.	✓ STUDENT RESOURCE p. 250, Example 3a	
5.4 Select a method for solving a quadratic equation, justify the choice, and verify the solution.	✓ Solve a quadratic equation, provide a partial justification of the method chosen, and verify the solution. STUDENT RESOURCE p. 254, #7	✓ Solve a quadratic equation, provide a complete justification of the method chosen, and verify the solution. STUDENT RESOURCE p. 254, #7
5.5 Explain, using examples, how the discriminant may be used to determine whether a quadratic equation has two, one or no real roots; and relate the number of zeros to the graph of the corresponding quadratic function.	✓ STUDENT RESOURCE p. 246, Example 1	

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Achievement Indicators	Acceptable Standard	Standard of Excellence
5.6 Identify and correct errors in a solution to a quadratic equation.	✓ Identify and correct errors. STUDENT RESOURCE p. 256, #21	
5.7 Solve a problem by: • analyzing a quadratic equation • determining and analyzing a quadratic equation.	✓ Provide a complete solution to a problem given an equation, or a partial solution to a problem when an equation is not given. STUDENT RESOURCE p. 259, #16	✓ Provide a complete solution to a problem when an equation is not given. STUDENT RESOURCE p. 262, #12

Relations and Functions (continued)

Specific Outcome

It is expected that students will:

6. Solve, algebraically and graphically, problems that involve systems of linear-quadratic and quadratic-quadratic equations in two variables.
   [CN, PS, R, T, V]
   [ICT: C6–4.1, C6–4.4]

Notes

- Prior knowledge from previous courses includes:
  - solving systems of linear equations, both graphically and algebraically (Mathematics 10C, Relations and Functions, SO9)
  - factoring quadratic expressions (Mathematics 10C, Algebra and Number, SO5).
- Students should be introduced to Relations and Functions SO3, SO4 and SO5 prior to studying this outcome.

Achievement Indicators

The following set of indicators ***may*** *be used to determine whether students have met the corresponding specific outcome.*
(It is intended that the quadratic equations be limited to those that correspond to quadratic functions.)

Achievement Indicators	Acceptable Standard	Standard of Excellence
6.1 Model a situation, using a system of linear-quadratic or quadratic-quadratic equations.	✓ Create a system of equations that models a familiar problem. STUDENT RESOURCE p. 437, #13a	✓ Create a system of equations that models an unfamiliar problem. STUDENT RESOURCE p. 453, #11a
6.2 Relate a system of linear-quadratic or quadratic-quadratic equations to the context of a given problem.	✓ STUDENT RESOURCE p. 454, #16	
6.3 Determine and verify the solution of a system of linear-quadratic or quadratic-quadratic equations graphically, with technology.	✓ STUDENT RESOURCE p. 435, #4	
6.4 Determine and verify the solution of a system of linear-quadratic or quadratic-quadratic equations algebraically.	✓ STUDENT RESOURCE p. 441, Example 1 p. 447, Example 4	
6.5 Explain the meaning of the points of intersection of a system of linear-quadratic or quadratic-quadratic equations.	✓ STUDENT RESOURCE p. 427, Example 1b p. 435, #1b	
6.6 Explain, using examples, why a system of linear-quadratic or quadratic-quadratic equations may have zero, one, two or an infinite number of solutions.	✓ STUDENT RESOURCE p. 439, #20	

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Achievement Indicators	Acceptable Standard	Standard of Excellence
6.7 Solve a problem that involves a system of linear-quadratic or quadratic-quadratic equations, and explain the strategy used.	✓ Provide a complete solution to a problem given an equation, or a partial solution to a problem when an equation is not given. STUDENT RESOURCE p. 455, #17 p. 457, #6	✓ Provide a complete solution to a problem when an equation is not given. STUDENT RESOURCE p. 457, #6

Relations and Functions (continued)

Specific Outcome

It is expected that students will:

7. Solve problems that involve linear and quadratic inequalities in two variables.
   [C, PS, T, V]
   [ICT: C6–4.1, C6–4.3]

Notes

- Prior knowledge from previous grade levels includes:
  - solving and graphing linear inequalities in one variable (Grade 9).
- Systems of linear inequalities is **not** a part of this outcome.

Achievement Indicators

The following set of indicators ***may*** *be used to determine whether students have met the corresponding specific outcome.*

Achievement Indicators	Acceptable Standard	Standard of Excellence
7.1 Explain, using examples, how test points can be used to determine the solution region that satisfies an inequality.	✓ STUDENT RESOURCE p. 509, #13	
7.2 Explain, using examples, when a solid or broken line should be used in the solution for an inequality.	✓ STUDENT RESOURCE p. 475, #19	
7.3 Sketch, with or without technology, the graph of a linear or quadratic inequality.	✓ STUDENT RESOURCE p. 505, #6, #8	
7.4 Solve a problem that involves a linear or quadratic inequality.	✓ Solve and interpret a problem, given the inequality. STUDENT RESOURCE p. 498, #10	✓ Create an inequality that models a given problem, and use it to solve and interpret the problem. STUDENT RESOURCE p. 473, #13 p. 509, #17

Relations and Functions (continued)

Specific Outcome

It is expected that students will:

8. Solve problems that involve quadratic inequalities in one variable.
   [CN, PS, V]

Notes

- Prior knowledge from previous grade levels includes:
  - solving and graphing linear inequalities in one variable (Grade 9).
- It is important that students are exposed to a variety of methods to solve quadratic inequalities.
- Technology [T] has not been identified as one of the mathematical processes to be emphasized in completing this outcome. Students are expected to solve problems involving quadratic inequalities algebraically, but they may investigate these inequalities by using technology.

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

Achievement Indicators	Acceptable Standard	Standard of Excellence
8.1 Determine the solution of a quadratic inequality in one variable, using strategies such as case analysis, graphing, roots and test points, or sign analysis; and explain the strategy used.	✓ Solve and verify a quadratic inequality, using graphing or roots and test points. STUDENT RESOURCE p. 478, Example 1, Methods 1 and 2	✓ Solve and verify a quadratic inequality, using sign analysis or case analysis. STUDENT RESOURCE pp. 478–479 Example 1, Method 3
8.2 Represent and solve a problem that involves a quadratic inequality in one variable.	✓ Solve a problem, given the quadratic inequality. STUDENT RESOURCE p. 483, Example 4	✓ Create and solve a quadratic inequality to represent a situation. STUDENT RESOURCE p. 486, #13
8.3 Interpret the solution to a problem that involves a quadratic inequality in one variable.	✓ STUDENT RESOURCE p. 486, #12b	

Relations and Functions (continued)

Specific Outcome

It is expected that students will:

9. Analyze arithmetic sequences and series to solve problems. [CN, PS, R]

Notes

- This is the first time the concept of arithmetic sequences and series is formally introduced to students.
- Teachers should note that the revised curriculum uses t_1 instead of a to denote the first term of an arithmetic sequence.
- In Achievement Indicator 9.3, deriving a rule for an arithmetic sequence should conclude with the general term formulas for t_n.
- In Achievement Indicator 9.4, discrete and continuous data should be discussed.

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

Achievement Indicators	Acceptable Standard	Standard of Excellence
9.1 Identify the assumption(s) made when defining an arithmetic sequence or series.	✓ STUDENT RESOURCE p. 18, #16c	
9.2 Provide and justify an example of an arithmetic sequence.	✓ STUDENT RESOURCE p. 66, #3	
9.3 Derive a rule for determining the general term of an arithmetic sequence.	✓ STUDENT RESOURCE p. 8, Investigate Arithmetic Sequences #9c	
9.4 Describe the relationship between arithmetic sequences and linear functions.	✓ STUDENT RESOURCE p. 17, #7	
9.5 Determine t_1, d, n or t_n in a problem that involves an arithmetic sequence.	✓ STUDENT RESOURCE p. 16, #3, #4, #5	

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Achievement Indicators	Acceptable Standard	Standard of Excellence
9.6 Derive a rule for determining the sum of n terms of an arithmetic series.	✓ Derive the rule partially. STUDENT RESOURCE p. 23, Investigate Arithmetic Series #8	✓ Derive the rule completely. STUDENT RESOURCE p. 23, Investigate Arithmetic Series #8
9.7 Determine t_1, d, n or S_n in a problem that involves an arithmetic series.	✓ STUDENT RESOURCE p. 27, #2, #3, #4, #5	
9.8 Solve a problem that involves an arithmetic sequence or series.	✓ STUDENT RESOURCE p. 67, #10	

Relations and Functions (continued)

Specific Outcome

It is expected that students will:

10. Analyze geometric sequences and series to solve problems. [PS, R]

Notes

- Teachers should note that the revised curriculum uses t_1 instead of a to denote the first term of a geometric sequence.
- In Achievement Indicator 10.3, deriving a rule for a geometric sequence should conclude with the general term formulas for t_n.
- Discrete and continuous data should be discussed.
- In Achievement Indicators 10.4 and 10.6, guess and check is an appropriate method to solve for n.
- In Achievement Indicator 10.7, the introduction of limits is beyond the scope of this course.

Achievement Indicators

The following set of indicators ***may*** *be used to determine whether students have met the corresponding specific outcome.*

Achievement Indicators	Acceptable Standard	Standard of Excellence
10.1 Identify assumptions made when identifying a geometric sequence or series.	✓ STUDENT RESOURCE p. 41, #14d	
10.2 Provide and justify an example of a geometric sequence.	✓ STUDENT RESOURCE p. 33, Investigate a Geometric Sequence #5a	
10.3 Derive a rule for determining the general term of a geometric sequence.	✓ STUDENT RESOURCE p. 33, Investigate a Geometric Sequence #5b	
10.4 Determine t_1, r, n or t_n in a problem that involves a geometric sequence.	✓ Determine t_1, r or t_n. STUDENT RESOURCE p. 36, Example 3 p. 39, #5	✓ Determine n. STUDENT RESOURCE p. 39, #6

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Achievement Indicators	Acceptable Standard	Standard of Excellence
10.5 Derive a rule for determining the sum of *n* terms of a geometric series.	✓ Derive the rule partially.	✓ Derive the rule completely.
10.6 Determine t_1, r, n or S_n in a problem that involves a geometric series.	✓ Determine t_1, r or t_n. STUDENT RESOURCE p. 53, #3, #4 p. 54, #5	✓ Determine n. STUDENT RESOURCE p. 54, #6
10.7 Generalize, using inductive reasoning, a rule for determining the sum of an infinite geometric series.	✓ STUDENT RESOURCE p. 59, Investigate an Infinite Series #10a, b	
10.8 Explain why a geometric series is convergent or divergent.	✓ STUDENT RESOURCE p. 64, #17	
10.9 Solve a problem that involves a geometric sequence or series.	✓ Determine t_1, r or t_n. STUDENT RESOURCE p. 40, #9a, b, c p. 55, #13	✓ Determine n. STUDENT RESOURCE p. 40, #9d

Relations and Functions (continued)

Specific Outcome

It is expected that students will:

11. Graph and analyze reciprocal functions (limited to the reciprocal of linear and quadratic functions).
    [CN, R, T, V]
    [ICT: C6–4.1, C6–4.3]

Notes

- The concept of asymptotes, including horizontal asymptotes, will be introduced to students for the first time.

Achievement Indicators

The following set of indicators ***may*** *be used to determine whether students have met the corresponding specific outcome.*

Achievement Indicators	Acceptable Standard	Standard of Excellence
11.1 Compare the graph of $y = \frac{1}{f(x)}$ to the graph of $y = f(x)$.	✓ STUDENT RESOURCE pp. 394–395, Example 1	
11.2 Identify, given a function $f(x)$, values of x for which $y = \frac{1}{f(x)}$ will have vertical asymptotes; and describe their relationship to the non-permissible values of the related rational expression.	✓ STUDENT RESOURCE p. 403, #2	
11.3 Graph, with or without technology, $y = \frac{1}{f(x)}$, given $y = f(x)$ as a function or a graph, and explain the strategies used.	✓ Graph the reciprocal function when the function $y = f(x)$ is linear. STUDENT RESOURCE p. 404, #6a	✓ Graph the reciprocal function when the function $y = f(x)$ is quadratic. STUDENT RESOURCE p. 404, #6b, c
11.4 Graph, with or without technology, $y = f(x)$, given $y = \frac{1}{f(x)}$ as a function or a graph, and explain the strategies used.	✓ Graph the original linear function when the graph of its reciprocal function is given. STUDENT RESOURCE p. 406, #10a	✓ Graph the original quadratic function when the graph of its reciprocal function is given. STUDENT RESOURCE p. 406, #10b

Appendix: Mathematics Directing Words

Discuss	The word “discuss” **will not** be used as a directing word on mathematics examinations because it is not used consistently to mean a single activity.
	The following words are specific in meaning.
Algebraically	Use mathematical procedures that involve letters or symbols to represent numbers.
Analyze	Make a mathematical or methodical examination of parts to determine aspects of the whole; e.g., nature, proportion, function, interrelationship.
Compare	Examine the character or qualities of two things by providing characteristics of both that point out their mutual similarities and differences.
Conclude	State a logical end based on reasoning and/or evidence.
Contrast/Distinguish	Point out the differences between two things that have similar or comparable natures.
Criticize	Point out the merits and demerits of an item or issue.
Define	Provide the essential qualities or meaning of a word or concept; make distinct and clear by marking out the limits.
Describe	Give a written account or represent the characteristics of something, using a figure, model or picture.
Design/Plan	Construct a plan, i.e., a detailed sequence of actions, for a specific purpose.
Determine	Find a solution, to a specified degree of accuracy, to a problem by showing appropriate formulas, procedures and calculations.
Enumerate	Specify one-by-one or list in a concise form and according to some order.
Evaluate	Give the significance or worth of something by identifying the good and bad points or the advantages and disadvantages.
Explain	Make clear what is not immediately obvious or entirely known; give the cause of or reason for; make known in detail.
Graphically	Use a drawing that is produced electronically or by hand and that shows a relation between certain sets of numbers.

How	Show in what manner or way, with what meaning.
Hypothesize	Form a tentative proposition intended as a possible explanation for an observed phenomenon; i.e., a possible cause for a specific effect. The proposition should be testable logically and/or empirically.
Identify	Recognize and select as having the characteristics of something.
Illustrate	Make clear by providing an example. The form of the example must be specified in the question; i.e., word description, sketch or diagram.
Infer	Form a generalization from sample data; arrive at a conclusion by reasoning from evidence.
Interpret	State the meaning of something; present information in a new form that adds meaning to the original data.
Justify/Show How	Show reasons for or give facts that support a position.
Model	Find a model (in mathematics, a model of a situation is a pattern that is supposed to represent or set a standard for a real situation) that does a good job of representing a situation.
Outline	Give, in an organized fashion, the essential parts of something. The form of the outline must be specified in the question; i.e., lists, flowcharts, concept maps.
Predict	State in advance on the basis of empirical evidence and/or logic.
Prove	Establish the truth or validity of a statement for the general case by providing factual evidence or a logical argument.
Relate	Show a logical or causal connection between things.
Sketch	Provide a drawing that represents the key features of an object or a graph.
Solve	Give a solution for a problem; i.e., explanation in words and/or numbers.
Summarize	Give a brief account of the main points.
Trace	Give a step-by-step description of the development.
Verify	Establish, by substitution for a particular case or by geometric comparison, the truth of a statement.
Why	Show the cause, reason or purpose.