Outcomes with Assessment Standards
for
Mathematics 20-2

2013

This resource is intended to assist teachers with the
provincial implementation of Mathematics 20-2.

Alberta
Government

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The primary audience for this resource is:

Teachers	✓
Administrators	
Students	
Parents	

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INTRODUCTION

Mathematics 20-2 was provincially implemented in September 2011. Teachers participating in focus groups during the development of the program of studies expressed a need for a common understanding of the curriculum and assessment standards. In response to this need, and in keeping with Alberta Education's goal of establishing and effectively communicating clear outcomes and high standards, this standards resource was developed.

This resource is designed to support the implementation of the Alberta Mathematics Grades 10–12 Program of Studies, which can be found at http://education.alberta.ca/media/655889/math10to12.pdf. Teachers are strongly encouraged to consult the program of studies for details about the philosophy of the program.

PURPOSE

Outcomes with Assessment Standards for Mathematics 20-2 links the achievement indicators for the specific outcomes from the program of studies with information and commentaries about standards. Its purpose is to provide teachers of Mathematics 20-2 with clearly stated standards to use as guidelines in their classroom instruction and assessment practices.

DEFINITIONS AND TERMINOLOGY

Standards

A *standard* is a reference point used in planning and evaluation. In evaluating educational performance, the following standards apply:

- *Curriculum* and *assessment standards* apply to the assessment of individual students.
- *Achievement standards* apply to the assessment of student populations.

In this resource, only curriculum and assessment standards are discussed.

Curriculum Standards

Curriculum standards are outcomes for a course within a program. The curriculum standards for Mathematics 20-2 are defined by the general and specific outcomes outlined in the program of studies. They are further clarified by the achievement indicators, which reflect the scope of each specific outcome.

Outcomes

General outcomes are concise statements identifying what it is that students are expected to know and be able to do upon completion of a course within a program.

Specific outcomes are statements identifying the component knowledge, skills and attitudes of a general outcome. Specific outcomes identify a range of contexts in which the general outcomes apply.

In the specific outcomes, the word *including* indicates that any ensuing items must be addressed to fully meet the learning outcome. The phrase *such as* indicates that the ensuing items are provided for clarification and are not requirements that must be addressed to fully meet the learning outcome.

The word *and* used in an outcome indicates that both ideas must be addressed to fully meet the learning outcome, although not necessarily at the same time or in the same question.

Achievement Indicators

Achievement indicators are samples of how students may demonstrate their achievement of the goals of a specific outcome. The range of samples provided is meant to reflect the scope of the specific outcome.

The word *and* used in an achievement indicator implies that both ideas should be addressed at the same time or in the same question.

Assessment Standards

Assessment standards are the criteria used for judging individual student achievement relative to the curriculum standards.

STANDARDS FOR MATHEMATICS 20-2

Mathematics 20-2 is designed to follow directly from Mathematics 10C, so students taking Mathematics 20-2 are presumed to have reached the acceptable standard or better in the outcomes of Mathematics 10C.

The assessment standards for Mathematics 20-2 include an acceptable and an excellent level of performance. Student performance should be measured on a range of tasks, some of which are routine and obvious tasks in familiar contexts, and others which are nonroutine tasks in unfamiliar contexts. In many cases, a correlated example from the authorized resources is referenced to assist in assessing student performance. The authorized resources for Mathematics 20-2, published by Nelson Canada, are:

- Principles of Mathematics 11: Student Resource
- Principles of Mathematics 11: Teacher Resource.

Acceptable Standard

The *acceptable standard* of achievement in Mathematics 20-2 is met by students who receive a course mark between and including 50 percent and 79 percent. Typically, these students have gained new skills and a basic knowledge of the concepts and procedures relative to the general and specific outcomes defined for Mathematics 20-2 in the program of studies. These students can apply this knowledge to a limited range of familiar problem contexts.

Standard of Excellence

The *standard of excellence* for achievement in Mathematics 20-2 is met by students who receive a course mark at or above 80 percent. Typically, these students have gained a breadth and depth of understanding regarding the concepts and procedures, as well as the ability to apply this knowledge to a broad range of familiar and unfamiliar problem contexts.

Description of Standards

The following statements describe what is expected of Mathematics 20-2 students who meet the *acceptable standard* or the *standard of excellence* on independent work. The statements represent the standards against which student achievement is measured.

Acceptable Standard	*Standard of Excellence*
Students who meet the *acceptable standard* in Mathematics 20-2 consistently perform acceptable work on routine and obvious tasks in familiar contexts.	Students who meet the *standard of excellence* in Mathematics 20-2 consistently perform excellent work on routine and obvious tasks in familiar contexts, and acceptable work on nonroutine tasks in unfamiliar contexts.
These students have a basic understanding of the concepts and procedures outlined in the program of studies. They demonstrate their understanding in concrete, pictorial and symbolic modes, and can translate from one mode to another. They perform the mathematical operations and procedures that are fundamental to Mathematics 20-2 and apply what they know in daily living contexts.	These students have a comprehensive understanding of the concepts and procedures outlined in the program of studies. They demonstrate their understanding in concrete, pictorial and symbolic modes, and can translate from one mode to another. They perform the mathematical operations and procedures that are fundamental to Mathematics 20-2, apply what they know in daily living contexts and provide alternative solution procedures to verify results.
To meet the *acceptable standard*, students communicate about mathematical situations in an understandable way, using appropriate everyday and mathematical terms. They understand mathematical questions containing objects, diagrams or numbers in familiar contexts, and they construct mathematical models.	To meet the *standard of excellence*, students communicate about mathematical situations in a clear way, using numbers, diagrams and appropriate mathematical terms. They understand mathematical questions containing objects, diagrams or numbers in familiar and unfamiliar contexts, and they construct mathematical models using multiple representations.
Students meeting the *acceptable standard* apply what they know in solving straightforward problems in familiar settings and in analyzing simple mathematical models. They describe the steps they used to solve a particular problem, and verify and defend their solution to the problem.	Students meeting the *standard of excellence* apply what they know in solving routine and nonroutine problems in a broad range of settings. They describe the steps they used to solve a particular problem, defend their solution to the problem, and, where appropriate, provide alternative solution procedures to verify results.
Students meeting the *acceptable standard* have a positive attitude toward mathematics and a sense of personal competence in using mathematics. They demonstrate confidence when using common mathematical procedures and when applying problem-solving strategies in familiar settings.	Students meeting the *standard of excellence* have a positive attitude toward mathematics and show confidence in using mathematics meaningfully. They are self-motivated risk takers who persevere when solving novel problems. They take initiative in trying new methods and are creative in their approach to problem solving.

GENERAL NOTES

- All mathematical processes should be used and integrated throughout the outcomes.
- Technology [T], including calculators and computers, has been listed as one of the mathematical processes to be emphasized for some outcomes, with the expectation that students will have access to technology when completing the outcomes. If technology has not been specifically listed for a particular outcome, teachers may, at their discretion, use it to assist students in exploring patterns and relationships when learning a concept. It is expected, however, that technology will not be considered when assessing students' understandings of such outcomes.
- Most specific outcomes are accompanied by notes that address some of the questions that may arise when teaching the concepts. The assessment standards for each outcome are described in a chart that indicates, for each achievement indicator, whether the acceptable standard, the standard of excellence or, in some cases, both standards may be applicable (✓). Some check marks are accompanied by qualifying statements. Shaded regions indicate that the standard does not apply for the given achievement indicator. In many cases, a correlated example from the authorized resources is referenced in the chart to illustrate the standards.
- A *partial solution* to a problem is a solution in which a student demonstrates a basic understanding of the problem and the mathematical concepts required in solving the problem. However, the student is unable to complete the solution correctly for a variety of reasons, such as not being able to correctly connect the concepts involved or not being able to avoid procedural errors. For example, in solving a problem using the cosine law, given the measure of the three sides of a triangle, a student may be able to draw a diagram to correctly represent the situation and identify the appropriate equation needed to solve the problem, but then makes procedural errors in solving for the measure of an angle. Note that assessment of student learning is the responsibility of the teacher, and what is considered a partial solution may vary according to the question or task presented.

Topic: Measurement

General Outcome: Develop spatial sense and proportional reasoning.

Specific Outcome

It is expected that students will:

1. Solve problems that involve the application of rates. [CN, PS, R]

Notes

- Prior knowledge from previous grade levels/courses includes:
  - solving rate problems (Grade 8)
  - the concept of slope and rates of change (Mathematics 10C)
  - estimation strategies and measurement strategies (Mathematics 10C)
  - proportional reasoning and conversions between SI and imperial (Mathematics 10C).
- Examples used should be limited to linear rates.
- The emphasis of this outcome should be on the interpretation, comparison and use of rates.
- Students should be encouraged to use personal strategies to represent rates in different ways.

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

Achievement Indicators	Acceptable Standard	Standard of Excellence
1.1 Interpret rates in a given context, such as the arts, commerce, the environment, medicine or recreation.	✓ STUDENT RESOURCE p. 451, #5	
1.2 Solve a rate problem that requires the isolation of a variable.	✓ STUDENT RESOURCE p. 459, #5	
1.3 Determine and compare rates and unit rates.	✓ STUDENT RESOURCE p. 450, #1	
1.4 Make and justify a decision, using rates.	✓ STUDENT RESOURCE p. 460, #15	

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Achievement Indicators	Acceptable Standard	Standard of Excellence
1.5 Represent a given rate pictorially.	✓ STUDENT RESOURCE p. 450, #3	
1.6 Draw a graph to represent a rate.	✓ STUDENT RESOURCE p. 465, #4	
1.7 Explain, using examples, the relationship between the slope of a graph and a rate.	✓ STUDENT RESOURCE p. 465, #5	
1.8 Describe a context for a given rate or unit rate.	✓ STUDENT RESOURCE p. 459, #6	
1.9 Identify and explain factors that influence a rate in a given context.	✓ Identify the factors, with partial explanation. STUDENT RESOURCE p. 448, Example 3	✓ Identify the factors, with full explanation. STUDENT RESOURCE p. 448, Example 3
1.10 Solve a contextual problem that involves rates or unit rates.	✓ Solve simple contextual problems. STUDENT RESOURCE p. 459, #4	✓ Solve complex contextual problems, such as problems involving a comparison of different rates. STUDENT RESOURCE p. 461, #19

Measurement (continued)

Specific Outcome

It is expected that students will:

2. Solve problems that involve scale diagrams, using proportional reasoning.
   [CN, PS, R, V]

Notes

- Prior knowledge from previous grade levels/courses includes:
  - proportional reasoning (Grade 8 and Mathematics 10C)
  - scale diagrams and 2-D scale factors (Grade 9).
- Students are not required to make drawings of 3-D objects; e.g., orthographic projections and orthogonal drawings are not required.

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

Achievement Indicators	Acceptable Standard	Standard of Excellence
2.1 Explain, using examples, how scale diagrams are used to model a 2-D shape or a 3-D object.	✓ STUDENT RESOURCE pp. 466–467, Investigation	
2.2 Determine, using proportional reasoning, the scale factor, given one dimension of a 2-D shape or a 3-D object and its representation.	✓ STUDENT RESOURCE p. 471, #3	
2.3 Determine, using proportional reasoning, an unknown dimension of a 2-D shape or a 3-D object, given a scale diagram or a model.	✓ STUDENT RESOURCE p. 472, #6	
2.4 Draw, with or without technology, a scale diagram of a given 2-D shape, according to a specified scale factor (enlargement or reduction).	✓ STUDENT RESOURCE p. 472, #8	
2.5 Solve a contextual problem that involves a scale diagram.	✓ Solve contextual problems where a diagram is provided. STUDENT RESOURCE p. 468, Example 2	✓ Solve contextual problems where a diagram is not provided. STUDENT RESOURCE p. 474, #17

Measurement (continued)

Specific Outcome

It is expected that students will:

3. Demonstrate an understanding of the relationships among scale factors, areas, surface areas and volumes of similar 2-D shapes and 3-D objects.
   [C, CN, PS, R, V]

Notes

- Prior knowledge from previous courses includes area, surface area and volume formulas (Mathematics 10C). However, students are not expected to memorize area, surface area and volume formulas.
- Teachers should be aware that manipulating some surface area formulas may evolve into quadratics; care should be exercised when specifying the variable to be isolated in Achievement Indicator 3.7.
- The emphasis of this outcome is on conceptual understanding, not algebraic manipulation.

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

Achievement Indicators	Acceptable Standard	Standard of Excellence
3.1 Determine the area of a 2-D shape, given the scale diagram, and justify the reasonableness of the result.	✓ STUDENT RESOURCE p. 476, Example 1	
3.2 Determine the surface area and volume of a 3-D object, given the scale diagram, and justify the reasonableness of the result.	✓ STUDENT RESOURCE p. 500, #1	
3.3 Explain, using examples, the effect of a change in the scale factor on the area of a 2-D shape.	✓ STUDENT RESOURCE p. 481, #14	
3.4 Explain, using examples, the effect of a change in the scale factor on the surface area of a 3-D object.	✓ STUDENT RESOURCE p. 496, Example 1	
3.5 Explain, using examples, the effect of a change in the scale factor on the volume of a 3-D object.	✓ STUDENT RESOURCE p. 496, Example 1	

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Achievement Indicators	Acceptable Standard	Standard of Excellence
3.6 Explain, using examples, the relationships among scale factor, area of a 2-D shape, surface area of a 3-D object and volume of a 3-D object.	✓ Explain simple relationships; e.g., between scale factor and either surface area or volume. STUDENT RESOURCE pp. 496–497, Example 2	✓ Explain complex relationships; e.g., between surface area and volume or among surface area, volume and scale factor. STUDENT RESOURCE pp. 476–477, Example 1 pp. 496-497, Example 2
3.7 Solve a spatial problem that requires the manipulation of formulas.		✓ STUDENT RESOURCE p. 503, #19
3.8 Solve a contextual problem that involves the relationships among scale factors, areas and volumes.	✓ Solve simple contextual problems; e.g., between scale factor and either surface area or volume. STUDENT RESOURCE p. 501, #8	✓ Solve complex contextual problems; e.g., between surface area and volume or among surface area, volume and scale factor. STUDENT RESOURCE p. 508, #15

Topic: Geometry

General Outcome: Develop spatial sense.

Specific Outcome

It is expected that students will:

1. Derive proofs that involve the properties of angles and triangles. [CN, R, V]

Notes

- Prior knowledge from previous grade levels/courses includes:
  - similarity of polygons (Grade 9)
  - trigonometry (Mathematics 10C)
  - parallel lines, perpendicular lines and transversals (Grade 7)
  - circle properties (Grade 9).
- Students are expected to recognize the difference between deductive and inductive reasoning as introduced in Number and Logic, Specific Outcome (SO) 1.
- Proofs can be presented in a variety of formats, such as two-column proofs, paragraph proofs or flow-chart proofs.
- Proofs should be limited to direct proofs.
- Although technology is not an indicated process for this outcome, dynamic geometry programs and apps may be used in the exploration and development of the properties.
- Teachers should encourage dialogue and discussion among students to support reasoning throughout the proof. The emphasis should be on explaining each step of the proof.

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome. (It is intended that deductive reasoning be limited to direct proof.)*

Achievement Indicators	Acceptable Standard	Standard of Excellence
1.1 Generalize, using inductive reasoning, the relationships between pairs of angles formed by transversals and parallel lines, with or without technology.	✓ STUDENT RESOURCE pp. 70–71, Explore the Math	
1.2 Prove, using deductive reasoning, properties of angles formed by transversals and parallel lines, including the sum of the angles in a triangle.	✓ STUDENT RESOURCE p. 78, #1	
1.3 Generalize, using inductive reasoning, a rule for the relationship between the sum of the interior angles and the number of sides (n) in a polygon, with or without technology.	✓ STUDENT RESOURCE p. 94, Part 1	
1.4 Identify and correct errors in a given proof of a property that involves angles.	✓ Identify the errors. STUDENT RESOURCE p. 91, #9	✓ Identify and correct the errors.
1.5 Verify, with examples, that if lines are not parallel, the angle properties do not apply.	✓ STUDENT RESOURCE pp. 70–71, Explore the Math	
1.6 Prove, using deductive reasoning, that two triangles are congruent.	✓ STUDENT RESOURCE p. 112, #1	

Geometry (continued)

Specific Outcome

It is expected that students will:

2. Solve problems that involve properties of angles and triangles. [CN, PS, V]

Notes

- Prior knowledge from previous grade levels includes:
  - construction of parallel and perpendicular lines (Grade 7)
  - perpendicular bisectors (Grade 7).
- Teachers are encouraged to allow students to make their own constructions, with or without technology.

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

Achievement Indicators	Acceptable Standard	Standard of Excellence
2.1 Determine the measures of angles in a diagram that includes parallel lines, angles and triangles, and justify the reasoning.	✓ Determine the measure(s) in a given diagram, and provide a partial justification. STUDENT RESOURCE p. 90, #3	✓ Determine the measures, and provide a full justification. STUDENT RESOURCE p. 92, #12
2.2 Identify and correct errors in a given solution to a problem that involves the measures of angles.	✓	
2.3 Solve a contextual problem that involves angles or triangles.	✓ Solve a problem where a diagram is given. STUDENT RESOURCE p. 101, #13	✓ Solve a problem where a diagram is not given. STUDENT RESOURCE p. 100, #6
2.4 Construct parallel lines, given a compass or a protractor, and explain the strategy used.	✓ Construct and give a partial explanation. STUDENT RESOURCE p. 72, #3	✓ Construct and give a complete explanation. STUDENT RESOURCE p. 72, #3
2.5 Determine if lines are parallel, given the measure of an angle at each intersection formed by the lines and a transversal.	✓ STUDENT RESOURCE p. 72, #5	

Geometry (continued)

Specific Outcome

It is expected that students will:

3. Solve problems that involve the cosine law and the sine law, excluding the ambiguous case.
[CN, PS, R]

Notes

- Prior knowledge from previous courses includes:
  - primary trigonometric ratios (Mathematics 10C).

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

Achievement Indicators	Acceptable Standard	Standard of Excellence
3.1 Draw a diagram to represent a problem that involves the cosine law or the sine law.	✓ STUDENT RESOURCE p. 140, #9a – sine law p. 152, #8a – cosine law	
3.2 Explain the steps in a given proof of the sine law or cosine law.	✓ Explain the steps in a given proof of the sine law. STUDENT RESOURCE pp. 132–133, Investigate the Math	✓ Explain the steps in a given proof of the cosine law. STUDENT RESOURCE pp. 144–145, Investigate the Math
3.3 Solve a contextual problem that requires the use of the sine law or cosine law, and explain the reasoning.	✓ Solve a problem, and provide a partial explanation. STUDENT RESOURCE p. 161, #3	✓ Solve a problem, and provide a complete explanation. STUDENT RESOURCE p. 162, #6

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Achievement Indicators	Acceptable Standard	Standard of Excellence
3.4 Solve a contextual problem that involves more than one triangle.	✓ Solve a problem involving more than one triangle in two dimensions, when given a diagram. STUDENT RESOURCE p. 161, #5	✓ Solve a problem involving more than one triangle in two dimensions when no diagram is given, or solve a problem involving more than one triangle in three dimensions. STUDENT RESOURCE p. 163, #14

Topic: Number and Logic

General Outcome: Develop number sense and logical reasoning.

Specific Outcome

It is expected that students will:

1. Analyze and prove conjectures, using inductive and deductive reasoning, to solve problems.
   [C, CN, PS, R]

Notes

- Teachers should be aware that many mathematical concepts are embedded in language that may be difficult or challenging for some students. Therefore, teachers should encourage dialogue and discussion among students.

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

Achievement Indicators	Acceptable Standard	Standard of Excellence
1.1 Make conjectures by observing patterns and identifying properties, and justify the reasoning.	✓ Make a conjecture, with partial justification. STUDENT RESOURCE p. 12, #3	✓ Make a conjecture, with complete justification STUDENT RESOURCE p. 18, Learn about the Math
1.2 Explain why inductive reasoning may lead to a false conjecture.	✓ STUDENT RESOURCE p. 21, Example 3	
1.3 Compare, using examples, inductive and deductive reasoning.	✓ STUDENT RESOURCE p. 35, #8	
1.4 Provide and explain a counterexample to disprove a given conjecture.	✓ STUDENT RESOURCE p. 22, #1 STUDENT RESOURCE p. 23, #14	

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Achievement Indicators	Acceptable Standard	Standard of Excellence
1.5 Prove algebraic and number relationships such as divisibility rules, number properties, mental mathematics strategies or algebraic number tricks.	✓ Write a proof using examples or numeric verification. STUDENT RESOURCE p. 33, #17 (Joan and Garnet's work)	✓ Write a proof using algebraic reasoning. STUDENT RESOURCE p. 33, #17 (Jamie's work)
1.6 Prove a conjecture, using deductive reasoning (not limited to two column proofs).	✓ Write a proof involving a simple relationship. STUDENT RESOURCE p. 31, #2	✓ Write a proof involving a complex relationship. STUDENT RESOURCE p. 33, #15
1.7 Determine if a given argument is valid, and justify the reasoning.	✓ Determine the validity, with partial justification. STUDENT RESOURCE p. 32, #8	✓ Determine the validity, with complete justification. STUDENT RESOURCE p. 35, #6; p. 44, #9
1.8 Identify errors in a given proof; e.g., a proof that ends with 2 = 1.		✓ STUDENT RESOURCE p. 44, #7
1.9 Solve a contextual problem that involves inductive or deductive reasoning.	✓ Write a complete solution that involves inductive reasoning, or a partial solution that involves deductive reasoning. STUDENT RESOURCE p. 51, #16	✓ Write a complete solution that involves deductive reasoning. STUDENT RESOURCE p. 50, #11

Number and Logic (continued)

Specific Outcome

It is expected that students will:

2. Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies. [CN, PS, R, V]

Notes:

- Online games should be used with caution, as games that automatically complete some steps can obscure the mathematics involved.
- A variety of puzzles and games that involve logical reasoning should be used. They may include commercial games, such as Sudoku, Einstein puzzles, Clue, Mancala, Factory Balls, Pebble Jump, Nim and Mastermind; cribbage, solitaire and other card games; chess; or puzzles and games designed by students.
- (It is intended that this outcome be integrated throughout the course by using sliding, rotation, construction, deconstruction and similar puzzles and games.)

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
(It is intended that this outcome be integrated throughout the course by using sliding, rotation, construction, deconstruction and similar puzzles and games.)

Achievement Indicators	Acceptable Standard	Standard of Excellence
2.1 Determine, explain and verify a strategy to solve a puzzle or to win a game; e.g., • guess and check • look for a pattern • make a systematic list • draw or model • eliminate possibilities • simplify the original problem • work backward • develop alternative approaches.	✓ STUDENT RESOURCE p. 52, Investigate the Math, A–C	✓ STUDENT RESOURCE p. 53, Example 1, Your Turn

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Achievement Indicators	Acceptable Standard	Standard of Excellence
2.2 Identify and correct errors in a solution to a puzzle or in a strategy for winning a game.	✓ Identify and correct obvious errors in a solution or strategy.	✓ Identify and correct less obvious errors in a solution or strategy.
2.3 Create a variation on a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.	✓ Create a variation and partially describe a new strategy.	✓ Create a variation and completely describe the winning strategy or solution to the puzzle or game. STUDENT RESOURCE p. 57, #15

Number and Logic (continued)

Specific Outcome

It is expected that students will:

3. Solve problems that involve operations on radicals and radical expressions with numerical and variable radicands (limited to square roots).
[CN, ME, PS, R]

Notes

- Prior knowledge from previous grades/courses includes:
  - simplifying radical expressions with numerical radicands (Mathematics 10C)
  - simplifying like terms in polynomials (Grade 9).
- Variable radicands should be limited to monomials.
- Students are not expected to be able to rationalize radical expressions with binomial denominators.
- Teachers may also wish to explore cube roots, as solving cube root equations is expected in SO4.

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

Achievement Indicators	Acceptable Standard	Standard of Excellence
3.1 Compare and order radical expressions with numerical radicands.	✓ STUDENT RESOURCE p. 183, #12	
3.2 Express an entire radical with a numerical radicand as a mixed radical.	✓ STUDENT RESOURCE p. 182, #4, #5	
3.3 Express a mixed radical with a numerical radicand as an entire radical.	✓ STUDENT RESOURCE p. 183, #11	
3.4 Perform one or more operations to simplify radical expressions with numerical or variable radicands.	✓ Perform operations on radical expressions that involve only numerical radicands. STUDENT RESOURCE p. 198, #5	✓ Perform operations on radical expressions whose radicands contain variables. STUDENT RESOURCE p. 212, #6

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Achievement Indicators	Acceptable Standard	Standard of Excellence
3.5 Rationalize the monomial denominator of a radical expression.	✓ STUDENT RESOURCE p. 199, #13	
3.6 Identify values of the variable for which the radical expression is defined.	✓ STUDENT RESOURCE p. 211, #1	

Number and Logic (continued)

Specific Outcome

It is expected that students will:

4. Solve problems that involve radical equations (limited to square roots or cube roots).
[C, PS, R]

Notes

- Prior knowledge from previous courses includes:
  - factoring polynomials (Mathematics 10C)
  - rational exponents (Mathematics 10C).
- Equations involving cube roots should be limited to the form $\sqrt[3]{ax} = b$.
- Equations involving a variable in the denominator are beyond the scope of this outcome.
- Equations should be limited to a single radical.

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
(It is intended that the equations have only one radical.)

Achievement Indicators	Acceptable Standard	Standard of Excellence
4.1 Determine any restrictions on values for the variable in a radical equation.	✓ STUDENT RESOURCE p. 222, #1	
4.2 Determine, algebraically, the roots of a radical equation, and explain the process used to solve the equation.	✓ Determine the roots of a radical equation and provide a partial explanation of the process. STUDENT RESOURCE p. 222, #2	✓ Determine the roots of a radical equation and provide a complete explanation of the process.
4.3 Verify, by substitution, that the values determined in solving a radical equation are roots of the equation.	✓ STUDENT RESOURCE p. 216, Example 1	

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Achievement Indicators	Acceptable Standard	Standard of Excellence
4.4 Explain why some roots determined in solving a radical equation are extraneous.	✓ Provide an explanation that is limited to verification of extraneous roots by substitution. STUDENT RESOURCE p. 216, Example 1	✓ Provide an explanation that includes restrictions of the variables in the radicand. STUDENT RESOURCE p. 222, #3
4.5 Solve problems by modelling a situation with a radical equation and solving the equation.	✓ Provide a partial solution to a problem. STUDENT RESOURCE p. 224, #14	✓ Provide a complete solution to a problem.

Topic: Statistics

General Outcome: Develop statistical reasoning.

Specific Outcome

It is expected that students will:

1. Demonstrate an understanding of normal distribution, including:
   - standard deviation
   - *z*-scores.

   [CN, PS, T, V]
   [ICT: C6–4.1, C7–4.2]

Notes

- Prior knowledge from previous grades includes:
  – measures of central tendency (Grade 7).

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

Achievement Indicators	Acceptable Standard	Standard of Excellence
1.1 Explain, using examples, the meaning of standard deviation.	✓ STUDENT RESOURCE p. 264, #13	
1.2 Calculate, using technology, the population standard deviation of a data set.	✓ STUDENT RESOURCE p. 261, #2	
1.3 Explain, using examples, the properties of a normal curve, including the mean, median, mode, standard deviation, symmetry and area under the curve.	✓ STUDENT RESOURCE p. 280, #9	
1.4 Determine if a data set approximates a normal distribution, and explain the reasoning.	✓ STUDENT RESOURCE pp. 276–277, Example 4a	

(continued)

(continued)

Achievement Indicators	Acceptable Standard	Standard of Excellence
1.5 Compare the properties of two or more normally distributed data sets.	✓ STUDENT RESOURCE p. 279, #2	
1.6 Explain, using examples representing multiple perspectives, the application of standard deviation for making decisions in situations such as warranties, insurance or opinion polls.	✓ STUDENT RESOURCE pp. 270–271, Example 1	
1.7 Solve a contextual problem that involves the interpretation of standard deviation.	✓ STUDENT RESOURCE p. 262, #8	
1.8 Determine, with or without technology, and explain the z-score for a given value in a normally distributed data set.	✓ STUDENT RESOURCE p. 292, #10	
1.9 Solve a contextual problem that involves normal distribution.	✓ Solve a problem that involves determining a probability, given a data point or a z-score. STUDENT RESOURCE p. 282, #16	✓ Solve a problem that involves determining a data point, given a probability or area under the normal curve. STUDENT RESOURCE p. 294, #20

Statistics (continued)

Specific Outcome

It is expected that students will:

2. Interpret statistical data, using:
   - confidence intervals
   - confidence levels
   - margin of error.

   [C, CN, R]
   [ICT: C1–4.2, C2–4.2, C7–4.2]

Notes

- Prior knowledge from previous grades includes:
  - measures of central tendency (Grade 7)
  - collecting, displaying and analyzing data (Grade 9)
  - making inferences from data (Grade 9).
- Students are not expected to calculate confidence intervals or margins of error. The emphasis of this outcome is intended to be on interpretation rather than statistical calculations.

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
(It is intended that the focus of this outcome be on interpretation of data rather than on statistical calculations.)

Achievement Indicators	Acceptable Standard	Standard of Excellence
2.1 Explain, using examples, how confidence levels, margin of error and confidence intervals may vary depending on the size of the random sample.	✓ STUDENT RESOURCE p. 302, #2	
2.2 Explain, using examples, the significance of a confidence interval, margin of error or confidence level.	✓ STUDENT RESOURCE p. 303, #7	
2.3 Make inferences about a population from sample data, using given confidence intervals, and explain the reasoning.	✓ STUDENT RESOURCE p. 305, #4	
2.4 Provide examples from print or electronic media in which confidence intervals and confidence levels are used to support a particular position.	✓ STUDENT RESOURCE p. 303, #7	
2.5 Interpret and explain confidence intervals and margin of error, using examples found in print or electronic media.	✓ STUDENT RESOURCE p. 302, #3	
2.6 Support a position by analyzing statistical data presented in the media.	✓ Argument is based on a partial analysis of the statistics for the data, such as only considering the mean. STUDENT RESOURCE p. 304, Math in Action	✓ Argument is based on a complete analysis of the data, including all relevant statistics. STUDENT RESOURCE p. 304, Math in Action

Topic: Relations and Functions

General Outcome: Develop algebraic and graphical reasoning through the study of relations.

Specific Outcome

It is expected that students will:

1. Demonstrate an understanding of the characteristics of quadratic functions, including:
   - vertex
   - intercepts
   - domain and range
   - axis of symmetry.

   [CN, PS, T, V]
   [ICT: C6–4.1, C6–4.3]

Notes

- Teachers should make students aware that different forms of the equation of a quadratic function will lead to the same graphical representation. It is intended that completion of the square not be required.
- Prior knowledge from previous grades includes:
  - Domain and range (Mathematics 10C)
  - Intercepts (Mathematics 10C)
  - Factoring quadratic expressions (Mathematics 10C)

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome. (It is intended that completion of the square not be required.)*

Achievement Indicators	Acceptable Standard	Standard of Excellence
1.1 Determine, with or without technology, the coordinates of the vertex of the graph of a quadratic function.	✓ STUDENT RESOURCE p. 332, #1	
1.2 Determine the equation of the axis of symmetry of the graph of a quadratic function, given the x-intercepts of the graph.	✓ STUDENT RESOURCE p. 334, #9	
1.3 Determine the coordinates of the vertex of the graph of a quadratic function, given the equation of the function and the axis of symmetry, and determine if the *y*-coordinate of the vertex is a maximum or a minimum.	✓ STUDENT RESOURCE p. 363, #1	
1.4 Determine the domain and range of a quadratic function.	✓ STUDENT RESOURCE p. 334, #11c	
1.5 Sketch the graph of a quadratic function.	✓ STUDENT RESOURCE p. 329, Example 3	
1.6 Solve a contextual problem that involves the characteristics of a quadratic function.	✓ Solve a problem when the quadratic function and/or its graph are given. STUDENT RESOURCE p. 366, #13	✓ Solve a problem when the quadratic function and its graph are not given. STUDENT RESOURCE p. 367, #18

Relations and Functions (continued)

Specific Outcome

It is expected that students will:

2. Solve problems that involve quadratic equations.
   [C, CN, PS, R, T, V]
   [ICT: C6–4.1, C6–4.3]

Notes

- Prior knowledge from previous courses includes:
  – factoring polynomials (Mathematics 10C).
- Students are not required to identify imaginary roots when solving quadratic equations.
- Students are not expected to derive the quadratic formula; however, teachers may wish to show the derivation to the students so that they understand where it comes from.
- Completing the square is not required for this specific outcome.

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

Achievement Indicators	Acceptable Standard	Standard of Excellence
2.1 Determine, with or without technology, the intercepts of the graph of a quadratic function.	✓ STUDENT RESOURCE p. 346, #4	
2.2 Determine, by factoring, the roots of a quadratic equation, and verify by substitution.	✓ Determine and verify the integral roots of an equation. STUDENT RESOURCE p. 411, #1a, 1b	✓ Determine and verify all rational roots of an equation. STUDENT RESOURCE p. 411, #1c, 1d
2.3 Determine, using the quadratic formula, the roots of a quadratic equation.	✓ Determine the roots in decimal or radical form. STUDENT RESOURCE p. 433, #5	✓ Determine the roots in simplest radical form. STUDENT RESOURCE p. 420, #6a
2.4 Explain the relationships among the roots of an equation, the zeros of the corresponding function and the x-intercepts of the graph of the function.	✓ STUDENT RESOURCE p. 399, Example 2	
2.5 Explain, using examples, why the graph of a quadratic function may have zero, one or two *x*-intercepts.	✓ STUDENT RESOURCE p. 399, Example 2	
2.6 Express a quadratic equation in factored form, given the zeros of the corresponding quadratic function or the *x*-intercepts of the graph of the function.	✓ Determine a possible equation. STUDENT RESOURCE p. 412, #7	✓ Determine a possible equation and include an explanation about why there are many correct equations. STUDENT RESOURCE p. 412, #12
2.7 Solve a contextual problem by modelling a situation with a quadratic equation and solving the equation.	✓ Provide a partial solution to a problem. STUDENT RESOURCE p. 421, #10	✓ Provide a complete solution to a problem.

Topic: Mathematics Research Project

General Outcome: Develop an appreciation of the role of mathematics in society.

Specific Outcome

It is expected that students will:

1. Research and give a presentation on a historical event or an area of interest that involves mathematics.

   [C, CN, ME, PS, R, T, V]
   [ICT: C1–4.2, C1–4.4, C2–4.1, C3–4.1, C3–4.2, C7–4.2, F2–4.7]

Notes

- Data collected may be numerical data or informational data.
- Teachers may wish to discuss the difference between primary data and secondary data.
- Statistics Canada is a resource for data. Other sources of data include sports, media, weather, financial markets, etc.
- Cross-curricular projects, such as population growth in social studies, are possible.
- It is the responsibility of the teacher to set criteria by which Acceptable Standard can be distinguished from Standard of Excellence. These criteria may vary depending on the question or topic presented.

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

Achievement Indicators	Acceptable Standard	Standard of Excellence
1.1 Collect primary or secondary data (statistical or informational) related to the topic.	✓	✓
1.2 Assess the accuracy, reliability and relevance of the primary or secondary data collected by: • identifying examples of bias and points of view • identifying and describing the data collection methods • determining if the data is relevant • determining if the data is consistent with information obtained from other sources on the same topic.	✓	✓
1.3 Interpret data, using statistical methods if applicable.	✓	✓
1.4 Identify controversial issues, if any, and present multiple sides of the issues with supporting data.	✓	✓
1.5 Organize and present the research project, with or without technology.	✓	✓

Appendix: Mathematics Directing Words

Discuss	The word "discuss" **will not** be used as a directing word on mathematics examinations because it is not used consistently to mean a single activity.
	The following words are specific in meaning.
Algebraically	Use mathematical procedures that involve letters or symbols to represent numbers.
Analyze	Make a mathematical or methodical examination of parts to determine aspects of the whole; e.g., nature, proportion, function, interrelationship.
Compare	Examine the character or qualities of two things by providing characteristics of both that point out their mutual similarities and differences.
Conclude	State a logical end based on reasoning and/or evidence.
Contrast/Distinguish	Point out the differences between two things that have similar or comparable natures.
Criticize	Point out the merits and demerits of an item or issue.
Define	Provide the essential qualities or meaning of a word or concept; make distinct and clear by marking out the limits.
Describe	Give a written account or represent the characteristics of something, using a figure, model or picture.
Design/Plan	Construct a plan, i.e., a detailed sequence of actions, for a specific purpose.
Determine	Find a solution, to a specified degree of accuracy, to a problem by showing appropriate formulas, procedures and calculations.
Enumerate	Specify one-by-one or list in a concise form and according to some order.
Evaluate	Give the significance or worth of something by identifying the good and bad points or the advantages and disadvantages.
Explain	Make clear what is not immediately obvious or entirely known; give the cause of or reason for; make known in detail.
Graphically	Use a drawing that is produced electronically or by hand and that shows a relation between certain sets of numbers.

How	Show in what manner or way, with what meaning.
Hypothesize	Form a tentative proposition intended as a possible explanation for an observed phenomenon; i.e., a possible cause for a specific effect. The proposition should be testable logically and/or empirically.
Identify	Recognize and select as having the characteristics of something.
Illustrate	Make clear by providing an example. The form of the example must be specified in the question; i.e., word description, sketch or diagram.
Infer	Form a generalization from sample data; arrive at a conclusion by reasoning from evidence.
Interpret	State the meaning of something; present information in a new form that adds meaning to the original data.
Justify/Show How	Show reasons for or give facts that support a position.
Model	Find a model (in mathematics, a model of a situation is a pattern that is supposed to represent or set a standard for a real situation) that does a good job of representing a situation.
Outline	Give, in an organized fashion, the essential parts of something. The form of the outline must be specified in the question; i.e., lists, flowcharts, concept maps.
Predict	State in advance on the basis of empirical evidence and/or logic.
Prove	Establish the truth or validity of a statement for the general case by providing factual evidence or a logical argument.
Relate	Show a logical or causal connection between things.
Sketch	Provide a drawing that represents the key features of an object or a graph.
Solve	Give a solution for a problem; i.e., explanation in words and/or numbers.
Summarize	Give a brief account of the main points.
Trace	Give a step-by-step description of the development.
Verify	Establish, by substitution for a particular case or by geometric comparison, the truth of a statement.
Why	Show the cause, reason or purpose.