Outcomes with Assessment Standards
for
Mathematics 20-3

2013

This resource is intended to assist teachers with the
provincial implementation of Mathematics 20-3.

For further information, contact:

Alberta Education
Programs of Study and Resources Sector: Mathematics, Arts and Communication
8th Floor, 44 Capital Boulevard
10044 – 108 Street NW
Edmonton, Alberta T5J 5E6
Telephone: 780–427–2984 in Edmonton or
toll-free in Alberta by dialling 310–0000
Fax: 780–422–3745

The primary audience for this resource is:

Teachers	✓
Administrators	
Students	
Parents	

ISBN 978–1–4601–1128–4 (PDF)

Acknowledgements

This resource was developed as a joint project of Alberta classroom teachers and staff at Alberta Education. The cooperation of the Alberta Teachers' Association, the Alberta Assessment Consortium and the following school jurisdictions is greatly appreciated.

Calgary Roman Catholic Separate School District No. 1
Edmonton School District No. 7
Elk Island Catholic Separate Regional Division No. 41
Fort McMurray Roman Catholic Separate School District No. 32
Greater North Central Francophone Education Region No. 2
Horizon School Division No. 67
Lethbridge School District No. 51
Northland School Division No. 61
Pembina Hills Regional Division No. 7
Red Deer Catholic Regional Division No. 39
Rocky View School Division No. 41
St. Albert Public School District No. 5565
Wild Rose School Division No. 66
Wolf Creek School Division No. 72

The Alberta Education team members were from the Programs of Study and Resources Sector, the Assessment Sector and the French and International Education Services Sector.

Table of Contents

INTRODUCTION

Mathematics 20-3 was provincially implemented in September 2011. Teachers participating in focus groups during the development of the program of studies expressed a need for a common understanding of the curriculum and assessment standards. In response to this need, and in keeping with Alberta Education's goal of establishing and effectively communicating clear outcomes and high standards, this standards resource was developed.

This resource is designed to support the implementation of the Alberta Mathematics Grades 10–12 Program of Studies, which can be found at http://education.alberta.ca/media/655889/math10to12.pdf. Teachers are strongly encouraged to consult the program of studies for details about the philosophy of the program.

PURPOSE

Outcomes with Assessment Standards for Mathematics 20-3 links the achievement indicators for the specific outcomes from the program of studies with information and commentaries about standards. Its purpose is to provide teachers of Mathematics 20-3 with clearly stated standards to use as guidelines in their classroom instruction and assessment practices.

DEFINITIONS AND TERMINOLOGY

Standards

A *standard* is a reference point used in planning and evaluation. In evaluating educational performance, the following standards apply:

- *Curriculum* and *assessment standards* apply to the assessment of individual students.
- *Achievement standards* apply to the assessment of student populations.

In this resource, only curriculum and assessment standards are discussed.

Curriculum Standards

Curriculum standards are outcomes for a course within a program. The curriculum standards for Mathematics 20-3 are defined by the general and specific outcomes outlined in the program of studies. They are further clarified by the achievement indicators, which reflect the scope of each specific outcome.

Outcomes

General outcomes are concise statements identifying what it is that students are expected to know and be able to do upon completion of a course within a program.

Specific outcomes are statements identifying the component knowledge, skills and attitudes of a general outcome. Specific outcomes identify a range of contexts in which the general outcomes apply.

In the specific outcomes, the word *including* indicates that any ensuing items must be addressed to fully meet the learning outcome. The phrase *such as* indicates that the ensuing items are provided for clarification and are not requirements that must be addressed to fully meet the learning outcome.

The word *and* used in an outcome indicates that both ideas must be addressed to fully meet the learning outcome, although not necessarily at the same time or in the same question.

Achievement Indicators

Achievement indicators are samples of how students may demonstrate their achievement of the goals of a specific outcome. The range of samples provided is meant to reflect the scope of the specific outcome.

The word *and* used in an achievement indicator implies that both ideas should be addressed at the same time or in the same question.

Assessment Standards

Assessment standards are the criteria used for judging individual student achievement relative to the curriculum standards.

STANDARDS FOR MATHEMATICS 20-3

Mathematics 20-3 is designed to follow directly from Mathematics 10-3, so students taking Mathematics 20-3 are presumed to have reached the acceptable standard or better in the outcomes of Mathematics 10-3.

The assessment standards for Mathematics 20-3 include an acceptable and an excellent level of performance. Student performance should be measured on a range of tasks, some of which are routine and obvious tasks in familiar contexts, and others which are nonroutine tasks in unfamiliar contexts. In many cases, a correlated example from the authorized resources is referenced to assist in assessing student performance. The authorized resources for Mathematics 20-3, published by Pacific Educational Press, are:

- MathWorks 11: Student Resource
- MathWorks 11: Teacher Resource.

Acceptable Standard

The *acceptable standard* of achievement in Mathematics 20-3 is met by students who receive a course mark between and including 50 percent and 79 percent. Typically, these students have gained new skills and a basic knowledge of the concepts and procedures relative to the general and specific outcomes defined for Mathematics 20-3 in the program of studies. These students can apply this knowledge to a limited range of familiar problem contexts.

Standard of Excellence

The *standard of excellence* for achievement in Mathematics 20-3 is met by students who receive a course mark at or above 80 percent. Typically, these students have gained a breadth and depth of understanding regarding the concepts and procedures, as well as the ability to apply this knowledge to a broad range of familiar and unfamiliar problem contexts.

Description of Standards

The following statements describe what is expected of Mathematics 20-3 students who meet the *acceptable standard* or the *standard of excellence* on independent work. The statements represent the standards against which student achievement is measured.

Acceptable Standard	***Standard of Excellence***
Students who meet the *acceptable standard* in Mathematics 20-3 consistently perform acceptable work on routine and obvious tasks in familiar contexts.	Students who meet the *standard of excellence* in Mathematics 20-3 consistently perform excellent work on routine and obvious tasks in familiar contexts, and acceptable work on nonroutine tasks in unfamiliar contexts.
These students have a basic understanding of the concepts and procedures outlined in the program of studies. They demonstrate their understanding in concrete, pictorial and symbolic modes, and can translate from one mode to another. They perform the mathematical operations and procedures that are fundamental to Mathematics 20-3 and apply what they know in daily living contexts.	These students have a comprehensive understanding of the concepts and procedures outlined in the program of studies. They demonstrate their understanding in concrete, pictorial and symbolic modes, and can translate from one mode to another. They perform the mathematical operations and procedures that are fundamental to Mathematics 20-3, apply what they know in daily living contexts and provide alternative solution procedures to verify results.
To meet the *acceptable standard*, students communicate about mathematical situations in an understandable way, using appropriate everyday and mathematical terms. They understand mathematical questions containing objects, diagrams or numbers in familiar contexts, and they construct mathematical models.	To meet the *standard of excellence*, students communicate about mathematical situations in a clear way, using numbers, diagrams and appropriate mathematical terms. They understand mathematical questions containing objects, diagrams or numbers in familiar and unfamiliar contexts, and they construct mathematical models using multiple representations.
Students meeting the *acceptable standard* apply what they know in solving straightforward problems in familiar settings and in analyzing simple mathematical models. They describe the steps they used to solve a particular problem, and verify and defend their solution to the problem.	Students meeting the *standard of excellence* apply what they know in solving routine and nonroutine problems in a broad range of settings. They describe the steps they used to solve a particular problem, defend their solution to the problem, and, where appropriate, provide alternative solution procedures to verify results.
Students meeting the *acceptable standard* have a positive attitude toward mathematics and a sense of personal competence in using mathematics. They demonstrate confidence when using common mathematical procedures and when applying problem-solving strategies in familiar settings.	Students meeting the *standard of excellence* have a positive attitude toward mathematics and show confidence in using mathematics meaningfully. They are self-motivated risk takers who persevere when solving novel problems. They take initiative in trying new methods and are creative in their approach to problem solving.

GENERAL NOTES

- All mathematical processes should be used and integrated throughout the outcomes.
- Teachers should try to provide students with illustrative examples and contextual problems that are representative of the trades.
- In problems that involve multiple steps, complete solutions include carrying full answers until the final solution.
- Technology [T], including calculators and computers, has been listed as one of the mathematical processes to be emphasized for some outcomes, with the expectation that students will have access to technology when completing the outcomes. If technology has not been specifically listed for a particular outcome, teachers may, at their discretion, use it to assist students in exploring patterns and relationships when learning a concept. It is expected, however, that technology will not be considered when assessing students' understandings of such outcomes.
- Most specific outcomes are accompanied by notes that address some of the questions that may arise when teaching the concepts. The assessment standards for each outcome are described in a chart that indicates, for each achievement indicator, whether the acceptable standard, the standard of excellence or, in some cases, both standards may be applicable (✓). Some check marks are accompanied by qualifying statements. Shaded regions indicate that the standard does not apply for the given achievement indicator. In many cases, a correlated example from the authorized resources is referenced in the chart to illustrate the standards.
- A *partial solution* to a problem is a solution in which a student demonstrates a basic understanding of the problem and the mathematical concepts required in solving the problem. However, the student is unable to complete the solution correctly for a variety of reasons, such as not being able to correctly connect the concepts involved or not being able to avoid procedural errors. For example, in solving a problem using right-angle trigonometry, given an angle and the measure of the side opposite that angle, a student may be able to draw a diagram to correctly represent the situation and identify the appropriate trigonometric ratio needed to solve the problem but then makes procedural errors in solving for the measure of the side adjacent. Note that assessment of student learning is the responsibility of the teacher, and what is considered a partial solution may vary according to the question or task presented.

Topic: Measurement

General Outcome: Develop spatial sense through direct and indirect measurement.

Specific Outcome

It is expected that students will:

1. Solve problems that involve SI and imperial units in surface area measurements and verify the solutions.
   [C, CN, ME, PS, V]

Notes

- Students have been working with SI units of measure since Grade 3. Imperial units were first introduced in Mathematics 10-3 and Mathematics 10C.
- Measurements in the imperial system are generally expressed in fraction form, while measurements in the SI system are expressed in decimal form.
- Denominators for fractional measurements should be contextually appropriate; e.g., halves, quarters, eighths, sixteenths.
- Students are expected to be able to apply the formulas for calculating surface area; memorization of formulas is not required.
- Proportional reasoning allows an individual to solve for an unknown value by comparing the unknown value to known values, using ratios and proportions. Students are expected to use proportional reasoning when converting units within and between the two measurement systems.
- Technology [T] has not been identified as one of the mathematical processes to be emphasized; however, where appropriate, technology may be used to solve problems.
- Students are not expected to memorize long lists of conversion factors. Basic conversion factors, especially those between the SI and imperial systems of measure, shall be provided.
- Formula manipulations that involve factoring, such as solving for r in the formula $SA = \pi r^2 + 2\pi rh$, is beyond the scope of this course.
- In Achievement Indicator 1.4, restrict estimations of surface area to simple polyhedra such as prisms, pyramids, spheres and right circular cylinders. The use of nets may be beneficial.
- Estimates should include surface areas relevant to common trades; e.g., surface areas of composite 3-D objects like houses.

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

Achievement Indicators	Acceptable Standard	Standard of Excellence
1.1 Explain, using examples, the difference between volume and surface area.	✓	
1.2 Explain, using examples, including nets, the relationship between area and surface area.	✓ Use given examples to explain the relationship. STUDENT RESOURCE pp. 116–117, Explore the Math	✓ Create examples to explain the relationship. STUDENT RESOURCE pp. 122–123, Example 3
1.3 Explain how a referent can be used to estimate surface area.	✓	
1.4 Estimate the surface area of a 3-D object.	✓ Estimate the surface area of a simple 3-D object. STUDENT RESOURCE p. 126, #6	✓ Estimate the surface area of composite 3-D objects. STUDENT RESOURCE pp. 134–135, Build Your Skills #2
1.5 Illustrate, using examples, the effect of dimensional changes on surface area.	✓ Describe the effect on simple polyhedra of changing one or more dimensions by a single factor. STUDENT RESOURCE p. 122, #2	✓ Describe the effect on simple polyhedra of changing two or more dimensions by different factors. STUDENT RESOURCE p. 140, Example 1
1.6 Solve a contextual problem that involves the surface area of 3-D objects, including spheres, and that requires the manipulation of formulas.	✓ Calculate the surface area of simple 3-D objects. STUDENT RESOURCE p. 130, Example 2 ✓ Calculate the missing dimension of a simple 3-D object, given the surface area. STUDENT RESOURCE p. 128, #2	✓ Calculate the surface area of composite 3-D objects. STUDENT RESOURCE p. 130, Activity 3.3 ✓ Calculate the surface area of a 3-D object that requires the manipulation of a formula. STUDENT RESOURCE p. 135, #6b

Measurement (continued)

Specific Outcome

It is expected that students will:

2. Solve problems that involve SI and imperial units in volume and capacity measurements.
   [C, CN, ME, PS, V]

Notes

- Students have been working with SI units of measure since Grade 3. Imperial units were first introduced in Mathematics 10-3 and Mathematics 10C.
- Measurements in the imperial system are expressed in fraction form, while measurements in the SI system are expressed in decimal form.
- When appropriate, students are expected to perform the four arithmetic operations on decimals and fractions without the use of technology.
- Denominators for fractional measurements should be contextually appropriate; e.g., halves, quarters, eighths, sixteenths, thirty-seconds and sixty-fourths.
- Students are expected to be able to apply the formulas for calculating volume and capacity; memorization of formulas is not required.
- Proportional reasoning allows an individual to solve for an unknown value by comparing the unknown value to known values, using ratios and proportions. Students are expected to use proportional reasoning when converting units within and between the two measurement systems.
- Technology [T] has not been identified as one of the mathematical processes to be emphasized in completing this outcome. Conversions between systems should not be completed by entering data into a conversion program. Technology may be used, where appropriate, in solving a proportion.
- Students are not expected to memorize long lists of conversion factors. Basic conversion factors, especially those between the SI and imperial systems of measure, shall be provided.
- Volume refers to the amount of space an object occupies at any given time. Capacity refers to the maximum quantity a container or receptacle can hold or carry. The volume of a container would generally be measured in units such as cm^3 and m^3, whereas capacity would be measured in mL or L.
- The use of imperial units should be limited to typical units used in trades.
- Students may have experience with unit analysis, also known as dimensional analysis, in previous science courses.
- In Achievement Indicator 2.11, students are asked only to describe the relationship. No algebraic proof is expected.

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

Achievement Indicators	Acceptable Standard	Standard of Excellence
2.1 Explain, using examples, the difference between volume and capacity.	✓ STUDENT RESOURCE p. 140, Example 1d	
2.2 Identify and compare referents for volume and capacity measurements in SI and imperial units.	✓	
2.3 Estimate the volume or capacity of a 3-D object or container, using a referent.	✓ STUDENT RESOURCE pp. 138–139, Explore the Math	
2.4 Identify a situation where a given SI or imperial volume unit would be used.	✓ STUDENT RESOURCE p. 141, Example 2	
2.5 Solve problems that involve the volume of 3-D objects and composite 3-D objects in a variety of contexts.	✓ Solve problems involving 3-D objects. STUDENT RESOURCE p. 150, Example 1	✓ Solve problems involving composite 3-D objects. STUDENT RESOURCE pp. 154–155, Example 4
2.6 Solve a problem that involves the capacity of containers.	✓ Solve problems involving 3-D objects. STUDENT RESOURCE p. 154, Example 3	✓ Solve problems involving composite 3-D objects. STUDENT RESOURCE p. 160, #3
2.7 Write a given volume measurement expressed in one SI unit cubed in another SI unit cubed.	✓ STUDENT RESOURCE p. 144, #2	
2.8 Write a given volume measurement expressed in one imperial unit cubed in another imperial unit cubed.	✓ STUDENT RESOURCE pp. 154–155, Example 4a	

(continued)

(continued)

Achievement Indicators	Acceptable Standard	Standard of Excellence
2.9 Determine the volume of prisms, cones, cylinders, pyramids, spheres and composite 3-D objects, using a variety of measuring tools such as rulers, tape measures, calipers and micrometers.	✓ Determine the volume of 3-D objects. STUDENT RESOURCE p. 150, Example 1a	✓ Determine the volume of composite 3-D objects. STUDENT RESOURCE p. 156, #1
2.10 Determine the capacity of prisms, cones, pyramids, spheres and cylinders, using a variety of measuring tools and methods, such as graduated cylinders, measuring cups, measuring spoons and displacement.	✓ Determine the capacity of 3-D objects. STUDENT RESOURCE p. 154, Example 3	✓ Determine the capacity of composite 3-D objects.
2.11 Describe the relationship between the volumes of: • cones and cylinders with the same base and height • pyramids and prisms with the same base and height.	✓	
2.12 Illustrate, using examples, the effect of dimensional changes on volume.	✓ STUDENT RESOURCE p. 140, Example 1b, c	
2.13 Solve a contextual problem that involves the volume of a 3-D object, including composite 3-D objects, or the capacity of a container.	✓ Solve a problem that involves the volume or capacity of simple 3-D objects. STUDENT RESOURCE p. 160, #2	✓ Solve a problem that involves the volume or capacity of composite 3-D objects. STUDENT RESOURCE p. 160, #3
2.14 Solve a contextual problem that involves the volume of a 3-D object and requires the manipulation of formulas.	✓ Provide a partial solution. STUDENT RESOURCE p. 158, #7c	✓ Provide a complete solution. STUDENT RESOURCE p. 158, #7c

Topic: Geometry

General Outcome: Develop spatial sense.

Specific Outcome

It is expected that students will:

1. Solve problems that involve two and three right triangles.
   [CN, PS, T, V]
   [ICT: C6–4.1]

Notes

- Providing opportunities for students to work with manipulatives and concrete materials connects mathematical ideas to the real world.
- Students are expected to create sketches to visualize information and assist in problem solving.
- In Achievement Indicator 1.5, single right triangle trigonometry problems have been solved in Mathematics 10-3.

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

Achievement Indicators	Acceptable Standard	Standard of Excellence
1.1 Identify all of the right triangles in a given illustration for a context.	✓ STUDENT RESOURCE p. 174, Example 3	
1.2 Determine if a solution to a problem that involves two or three right triangles is reasonable.	✓ STUDENT RESOURCE p. 177, Build Your Skills #2	
1.3 Sketch a representation of a given description of a problem in a 2-D or 3-D context.	✓ Sketch 2-D representations. STUDENT RESOURCE p. 196, #3	✓ Sketch 3-D representations. STUDENT RESOURCE p. 197, #6
1.4 Solve a contextual problem that involves angles of elevation or angles of depression.	✓ Solve 2-D problems. STUDENT RESOURCE p. 191, Example 3	✓ Solve 3-D problems. STUDENT RESOURCE p. 197, #5

(continued)

(continued)

Achievement Indicators	Acceptable Standard	Standard of Excellence
1.5 Solve a contextual problem that involves two or three right triangles, using the primary trigonometric ratios.	✓ Solve problems that involve two right triangles in 2-D. STUDENT RESOURCE p. 178, #5	✓ Solve problems that involve three right triangles in 2-D. STUDENT RESOURCE p. 199, #9 ✓ Solve problems that involve two or three right triangles in 3-D. STUDENT RESOURCE p. 198, #8

Geometry (continued)

Specific Outcome

It is expected that students will:

2. Solve problems that involve scale. [PS, R, V]

Notes

- Providing opportunities for students to work with manipulatives and concrete materials connects mathematical ideas to the real world.
- Prior knowledge includes drawing and interpreting scale diagrams in two dimensions (Grade 9).
- Achievement Indicator 2.3 is limited to simple polyhedra.

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

Achievement Indicators	Acceptable Standard	Standard of Excellence
2.1 Describe contexts in which a scale representation is used.	✓ STUDENT RESOURCE p. 215, #1, #2	
2.2 Determine, using proportional reasoning, the dimensions of an object from a given scale drawing or model.	✓ STUDENT RESOURCE p. 212, Example 2	
2.3 Construct a model of a 3-D object, given the scale.	✓ STUDENT RESOURCE p. 214, Activity 5.1, Part 2	
2.4 Draw, with and without technology, a scale diagram of a given object.	✓ Draw scale diagrams of simple polygons. STUDENT RESOURCE p. 214, Activity 5.1, Part 1	✓ Draw scale diagrams of composite shapes that include triangles, non-right angles and/or a semicircle. STUDENT RESOURCE p. 218, Math on the Job #2
2.5 Solve a contextual problem that involves scale.	✓ STUDENT RESOURCE p. 217, #8	

Geometry (continued)

Specific Outcome

It is expected that students will:

3. Model and draw 3-D objects and their views.
   [CN, R, V]

Notes

- Providing opportunities for students to work with manipulatives and concrete materials connects mathematical ideas to the real world.
- Achievement Indicator 3.3 may include completing a scaled isometric drawing.
- Students should spend time constructing 3-D objects with concrete materials.
- When students create a one-point perspective, the placement of the horizon line, the location of the vanishing point on the horizon line and the depth of the object represented do not affect the integrity of the drawing.

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

Achievement Indicators	Acceptable Standard	Standard of Excellence
3.1 Draw a 2-D representation of a given 3-D object.	✓ STUDENT RESOURCE p. 221, Example 2	
3.2 Draw, using isometric dot paper, a given 3-D object.	✓ Draw simple 3-D objects. STUDENT RESOURCE pp. 234–237, Example 1	✓ Draw complex 3-D objects. STUDENT RESOURCE p. 248, #3b
3.3 Draw to scale top, front and side views of a given 3-D object.	✓ Draw views of simple 3-D objects. STUDENT RESOURCE p. 223, Activity 5.2, Part 1	✓ Draw views of complex 3-D objects (may include holes or hidden parts). STUDENT RESOURCE p. 228, #5
3.4 Construct a model of a 3-D object, given the top, front and views.	✓ Construct right prisms.	✓ Construct models of objects with layers and/or hidden lines. STUDENT RESOURCE p. 224, Activity 5.3, Part 2
3.5 Draw a 3-D object, given the top, front and side views.	✓ Draw right prisms. STUDENT RESOURCE p. 229, #7a	✓ Draw objects with layers and/or hidden lines. STUDENT RESOURCE p. 248, #3
3.6 Determine if given views of a 3-D object represent a given object, and explain the reasoning.	✓ Determine if the views represent a given object, and provide a partial explanation of the reasoning. STUDENT RESOURCE p. 226, #2	✓ Determine if the views represent a given object, and provide a complete explanation of the reasoning. STUDENT RESOURCE p. 226, #2

(continued)

(continued)

Achievement Indicators	Acceptable Standard	Standard of Excellence
3.7 Identify the point of perspective of a given one-point perspective drawing of a 3-D object.	✓ STUDENT RESOURCE p. 244, #1	
3.8 Draw a one-point perspective view of a given 3-D object.		✓ STUDENT RESOURCE p. 244, #2

Geometry (continued)

Specific Outcome

It is expected that students will:

4. Draw and describe exploded views, component parts and scale diagrams of simple 3-D objects.
   [CN, V]

Notes

- Providing opportunities for students to work with manipulatives and concrete materials connects mathematical ideas to the real world.
- When explaining the relationship between an original 3-D object and its exploded diagram, students should include a description of elements that may be hidden from view.
- Exploded views should be limited to two or three components.
- It is intended that the simple 3-D objects come from contexts such as flat-packed furniture or sewing patterns.

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
(It is intended that the simple 3-D objects come from contexts such as flat-packed furniture or sewing patterns.)

Achievement Indicators	Acceptable Standard	Standard of Excellence
4.1 Draw the components of a given exploded diagram, and explain their relationship to the original 3-D object.	✓ STUDENT RESOURCE p. 249, #4a	
4.2 Sketch an exploded view of a 3-D object to represent the components.	✓ Sketch an object that has no hidden surfaces. STUDENT RESOURCE p. 241, Example 3	✓ Sketch an object that has hidden surfaces. STUDENT RESOURCE p. 246, #6c
4.3 Draw to scale the components of a 3-D object.		✓ STUDENT RESOURCE p. 246, #6a
4.4 Sketch a 2-D representation of a 3-D object, given its exploded view.	✓ Sketch a 2-D representation of a 3-D object, given its exploded view. STUDENT RESOURCE p. 245, #4	✓ Sketch a 2-D representation of a 3-D object, given its exploded view that has hidden surfaces. STUDENT RESOURCE p. 245, #5

Topic: Number

General Outcome: Develop number sense and critical thinking skills.

Specific Outcome

It is expected that students will:

1. Analyze puzzles and games that involve numerical reasoning, using problem-solving strategies.
   [C, CN, PS, R]

Notes

- Verifying a strategy to solve a puzzle or game could include successfully using the strategy.
- The incorporation of games involving money sense will support student achievement of other outcomes for this topic.
- It is expected that students will be exposed to a wide variety of games that involve different skills and that can be played alone, in pairs or in groups.

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
(It is intended that this outcome be integrated throughout the course by using puzzles and games such as cribbage, magic squares and Kakuro.)

Achievement Indicators	Acceptable Standard	Standard of Excellence
1.1 Determine, explain and verify a strategy to solve a puzzle or to win a game; e.g., • guess and check • look for a pattern • make a systematic list • draw or model • eliminate possibilities • simplify the original problem • work backward • develop alternative approaches.	✓ Determine and verify a strategy, and provide a partial explanation. STUDENT RESOURCE p. 17, Puzzle It Out (draw or model) p. 101, Puzzle It Out p. 147, Puzzle It Out (guess and check, systematic list) p. 184, Puzzle It Out p. 225, Puzzle It Out (draw or model) p. 287, Puzzle It Out (eliminate possibilities) p. 337, Puzzle It Out	✓ Determine and verify a strategy, and provide a complete explanation. STUDENT RESOURCE p. 17, Puzzle It Out (draw or model) p. 101, Puzzle It Out p. 147, Puzzle It Out (guess and check, systematic list) p. 184, Puzzle It Out p. 225, Puzzle It Out (draw or model) p. 287, Puzzle It Out (eliminate possibilities) p. 337, Puzzle It Out
1.2 Identify and correct errors in a solution to a puzzle or in a strategy for winning a game.	✓ Identify errors.	✓ Identify and correct errors.
1.3 Create a variation on a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.		✓

Number (continued)

Specific Outcome

It is expected that students will:

2. Solve problems that involve personal budgets.
   [CN, PS, R, T]
   [ICT: C6–4.2, C6–4.4]

Notes

- When modifying budgets, students are expected to investigate both an increase and a decrease in income.
- Illustrative examples and contextual problems should use current and relevant income and expense data.

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

Achievement Indicators	Acceptable Standard	Standard of Excellence
2.1 Identify income and expenses that should be included in a personal budget.	✓ STUDENT RESOURCE p. 314, #4	
2.2 Explain considerations that must be made when developing a budget; e.g., prioritizing, recurring and unexpected expenses.	✓ STUDENT RESOURCE p. 309, #1	
2.3 Create a personal budget based on given income and expense data.	✓ STUDENT RESOURCE p. 321, Build Your Skills #2	
2.4 Collect income and expense data, and create a budget.	✓ STUDENT RESOURCE p. 315, Example 1c	
2.5 Modify a budget to achieve a set of personal goals.	✓ STUDENT RESOURCE p. 324, #8	
2.6 Investigate and analyze, with or without technology, “what if …” questions related to personal budgets.	✓ STUDENT RESOURCE p. 311, #6c	

Number (continued)

Specific Outcome

It is expected that students will:

3. Demonstrate an understanding of compound interest.
   [CN, ME, PS, T]
   [ICT: C6–4.1]

Notes

- Students are not expected to memorize the simple interest or compound interest formulas.
- In addition to the formulas, calculators (TVM Solver), tables, spreadsheets and websites may be used to enhance teaching and learning of this outcome.
- In Achievement Indicator 3.3, solving for the interest per compounding period (i) or the number of compounding periods (n) is outside the scope of this course.

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

Achievement Indicators	Acceptable Standard	Standard of Excellence
3.1 Solve a problem that involves simple interest, given three of the four values in the formula $I=Prt$.	✓ Determine the value of I. STUDENT RESOURCE p. 272, Build Your Skills #1	✓ Determine the value of P, r or t. STUDENT RESOURCE pp. 272–273, #2
3.2 Compare simple and compound interest, and explain their relationship.	✓ STUDENT RESOURCE p. 269, Activity 6.4	
3.3 Solve, using a formula, a contextual problem that involves compound interest.	✓ Determine the accumulated value, given the principal amount. STUDENT RESOURCE p. 273, #5	✓ Determine the principal amount, given the accumulated value.
3.4 Explain, using examples, the effect of different compounding periods on calculations of compound interest.	✓ STUDENT RESOURCE p. 273, #7	
3.5 Estimate, using the Rule of 72, the time required for a given investment to double in value.	✓ STUDENT RESOURCE p. 273, #4	

Number (continued)

Specific Outcome

It is expected that students will:

4. Demonstrate an understanding of financial institution services used to access and manage finances.
   [C, CN, R, T]
   [ICT: F2–4.6]

Notes

- A project that incorporates many of the achievement indicators would allow students to meet the specific outcome.
- Accessing the financial expertise of a banking official would connect students to the real world and make mathematical ideas more meaningful.

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

Achievement Indicators	Acceptable Standard	Standard of Excellence
4.1 Describe the type of banking services available from various financial institutions, such as online services.	✓ STUDENT RESOURCE p. 258, Activity 6.1	
4.2 Describe the types of accounts available at various financial institutions.	✓ STUDENT RESOURCE p. 258, Activity 6.1	
4.3 Identify the type of account that best meets the needs for a given set of criteria.	✓ STUDENT RESOURCE p. 262, Build Your Skills #2	
4.4 Identify and explain various automated teller machine (ATM) service charges.	✓ STUDENT RESOURCE pp. 258–259, Activity 6.2	
4.5 Describe the advantages and disadvantages of online banking.	✓ STUDENT RESOURCE p. 258, Activity 6.1, #2	

(continued)

(continued)

Achievement Indicators	Acceptable Standard	Standard of Excellence
4.6 Describe the advantages and disadvantages of debit card purchases.	✓ STUDENT RESOURCE p. 259, Activity 6.2, #2	
4.7 Describe ways that ensure the security of personal and financial information; e.g., passwords, encryption, protection of personal identification number (PIN) and other personal identity information.	✓ STUDENT RESOURCE p. 263, #4	

Number (continued)

Specific Outcome

It is expected that students will:

5. Demonstrate an understanding of credit options, including:
   - credit cards
   - loans.

   [CN, ME, PS, R]
   [ICT: F2–4.7]

Notes

- Specific Outcome 5 may be taught in conjunction with Specific Outcome 4.
- Accessing the financial expertise of a banking official would connect students to the real world and make mathematical ideas more meaningful.

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

Achievement Indicators	Acceptable Standard	Standard of Excellence
5.1 Compare advantages and disadvantages of different types of credit options, including bank and store credit cards, personal loans, lines of credit, overdraft.	✓ STUDENT RESOURCE p. 285, #7	
5.2 Make informed decisions and plans related to the use of credit, such as service charges, interest, payday loans and sales promotions, and explain the reasoning.	✓ Make informed decisions and plans. STUDENT RESOURCE p. 283, Activity 6.9, #1, #2	✓ Explain reasoning. STUDENT RESOURCE p. 283, Activity 6.9, #3
5.3 Describe strategies to use credit effectively, such as negotiating interest rates, planning payment timelines, reducing accumulated debt and timing purchases.	✓ STUDENT RESOURCE p. 294, Activity 6.11	
5.4 Compare credit card options from various companies and financial institutions.	✓ STUDENT RESOURCE p. 285, #7	
5.5 Solve a contextual problem that involves credit cards or loans.	✓ Solve one-step problems. STUDENT RESOURCE p. 295, #1	✓ Solve multistep problems. STUDENT RESOURCE p. 295, #3

(continued)

(continued)

Achievement Indicators	Acceptable Standard	Standard of Excellence
5.6 Solve a contextual problem that involves credit linked to sales promotions.	✓ Solve one-step problems. STUDENT RESOURCE p. 282, Example 3a	✓ Solve multistep problems. STUDENT RESOURCE p. 296, #7

Topic: Algebra

General Outcome: Develop algebraic reasoning.

Specific Outcome

It is expected that students will:

1. Solve problems that require the manipulation and application of formulas related to:
   - volume and capacity
   - surface area
   - slope and rate of change.
   - simple interest
   - finance charges.

   [CN, PS, R]

Notes

- Integration of this specific outcome into the Measurement and Number topics of the course is recommended.
- In Achievement Indicators 1.2 and 1.3, formula manipulations are limited to a variable that appears in one term only. For example, factoring for r in the formula $SA = \pi r^2 + 2\pi rh$ is beyond the scope of this course.

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

Achievement Indicators	Acceptable Standard	Standard of Excellence
1.1 Solve a contextual problem involving the application of a formula that does not require manipulation.	✓ STUDENT RESOURCE p. 266, Example 2a	
1.2 Solve a contextual problem involving the application of a formula that requires manipulation.	✓ Provide a partial solution. STUDENT RESOURCE p. 135, #6	✓ Provide a complete solution. STUDENT RESOURCE p. 135, #6
1.3 Explain and verify why different forms of the same formula are equivalent.	✓ Verify equivalency with a partial explanation. STUDENT RESOURCE p. 129, Example 1	✓ Verify equivalency with a complete explanation. STUDENT RESOURCE p. 129, Example 1
1.4 Describe, using examples, how a given formula is used in a trade or an occupation.	✓ STUDENT RESOURCE p. 138, Math on the Job	

(continued)

(continued)

Achievement Indicators	Acceptable Standard	Standard of Excellence
1.5 Create and solve a contextual problem that involves a formula.		✓
1.6 Identify and correct errors in a solution to a problem that involves a formula.	✓ Identify errors in a solution. STUDENT RESOURCE p. 46, #2	✓ Identify and correct errors in a solution. STUDENT RESOURCE p. 46, #3

Algebra (continued)

Specific Outcome

It is expected that students will:

2. Demonstrate an understanding of slope:
   - as rise over run
   - as rate of change
   - by solving problems.

   [C, CN, PS, V]

Notes

- The use of the slope formula is beyond the scope of this course $m = \frac{y_2 - y_1}{x_2 - x_1}$.
- Rates of change should be limited to linear rates. The slope of waste water lines, the grade of a mountain or hill, the transfer of heat through insulation and the pitch of a conveyor belt are some examples that can be used.
- In Achievement Indicator 2.6, a variety of different methods could be used to explain the relationship between the slope and the angle of elevation. Some methods may include using the tangent ratio, patterning or drawing a scale diagram and using a protractor. Students will have seen primary trigonometric ratios in Mathematics 10-3 or in Mathematics 10C.

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

Achievement Indicators	Acceptable Standard	Standard of Excellence
2.1 Describe contexts that involve slope; e.g., ramps, roofs, road grade, flow rates within a tube, skateboard parks, ski hills.	✓ STUDENT RESOURCE p. 13, Discuss the Ideas	
2.2 Explain, using diagrams, the difference between two given slopes (e.g., a 3:1 and a 1:3 roof pitch), and describe the implications.	✓ STUDENT RESOURCE p. 21, #5	
2.3 Describe the conditions under which a slope will be either 0 or undefined.	✓ STUDENT RESOURCE p. 19, Activity 1.3, Part B	
2.4 Explain, using examples and illustrations, slope as rise over run.	✓ STUDENT RESOURCE p. 14, Example 1	
2.5 Verify that the slope of an object, such as a ramp or a roof, is constant.	✓ STUDENT RESOURCE p. 21, #6	
2.6 Explain, using illustrations, the relationship between slope and angle of elevation; e.g., for a ramp with a slope of 7:100, the angle of elevation is approximately 4º.	✓ STUDENT RESOURCE p. 27, Example 1	
2.7 Explain the implications, such as safety and functionality, of different slopes in a given context.	✓ STUDENT RESOURCE p. 22, #9	
2.8 Explain, using examples and illustrations, slope as rate of change.	✓ STUDENT RESOURCE p. 46, Build Your Skills #1	
2.9 Solve a contextual problem that involves slope or rate of change.	✓ Solve a problem when a diagram is given. STUDENT RESOURCE p. 52, #3	✓ Solve a problem when no diagram is given. STUDENT RESOURCE p. 53, #6

Algebra (continued)

Specific Outcome

It is expected that students will:

3. Solve problems by applying proportional reasoning and unit analysis.
   [C, CN, PS, R]

Notes

- Integration of this specific outcome into the Measurement and Number topics of the course is recommended.

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

Achievement Indicators	Acceptable Standard	Standard of Excellence
3.1 Explain the process of unit analysis used to solve a problem (e.g., given km/h and time in hours, determine how many km; given revolutions per minute, determine the number of seconds per revolution).	✓ Explain the process of unit analysis involving one step.	✓ Explain the process of unit analysis involving two or more steps. STUDENT RESOURCE p. 161, #6b
3.2 Solve a problem, using unit analysis.	✓ Solve problems involving one step using unit analysis. STUDENT RESOURCE p. 53, #6c	✓ Solve problems involving two or more steps using unit analysis. STUDENT RESOURCE p. 46, Build Your Skills #1f
3.3 Explain, using an example, how unit analysis and proportional reasoning are related; e.g., to change km/h to km/min, multiply by 1h/60min because hours and minutes are proportional (constant relationship).	✓ Explain a situation involving one step. STUDENT RESOURCE p. 53, #6c	✓ Explain a situation involving two or more steps. STUDENT RESOURCE p. 143, Example 3
3.4 Solve a problem within and between systems, using proportions or tables; e.g., km to m or km/h to ft/sec.	✓ Solve a situation involving one set of units (kilometres to metres or kilometres to miles). STUDENT RESOURCE p. 160, #2a, b, c	✓ Solve a situation involving two sets of units (km/h to ft/sec). STUDENT RESOURCE p. 154, Example 3

Topic: Statistics

General Outcome: Develop statistical reasoning.

Specific Outcome

It is expected that students will:

1. Solve problems that involve creating and interpreting graphs, including:
   - bar graphs
   - histograms
   - line graphs
   - circle graphs.

   [C, CN, PS, R, T, V]
   [ICT: C6–4.1, C6–4.2, C6–4.3, P2–4.1]

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

Achievement Indicators	Acceptable Standard	Standard of Excellence
1.1 Determine the possible graphs that can be used to represent a given data set, and explain the advantages and disadvantages of each.	✓ STUDENT RESOURCE p. 111, #5	
1.2 Create, with and without technology, a graph to represent a given data set.	✓ STUDENT RESOURCE p. 75, Example 1a	
1.3 Describe the trends in the graph of a given data set.	✓ Describe the trend. STUDENT RESOURCE p. 67, #2c	✓ Provide possible reasons for trends. STUDENT RESOURCE p. 63, Example 3c
1.4 Interpolate and extrapolate values from a given graph.	✓ Determine an interpolated value(s). STUDENT RESOURCE p. 60, #2	✓ Determine an extrapolated value(s). STUDENT RESOURCE p. 61, Mental Math and Estimation
1.5 Explain, using examples, how the same graph can be used to justify more than one conclusion.	✓ Provide a partial explanation. STUDENT RESOURCE pp. 104–105, #1	✓ Provide a complete explanation. STUDENT RESOURCE pp. 104–105, #1
1.6 Explain, using examples, how different graphic representations of the same data set can be used to emphasize a point of view.	✓ Explain when graphs are provided. STUDENT RESOURCE p. 111, #7c	✓ Explain when graphs are not provided. STUDENT RESOURCE p. 111, #5
1.7 Solve a contextual problem that involves the interpretation of a graph.	✓ Provide a partial solution. STUDENT RESOURCE p. 112, #8	✓ Provide a complete solution. STUDENT RESOURCE p. 112, #8

Appendix: Mathematics Directing Words

Discuss	The word "discuss" **will not** be used as a directing word on mathematics examinations because it is not used consistently to mean a single activity.
	The following words are specific in meaning.
Algebraically	Use mathematical procedures that involve letters or symbols to represent numbers.
Analyze	Make a mathematical or methodical examination of parts to determine aspects of the whole; e.g., nature, proportion, function, interrelationship.
Compare	Examine the character or qualities of two things by providing characteristics of both that point out their mutual similarities and differences.
Conclude	State a logical end based on reasoning and/or evidence.
Contrast/Distinguish	Point out the differences between two things that have similar or comparable natures.
Criticize	Point out the merits and demerits of an item or issue.
Define	Provide the essential qualities or meaning of a word or concept; make distinct and clear by marking out the limits.
Describe	Give a written account or represent the characteristics of something, using a figure, model or picture.
Design/Plan	Construct a plan, i.e., a detailed sequence of actions, for a specific purpose.
Determine	Find a solution, to a specified degree of accuracy, to a problem by showing appropriate formulas, procedures and calculations.
Enumerate	Specify one-by-one or list in a concise form and according to some order.
Evaluate	Give the significance or worth of something by identifying the good and bad points or the advantages and disadvantages.
Explain	Make clear what is not immediately obvious or entirely known; give the cause of or reason for; make known in detail.
Graphically	Use a drawing that is produced electronically or by hand and that shows a relation between certain sets of numbers.

How	Show in what manner or way, with what meaning.
Hypothesize	Form a tentative proposition intended as a possible explanation for an observed phenomenon; i.e., a possible cause for a specific effect. The proposition should be testable logically and/or empirically.
Identify	Recognize and select as having the characteristics of something.
Illustrate	Make clear by providing an example. The form of the example must be specified in the question; i.e., word description, sketch or diagram.
Infer	Form a generalization from sample data; arrive at a conclusion by reasoning from evidence.
Interpret	State the meaning of something; present information in a new form that adds meaning to the original data.
Justify/Show How	Show reasons for or give facts that support a position.
Model	Find a model (in mathematics, a model of a situation is a pattern that is supposed to represent or set a standard for a real situation) that does a good job of representing a situation.
Outline	Give, in an organized fashion, the essential parts of something. The form of the outline must be specified in the question; i.e., lists, flowcharts, concept maps.
Predict	State in advance on the basis of empirical evidence and/or logic.
Prove	Establish the truth or validity of a statement for the general case by providing factual evidence or a logical argument.
Relate	Show a logical or causal connection between things.
Sketch	Provide a drawing that represents the key features of an object or a graph.
Solve	Give a solution for a problem; i.e., explanation in words and/or numbers.
Summarize	Give a brief account of the main points.
Trace	Give a step-by-step description of the development.
Verify	Establish, by substitution for a particular case or by geometric comparison, the truth of a statement.
Why	Show the cause, reason or purpose.